

A02 - Assignment 2: Constraint Satisfaction Problems

Students:

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Program's solutions

```
[simonedeidier@dhcp-10-24-147-88 assignment 2 % python3 sudoku.py
True
7 8 4 | 9 3 2 | 1 5 6
6 1 9 | 4 8 5 | 3 2 7
2 3 5 | 1 7 6 | 4 8 9
-----+-----+-----
5 7 8 | 2 6 1 | 9 3 4
3 4 1 | 8 9 7 | 5 6 2
9 2 6 | 5 4 3 | 8 7 1
-----+-----+-----
4 5 3 | 7 2 9 | 6 1 8
8 6 2 | 3 1 4 | 7 9 5
1 9 7 | 6 5 8 | 2 4 3
```

Figure 1: *Solution of our program for the easy Sudoku problem.*

```
[simonedeidier@dhcp-10-24-147-88 assignment 2 % python3 sudoku.py
True
8 7 5 | 9 3 6 | 1 4 2
1 6 9 | 7 2 4 | 3 8 5
2 4 3 | 8 5 1 | 6 7 9
-----+-----+-----
4 5 2 | 6 9 7 | 8 3 1
9 8 6 | 4 1 3 | 2 5 7
7 3 1 | 5 8 2 | 9 6 4
-----+-----+-----
5 1 7 | 3 6 9 | 4 2 8
6 2 8 | 1 4 5 | 7 9 3
3 9 4 | 2 7 8 | 5 1 6
```

Figure 2: *Solution of our program for the medium Sudoku problem.*

```

[simonedeidier@dhcp-10-24-147-88 assignment 2 % python3 sudoku.py
True
1 5 2 | 3 4 6 | 8 9 7
4 3 7 | 1 8 9 | 6 5 2
6 8 9 | 5 7 2 | 3 1 4
-----+-----+-----
8 2 1 | 6 3 7 | 9 4 5
5 4 3 | 8 9 1 | 7 2 6
9 7 6 | 4 2 5 | 1 8 3
-----+-----+-----
7 9 8 | 2 5 3 | 4 6 1
3 6 5 | 9 1 4 | 2 7 8
2 1 4 | 7 6 8 | 5 3 9

```

Figure 3: *Solution of our program for the hard Sudoku problem.*

```

[simonedeidier@dhcp-10-24-147-88 assignment 2 % python3 sudoku.py
True
4 3 1 | 8 6 7 | 9 2 5
6 5 2 | 4 9 1 | 3 8 7
8 9 7 | 5 3 2 | 1 6 4
-----+-----+-----
3 8 4 | 9 7 6 | 5 1 2
5 1 9 | 2 8 4 | 7 3 6
2 7 6 | 3 1 5 | 8 4 9
-----+-----+-----
9 4 3 | 7 2 8 | 6 5 1
7 6 5 | 1 4 3 | 2 9 8
1 2 8 | 6 5 9 | 4 7 3

```

Figure 4: *Solution of our program for the very hard Sudoku problem.*

```

simonedeidier@dhcp-10-24-147-88 assignment 2 % python3 sudoku.py
Domains before and after ac_3:
X11: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 7, 8}
X12: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 7, 8}
X14: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 6, 7, 9}
X16: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 6, 8}
X17: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3}
X19: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 6, 7}
X22: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 6, 7, 8}
X25: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 7, 8, 9}
X26: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 5, 6, 8}
X27: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 5}
X28: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 5, 6, 7}
X29: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 6, 7}
X31: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 6, 7}
X32: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 6, 7}
X35: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 7}
X36: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 5, 6}
X41: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 5, 7, 8}
X42: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 4, 7, 8}
X43: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 4, 5, 7, 8}
X44: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 5}
X46: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 5, 6}
X49: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 6, 7, 8, 9}
X52: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4}
X53: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 4, 5, 9}
X55: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 6, 8, 9}
X57: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 8, 9}
X58: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 6, 7, 8, 9}
X61: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 5, 7, 8, 9}
X64: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 5, 6, 7, 8, 9}
X66: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 5, 6, 7, 8}
X67: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 8, 9}
X68: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}
X69: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 6, 7, 8, 9}
X74: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 7, 8}
X75: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 7, 8}
X78: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 6, 8, 9}
X79: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 6, 8, 9}
X81: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 6, 8}
X82: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 6, 8}
X83: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 6, 8}
X84: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 6, 8, 9}
X85: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 6, 8, 9}
X88: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 6, 7, 8, 9}
X91: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 6, 7, 8}
X93: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 6, 7, 8, 9}
X94: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 6, 7, 8, 9}
X96: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 6, 7, 8, 9}
X98: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}
X99: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}

Reduction in domains: 40.59%

```

Figure 5: Domain values for the unknown variabls before and after the *AC3* algorithm for the easy Sudoku problem.

```

simonedeidier@dhcp-10-24-147-88 assignment 2 % python3 sudoku.py
Domains before and after ac_3:
X11: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 5, 6, 7, 8}
X12: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 5, 6, 7, 8}
X13: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {5, 7, 8}
X14: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 9}
X16: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 6}
X17: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 5, 6}
X19: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 5, 6, 8, 9}
X22: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 4, 5, 6, 8}
X25: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 4, 9}
X26: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 4, 6}
X27: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 6}
X28: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 5, 6, 8, 9}
X29: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 5, 6, 8, 9}
X31: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 4, 6}
X32: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 4, 6}
X33: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 4, 9}
X37: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 6}
X39: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 5, 6, 8, 9}
X41: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {4, 5}
X42: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {4, 5}
X45: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {4, 9}
X49: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 4, 5, 6, 8}
X52: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {4, 5, 8}
X54: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {4, 6, 8, 9}
X56: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 6, 8}
X58: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 8}
X61: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {4, 6, 7, 8}
X65: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 8}
X68: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}
X69: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 5, 6, 7, 8, 9}
X71: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 4, 5, 7, 8}
X73: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 5, 7, 8}
X77: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 5, 6, 8, 9}
X78: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 5, 6, 8, 9}
X79: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 5, 6, 8, 9}
X81: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 6, 8}
X82: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 6, 8}
X83: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 6, 8}
X84: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 6, 8, 9}
X85: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 6, 8, 9}
X88: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 5, 6, 7, 8, 9}
X91: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 8}
X93: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 8, 9}
X94: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 8, 9}
X96: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}
X97: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}
X98: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}
X99: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}

Reduction in domains: 39.12%

```

Figure 6: Domain values for the unknown variabls before and after the *AC3* algorithm for the medium Sudoku problem.

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simonedeidier@dhcp-10-24-147-88 assignment 2 % python3 sudoku.py
Domains before and after ac_3:
X12: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 5, 6, 8}
X14: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 9}
X16: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 6, 9}
X17: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 6, 8}
X18: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 5, 6, 8, 9}
X21: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 6}
X22: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 5, 6, 7}
X23: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 7}
X24: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 7, 9}
X26: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 6, 8, 9}
X27: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 6, 7, 8}
X28: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 5, 6, 7, 8, 9}
X29: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 5, 7, 8}
X31: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 6, 8}
X32: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 6, 7, 8}
X35: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 5, 7, 8, 9}
X36: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 6, 8, 9}
X38: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 5, 6, 7, 8, 9}
X41: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 8}
X42: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 8}
X43: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3}
X45: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 8}
X48: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 7, 8, 9}
X49: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 7, 8}
X53: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 7, 9}
X54: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 7, 8, 9}
X55: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 7, 8, 9}
X56: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 7, 8, 9}
X57: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 7, 8, 9}
X61: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 8, 9}
X62: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 7, 8, 9}
X65: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 4, 7, 8, 9}
X67: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 6, 7, 8, 9}
X68: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}
X69: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8}
X72: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 5, 6, 7, 9}
X74: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 5, 7, 8, 9}
X75: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 5, 7, 8, 9}
X78: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 6, 7, 8}
X79: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 6, 7, 8}
X81: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 4, 5, 6, 7, 8, 9}
X82: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 4, 5, 6, 7, 8, 9}
X83: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 4, 5, 6, 7, 8, 9}
X84: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 4, 5, 7, 8, 9}
X86: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 7, 8, 9}
X87: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 6, 7, 8}
X88: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 6, 7, 8}
X89: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 6, 7, 8}
X92: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 7, 8}
X93: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 7, 8}
X94: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 7, 8}
X96: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 6, 7, 8}
X98: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8}

Reduction in domains: 37.11%

```

Figure 7: Domain values for the unknown variables before and after the AC3 algorithm for the hard Sudoku problem.⁵

```

[simoneideidier@dhcp-10-24-147-88 assignment 2 % python3 sudoku.py
Domains before and after ac_3:
X11: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {4, 5, 8, 9}
X12: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 4, 5, 9}
X14: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 5, 6, 8, 9}
X15: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {5, 6, 8, 9}
X17: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 5, 7, 8, 9}
X18: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 7, 8, 9}
X19: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 5, 7, 8, 9}
X22: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 5, 6, 9}
X23: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 5, 6, 7, 9}
X25: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {4, 5, 6, 8, 9}
X26: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 7, 8}
X28: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 7, 8, 9}
X29: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 5, 7, 8, 9}
X31: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 5, 8, 9}
X32: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 5, 9}
X33: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 5, 7, 9}
X34: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 5, 7, 8, 9}
X36: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 7, 8}
X37: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 5, 7, 8, 9}
X43: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 9}
X44: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 4, 8, 9}
X47: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 4, 5, 7, 8, 9}
X48: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 4, 7, 8, 9}
X49: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 4, 5, 7, 8, 9}
X51: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 4, 5, 8, 9}
X52: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 4, 5, 8, 9}
X53: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 4, 5, 9}
X54: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 4, 7, 8, 9}
X55: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {4, 7, 8, 9}
X56: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 4, 7, 8}
X57: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 4, 5, 7, 8, 9}
X63: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 4, 6, 7, 9}
X64: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 4, 6, 7, 8, 9}
X67: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 6, 7, 8, 9}
X68: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 6, 7, 8, 9}
X69: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 4, 5, 6, 7, 8, 9}
X71: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 4, 6, 9}
X72: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 4, 6, 9}
X73: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 4, 6, 9}
X74: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 4, 6, 7, 8}
X76: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {2, 3, 4, 6, 7, 8}
X77: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 4, 6, 7, 8, 9}
X82: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 4, 5, 6, 7, 9}
X83: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {3, 4, 5, 6, 7, 9}
X85: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 6, 7, 8}
X86: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 3, 4, 5, 6, 7, 8}
X88: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}
X89: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}
X91: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7}
X92: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7}
X94: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8}
X95: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8}
X97: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}
X98: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}
X99: before -> {1, 2, 3, 4, 5, 6, 7, 8, 9}, after -> {1, 2, 3, 4, 5, 6, 7, 8, 9}

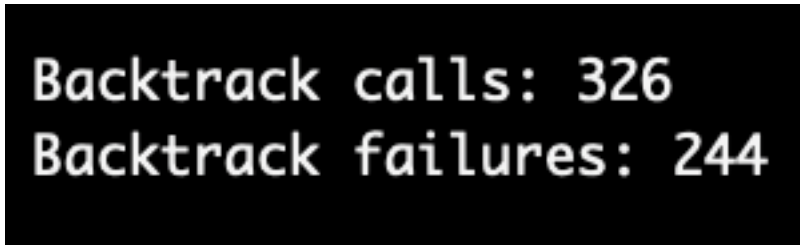
Reduction in domains: 34.95%

```

Figure 8: Domain values for the unknown variables before and after the *AC3* algorithm for the very hard Sudoku problem.

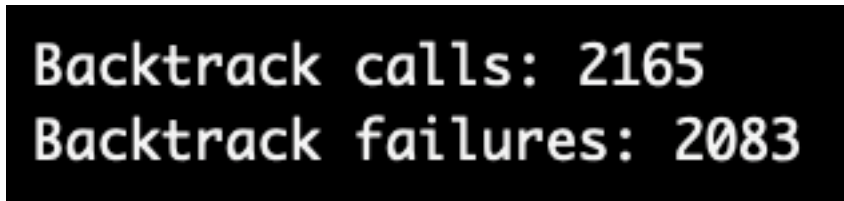
Domains

`backtrack()` calls



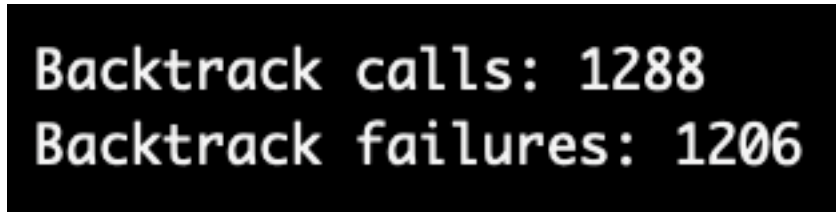
Backtrack calls: 326
Backtrack failures: 244

Figure 9: *Count of calls of the `backtrack()` function and count of failures for the easy Sudoku.*



Backtrack calls: 2165
Backtrack failures: 2083

Figure 10: *Count of calls of the `backtrack()` function and count of failures for the medium Sudoku.*



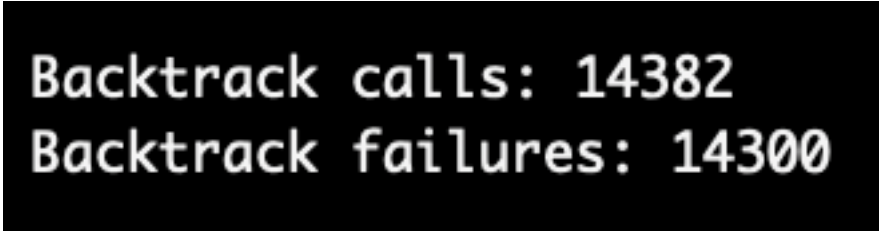
Backtrack calls: 1288
Backtrack failures: 1206

Figure 11: *Count of calls of the `backtrack()` function and count of failures for the hard Sudoku.*

Runtime of the backtracking search algorithm, AC-3 and total runtime

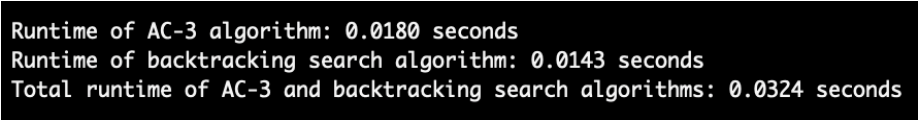
Discussion of the results

The **AC-3** algorithm drastically reduces the total runtime when solving some Sudoku problems, especially in more complex configurations, due to its ability to efficiently prune the search space by enforcing arc-consistency early in the process. In simpler terms, **AC-3** eliminates values from variable domains that



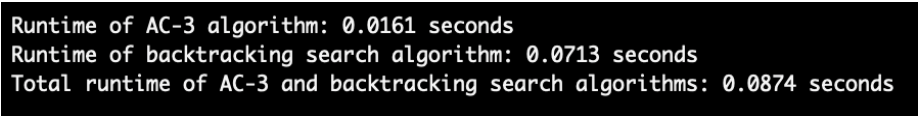
Backtrack calls: 14382
Backtrack failures: 14300

Figure 12: *Count of calls of the backtrack() function and count of failures for the very hard Sudoku.*



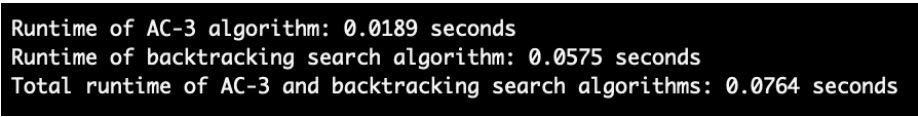
Runtime of AC-3 algorithm: 0.0180 seconds
Runtime of backtracking search algorithm: 0.0143 seconds
Total runtime of AC-3 and backtracking search algorithms: 0.0324 seconds

Figure 13: *Runtime of both the backtrack() * and AC-3 algorithm, with also the total runtime, for the easy Sudoku.**



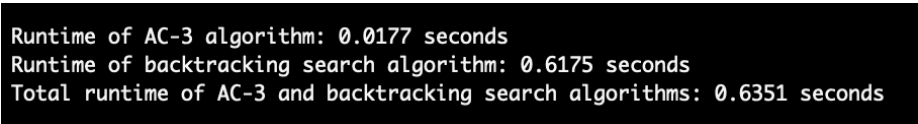
Runtime of AC-3 algorithm: 0.0161 seconds
Runtime of backtracking search algorithm: 0.0713 seconds
Total runtime of AC-3 and backtracking search algorithms: 0.0874 seconds

Figure 14: *Runtime of both the backtrack() * and AC-3 algorithm, with also the total runtime, for the medium Sudoku.**



Runtime of AC-3 algorithm: 0.0189 seconds
Runtime of backtracking search algorithm: 0.0575 seconds
Total runtime of AC-3 and backtracking search algorithms: 0.0764 seconds

Figure 15: *Runtime of both the backtrack() * and AC-3 algorithm, with also the total runtime, for the hard Sudoku.**



Runtime of AC-3 algorithm: 0.0177 seconds
Runtime of backtracking search algorithm: 0.6175 seconds
Total runtime of AC-3 and backtracking search algorithms: 0.6351 seconds

Figure 16: *Runtime of both the backtrack() * and AC-3 algorithm, with also the total runtime, for the very hard Sudoku.**

are inconsistent with their neighboring constraints, which simplifies the problem before more computationally expensive search algorithms, such as **backtracking**, are applied.

For easier Sudoku puzzles, fewer constraints lead to quicker reductions, as there are more obvious domain eliminations. However, even in more complex configurations, the algorithm still cuts down unnecessary branches of the search tree, speeding up the solving process by reducing the number of possible variable assignments the solver needs to explore. This pre-processing step significantly lowers the runtime in cases where otherwise the problem would involve a larger number of constraint checks or deep recursive searches.

The **backtracking** algorithm, on the other hand, is a depth-first search algorithm that incrementally builds candidates to the solutions and abandons a candidate (“backtracks”) as soon as it determines that the candidate cannot possibly be completed to a valid solution. While **backtracking** is powerful and can solve many problems, it can be slow for large and complex problems due to its exhaustive nature. However, when combined with **AC-3**, the search space is significantly reduced, making **backtracking** more efficient and faster in finding solutions. *In summary, the combination of these two algorithms leverages the strengths of both: the pre-processing efficiency of **AC-3** and the exhaustive search capability of **backtracking**.*