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Measurement base change for localization encoded dual rail photonic quantum computing

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ABSTRACT

We study behavior of number states in systems of coupled modes. Using an auxiliary mode and linear transformations, we propose a simple method for measuring on orthogonal bases other than 0 and 1 in the case of dual rail photonic computing which may act as a simple filter too. Using two auxiliary modes it is shown that an equivalent polarizing beam splitter can be realized for dual rail encoding. Simple structures based on coupled waveguides are presented for these purposes.

1. Introduction

Lack of a strong nonlinear interaction among photons was considered as a fatal obstacle in utilizing photons as qbits for efficient quantum computing. After the famous work by Knill et al. [1], linear optical quantum computing (LOQC) became a rapidly growing area of research due to its advantages from realization point of view. Considering two versions for implementations of LOQC [2], dual rail and polarization encoded quantum computing, have been of equal interest since their proposal where the simple polarizer and beam splitters have been a motivation for polarization encoding and simple waveguide structures and their easy fabrication have supported dual rail quantum computing. Dual rail logic's unique feature is integration feasibility by modern available technology, even on silicon systems with ease. Politi et al. in their paper [3] demonstrated basic operations on silicon waveguide circuits with high fidelity. Very recently they demonstrated Shor's algorithm, based on dual rail photonic quantum computing, on a photonic chip just utilizing waveguides and coupling between them [4]. Dual rail photonic quantum computing relies on the different excitation combinations of two spatially separated modes as states of a single bit [2]. On each case just one mode is excited with a single photon and depending on the excited mode it is referred as 0 or 1. This simple encoding offers unique features. Till now it has been merely possible to measure the presence and absence of the photon on each of the rails using photon counting methods. So, the result for the whole bit was $|1\rangle$ or $|0\rangle$ ($|1\rangle$ or $|10\rangle$).

In polarization encoded quantum computing, the task of rotating the measurement basis is done using the lately known polarizer and polarizing beam splitters. For long time it has been known that arbitrary single bit gates can be realized for both encodings of photonic quantum computing using simple multiport interferometers. But to the knowledge of authors there has not been any explicit demonstration of beam splitters or detectors for dual rail encoding possessing properties of those in this paper. So, in what follows, we discuss that utilizing auxiliary modes and linear coupling between the modes it is possible to do measurements and splitting on mutually orthogonal bases of the form $(\alpha|01\rangle+\beta|10\rangle)$ and $(\alpha|10\rangle-\beta|01\rangle)$, or simply $(\alpha|0\rangle+\beta|1\rangle)$ and $(\alpha|1\rangle-\beta|0\rangle)$, as new orthogonal bases where $\alpha^2+\beta^2=1$. Our simple method makes it possible to measure and split mixed states on arbitrary basis of choice. The paper is organized as follows. In Section 2, a short mathematical discussion is given for treating single photons in coupled systems, where, we follow the method of Rai et al. for evolution of nonclassical states of light, namely number states, in coupled systems with some modifications [5]. Then the method of utilizing an auxiliary mode is proposed and analyzed. The last section includes a proposal for implementation of the proposed system using the coupled waveguides and the very simple method for tuning the coefficients α and β in coupled waveguides is proposed.

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2. Mathematical formalism

As it is obvious, this paper deals with nonclassical states of light, namely photon number states. These states call for a quantum mechanical treatment as they exhibit quantum features [5,6] which are of interest for quantum computers. The most famous of these is the two photon interference effect or Hong-Ou-Mandel effect. In what follows, we briefly present a general method to treat the evolution of number states in systems of linearly coupled modes. The Hamiltonian, for an arbitrary system of linearly coupled optical modes, has the general form of

$$H=\hbar\omega\sum_{i=1}^{n}a_i^{\dagger}a_i+\frac{1}{2}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\hbar C_{ij}(a_i^{\dagger}a_j+cc), \tag{1}$$

where C_{ij} is the coupling rate between modes i and j, a_i^{+} and a_i are creation and annihilation operators associated with ith mode which obey the following commutation relations [7]

$$\left[a_i, a_j^{\dagger}\right]=\delta_{ij}, \qquad \left[a_i, a_j\right]=0, \tag{2}$$

It is to be mentioned that the first term in Eq. (1) is the energy of pure modes and the second term rises from the coupling between the modes. Using the Heisenberg equation of motion, we can evaluate the evolution of creation and annihilation operator a_i^{+} and a_i which those are Hermitian conjugates as

$$\frac{d}{dt}a_i^{\dagger}(t)=\hbar\omega a_i^{\dagger}(t)+\sum_{\substack{j=1\\ j\neq i}}^{n}\hbar C_{ij}a_j^{\dagger}(t), \tag{3}$$

Although for the case of propagating coupled modes, one should derive proper momentum operator and use the equation of motion to derive the evolution equation, here we prefer to use the conventional $Z=Vt$ transformation. Where, this transformation is used to obtain the evaluation without deriving the momentum operator [5]. The Eq. (2) can be solved analytically or numerically to obtain the answers which will simply be of the form

$$a_i^{\dagger}(t)=\sum_{j=1}^{n}K_{ij}(t)a_j^{\dagger}(0), \tag{4}$$

The coefficients K_{ij} are interpreted as the probability of a photon starting in mode i and being found at time t in mode j. Eq. (4) will be used to derive evaluation of wave function of system due to coupling. Suppose that we have a system of n modes with n_i photons in each at $t=0$ or $z=0$ represented as $\psi(0)=|n_1, n_2, \ldots, n_n\rangle$.

This in the same manner may be written as $\left(1/\sqrt{n_1!n_2!\ldots n_n!}\right)a_1^{\dagger n_1}a_2^{\dagger n_2}\ldots a_n^{\dagger n_n}\left|0,0,\ldots 0\right\rangle$ for states of interest in this work, Fock sates. The $\psi(t)$ is found to be

$$\frac{1}{\sqrt{n_1!n_2!\ldots n_n!}}a_1^{\dagger n_1}(-t)a_2^{\dagger n_2}(-t)\ldots a_n^{\dagger n_n}(-t)\left|0,0,\ldots 0\right\rangle$$

Simply applying the evolution operator $U(t)$ to $\psi(0)$. By this approach it is clear that different combinations of input states will yield different output states, which really differ from the case of classical input states, e.g. Hong-Ou-Mandel effect. In studying the behavior of number states for quantum mechanical effects, one should consider that the most of these effects arise from the interference of transition amplitudes K_{jk}. At this point one should notice that for dual rail quantum computing the state of input can be written as $R_1|01\rangle+R_2|10\rangle$, where R_1 and R_2 are complex numbers. With a simple mathematical discussion one can prove that the states $(\alpha|01\rangle+\beta|10\rangle)$ and $(\alpha|10\rangle-\beta|01\rangle)$ form an orthogonal complete set of states, as base states, for these input states. The system of our interest for measuring on the bases $(\alpha|01\rangle+\beta|10\rangle)$ and $(\alpha|10\rangle-\beta|01\rangle)$ is composed of two modes of the dual rail encoding, and an auxiliary mode named as *au* mode, which will be photon counting measured. In this structure the two modes of dual rail encoding, named as 1 and 2, are linearly coupled to the auxiliary mode, but not coupled to each other. That means

$$C_{12}=C_{21}=0, \tag{5}$$

and the coefficients $C_{1,au}$ and $C_{2,au}$ are to be determined by the choice of α and β. Here we should emphasize, for the sake of energy conservation, we have [8]

$$C_{ij}=C_{ji}^{*}, \tag{6}$$

Eq. (3) solved for the system under consideration would yield the answer as [5]

$$K_{1,au}=\frac{1}{2}\frac{C_{1,au}}{\sqrt{(C_{1,au}^2+C_{2,au}^2)}}\times\left(e^{-i(w+\sqrt{(C_{1,au}^2+C_{2,au}^2)})t}-e^{i(w-\sqrt{(C_{1,au}^2+C_{2,au}^2)})t}\right) \tag{7}$$

Further investigation of Eqs. (7) and (1) show that the coefficients $C_{1,au}$ and $C_{2,au}$ can be chosen so that the state $(\alpha|10\rangle-\beta|01\rangle)$ be an eigenstate of coupled system and pass through system with merely a phase change. Moreover this choice of $C_{1,au}$ and $C_{2,au}$ results in state $(\alpha|01\rangle+\beta|10\rangle)$ as an input state yielding a photon in the auxiliary mode at a coupling length. Consequently, a photon counting event on this mode can simply act as filtering or as a measurement on $(\alpha|01\rangle+\beta|10\rangle)$ base. The second structure, we propose here, will operate as an analog to polarizing beam splitter for dual rail quantum computing. The structure will act to separate the input bit, according to its wave function, to two bits, one being orthogonal to $\alpha|01\rangle+\beta|10\rangle$ (expanded as $C(\alpha|10\rangle-\beta|01\rangle)$) and the other orthogonal to $\alpha|10\rangle-\beta|01\rangle$. This system includes two auxiliary modes which will function as the modes of $\alpha|01\rangle+\beta|10\rangle$ bit and the two modes of the input beam which will function as the modes of $\alpha|10\rangle-\beta|01\rangle$ bit. For this purpose the two input modes are linearly coupled to auxiliary modes without coupling with each other. The auxiliary modes are not coupled to each other too. So

$$C_{au1,au2}=C_{1,2}=0, \tag{8}$$

In such a configuration the coefficients K_{ij} are too long for an arbitrary case to be mentioned here, but assuming equal coupling constants they simplifies to

$$K_{1,au1}=\frac{1}{4}\left(e^{-i(w+2c)t}-e^{-i(w-2c)t}\right) \tag{9}$$

Using Eq. (9), one finds out that the state $\alpha|10\rangle-\beta|01\rangle$ remains as the eigenstate of system and appears at output with only a phase change. Further it is possible to show that the input state $\alpha|01\rangle+\beta|10\rangle$ will yield at appropriate distance or time as $\alpha|01\rangle+\beta|10\rangle$ in auxiliary modes though the separation of the states or filtering is achieved.

3. Simulation results

In this part, we discuss the simulation results for proposed structure realization by simple waveguide structures as in Fig. 1. Here, the coefficients α and β may easily be adjusted by the distance between the waveguides, for instance. One should also note that the results below can be expected from the coupled cavities and there is no great difference in the principles of operation.

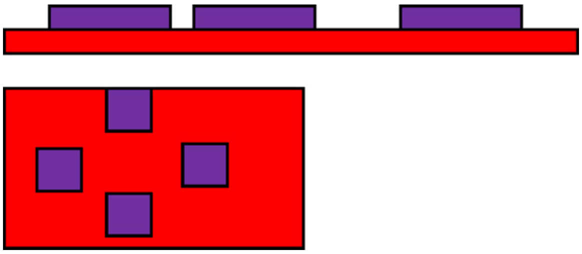

Fig. 1. The cross section of proposed structures.

Fig. 2(a) demonstrates evolution of probability amplitude of the system in Fig. 1(a), for the case of excitation with $1/\sqrt{2}$ $(|01\rangle + |10\rangle)$ as its input state, while the waveguides are assumed identical and equidistance. By this arrangement of system the coefficients are arranged to give $1/\sqrt{2}$ $(|01\rangle + |10\rangle)$ as the measurement base. Fig. 2(b) shows another structure's result designed to detect with $(\sqrt{0.5525}|01\rangle + \sqrt{0.4475}|10\rangle)$ as base state excited with $(\sqrt{0.5525}|01\rangle + \sqrt{0.4475}|10\rangle)$.

Fig. 2 demonstrates the appearance of a photon in the auxiliary at the appropriate distance for excitation with the base states. The anti symmetric base state, $(\alpha|10\rangle - \beta|01\rangle)$, as an input state would yield no photon in the auxiliary mode at all which is not plotted. Fig. 3(a) shows the results for the excitation of arrangement in Fig. 1(b) with the input state $1/\sqrt{2}$ $(|10\rangle + |01\rangle)$. In this system, too, the waveguides are assumed identical and equidistance so that the base states are $1/\sqrt{2}$ $(|10\rangle + |01\rangle)$ and $1/\sqrt{2}$ $(|10\rangle - |01\rangle)$. Fig. 3(b) shows evolution of this system designed for $(\sqrt{0.5525}|01\rangle + \sqrt{0.4475}|10\rangle)$ and $(\sqrt{0.5525}|10\rangle - \sqrt{0.4475}|01\rangle)$ states as base states and excited with the $(\sqrt{0.5525}|01\rangle + \sqrt{0.4475}|10\rangle)$ state as input.

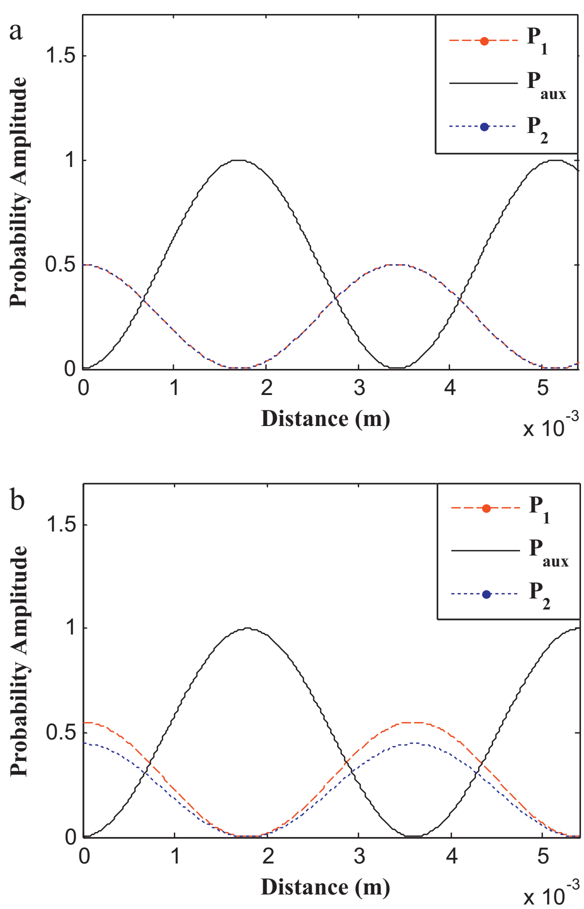

Fig. 2. The probability amplitude of photon for the three waveguides. At around 2 mm the complete transfer of the photon to the auxiliary mode is obvious.

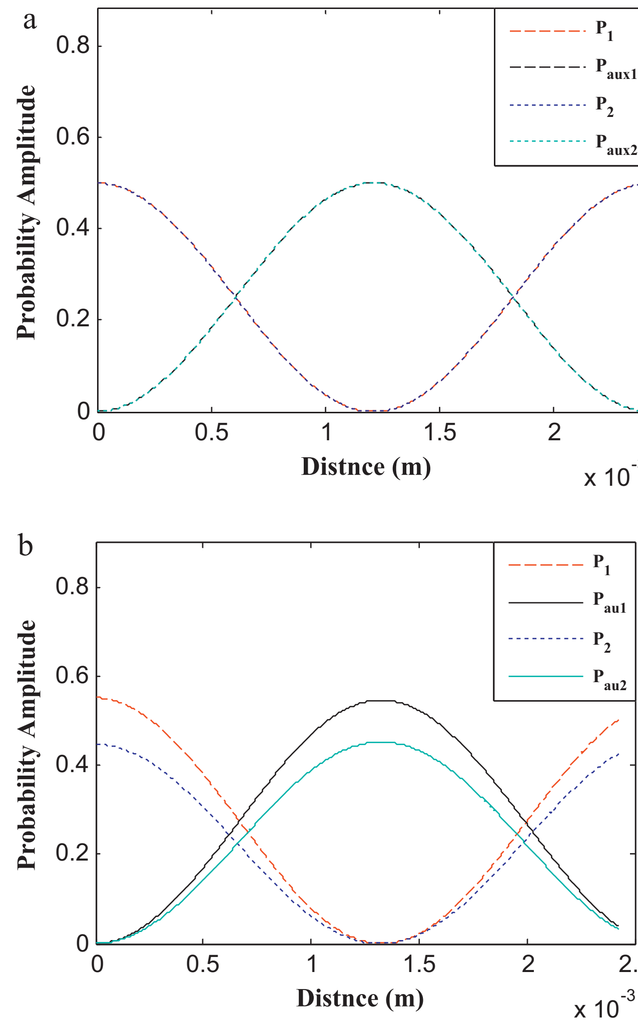

Fig. 3. The evolution of probability amplitude with distance for Fig. 1(b). The transfer of the input state to the two auxiliary modes is obvious at about 1.25 mm for Fig. 3(a) and 1.4 for Fig. 2(b).

As a result, Fig. 3 demonstrates the transfer of the input states $(\alpha|01\rangle + \beta|10\rangle)$ to the two auxiliary modes correspondingly. The input state $(\alpha|10\rangle - \beta|01\rangle)$ passes through the system with only a phase change which is not visualized in Fig. 3.

4. Conclusion

In this paper using auxiliary modes and linear coupling between modes, we demonstrated the possibility of measurement and beam splitting for dual rail photonic bit encoding on bases other than the lately known absence and presence of photons. We showed that by correct choice of the linear couplings of modes it is possible to rotate the base of measurement and splitting. We expect this proposal to be efficient in expanding linear optical quantum computing.

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