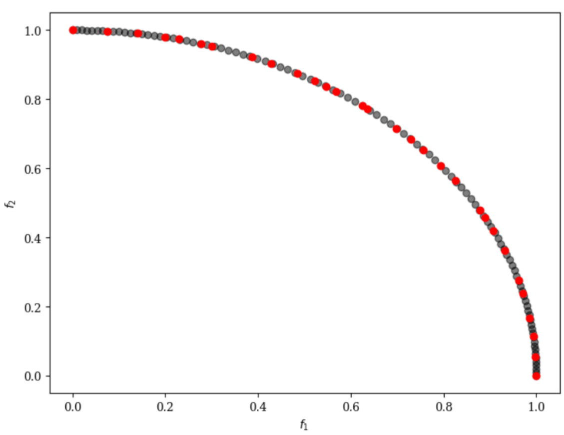
Comparison between NSGAII and NSGAIII on DTLZ problems

We consider each problem with 2, 3 and 4 objectives when possible. We performed 10 runs with different initial populations for both algorithms, to test their consistency on multiple runs. We report median, max and min IGD values for the 10 runs in each case. For the run with minimal IGD value we also report the hypervolume and (for comparison) the hypervolume of the pareto front when known. For 2 and 3 objectives we plot the result compared to the Pareto front for the run with lowest IGD value.

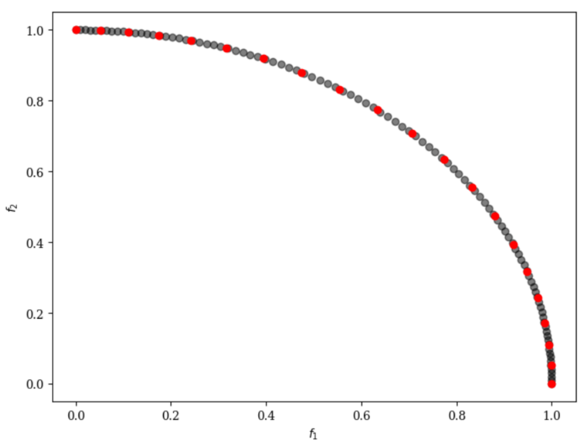
1 DTLZ2 problem

1.1 Two Objectives

	NSGAII	NSGAIII
IGD Values	Median: 0.01817 Max: 0.01973 Min: 0.01543	Median: 0.01949 Max: 0.01955 Min: 0.01947
HyperVolume	Min IGD: 0.1994 Pareto: 0.21026	Min IGD: 0.19328 Pareto: 0.21057



NSGAII



NSGAIII

Figure 1: Non-dominated front for the two algorithms in red and Pareto front in black.

Both algorithms perform well on this simple problem. NSGAII has slightly lower IGD values but the variance of IGD values for NSGAIII is very low.

1.2 Three Objectives

	NSGAII	NSGAIII
IGD Values	Median: 0.06999 Max: 0.07403 Min: 0.06817	Median: 0.05157 Max: 0.05223 Min: 0.05101
HyperVolume	Min IGD: 0.38703 Pareto: 0.43179	Min IGD: 0.41499 Pareto: 0.43016

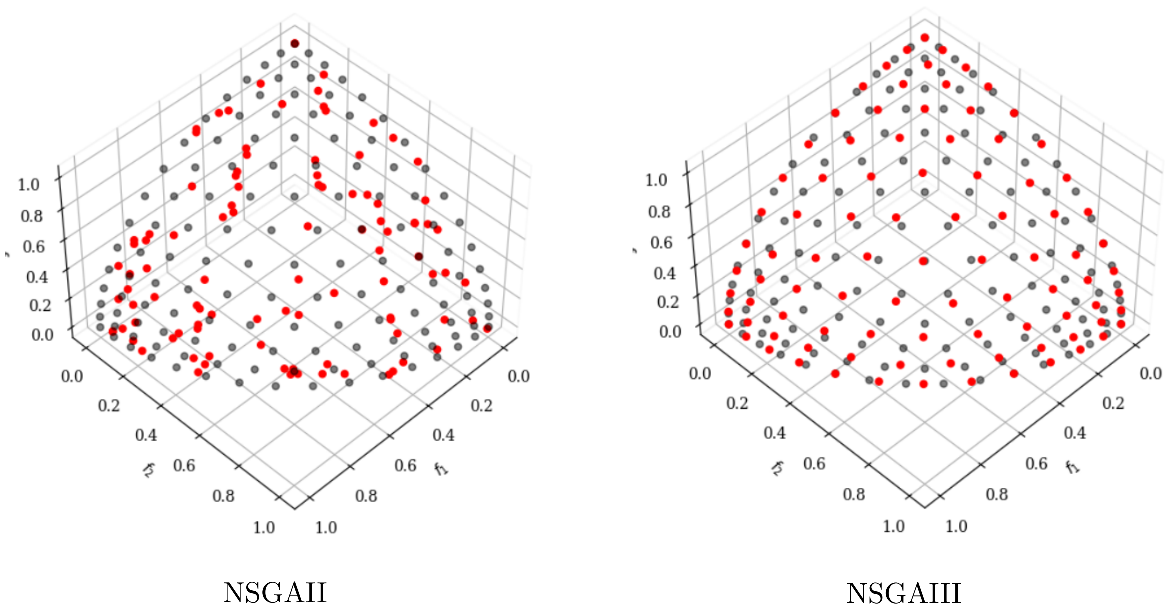


Figure 2: Non-dominated front for the two algorithms in red and Pareto front in black.

Both algorithms perform well but NSGAIII has a better distribution, covering the entirety of the pareto front. This is also reflected in lower IGD values with less variance and hypervolume closer to that of the Pareto front. NSGAIII is therefore better in this case.

1.3 Four Objectives

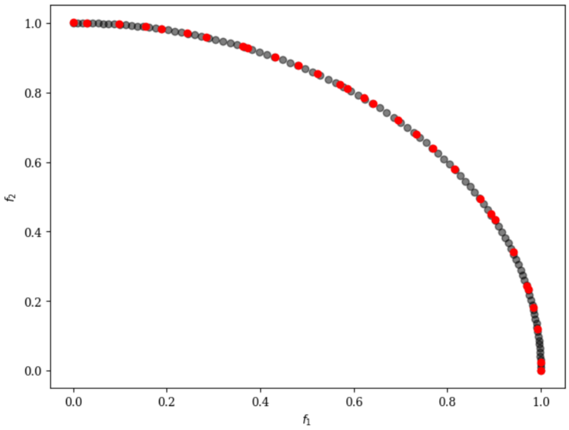
	NSGAII	NSGAIII
IGD Values	Median: 0.1603 Max: 0.17403 Min: 0.15529	Median: 0.00047 Max: 0.00068 Min: 0.00038
HyperVolume	Min IGD: 0.47863 Pareto: 0.55467	Min IGD: 0.54843 Pareto: 0.54858

In this case we cannot plot the four-dimensional fronts and we have to rely on the metrics only. NSGAIII clearly outperformes NSGAII: The IGD values are orders of magnitude better and also the hypervolume is very close to the reference value.

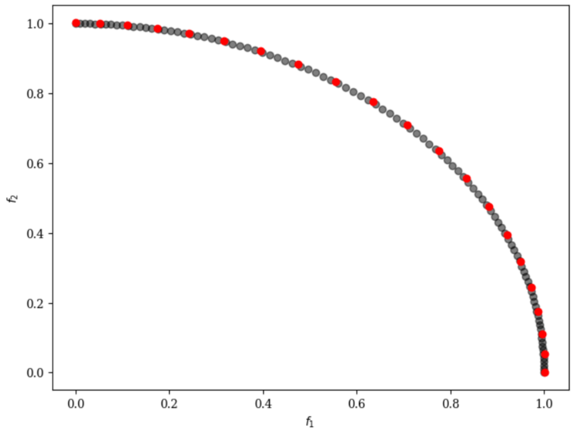
2 DTLZ3 problem

2.1 Two Objectives

	NSGAII	NSGAIII
IGD Values	Median: 0.0228 Max: 1.02515 Min: 0.01655	Median: 0.02278 Max: 1.00595 Min: 0.01979
HyperVolume	Min IGD: 0.19894 Pareto: 0.21214	Min IGD: 0.19407 Pareto: 0.21466



NSGAII



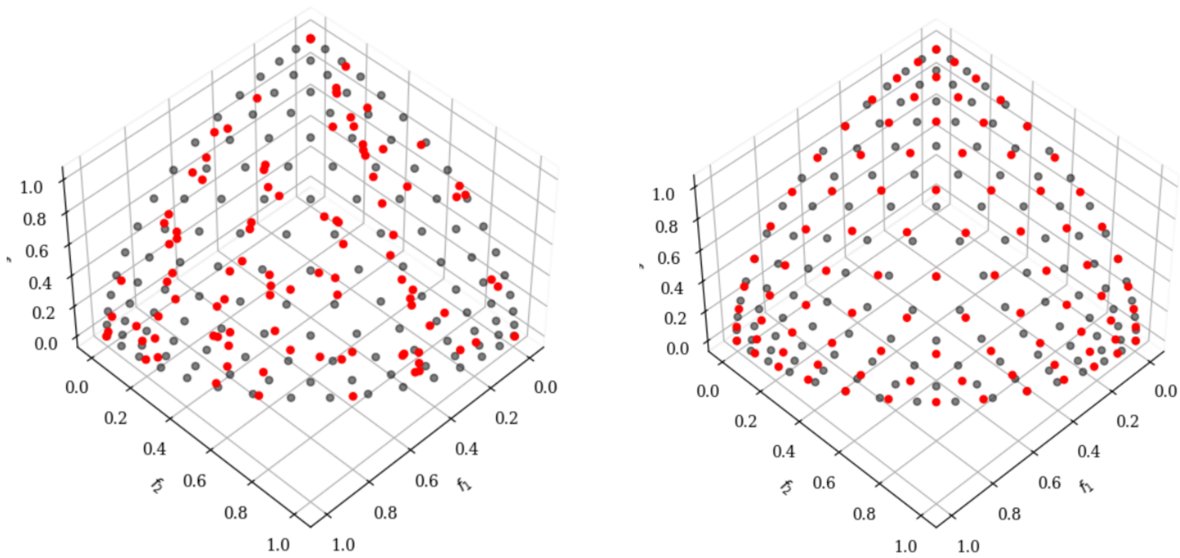
NSGAIII

Figure 3: Non-dominated front for the two algorithms in red and Pareto front in black.

Both algorithms perform well on average but NSGAII has slightly lower IGD values. Notice that in this case the IGD values have high variance for both algorithms, indicating that there is a significant chance that a single run does not perform well.

2.2 Three Objectives

	NSGAII	NSGAIII
IGD Values	Median: 0.07844 Max: 2.01509 Min: 0.07161	Median: 0.05158 Max: 2.01038 Min: 0.0512
HyperVolume	Min IGD: 0.39356 Pareto: 0.43604	Min IGD: 0.41393 Pareto: 0.42613



NSGAII NSGAIII

Figure 4: Non-dominated front for the two algorithms in red and Pareto front in black.

Again, as in the previous problem with 3 objectives, NSGAIII has a better distribution covering the entirety of the pareto front and it has better metrics with respect to NSGAII. As in the case with 2 objectives, we have for both algorithms high variance for IGD values.

2.3 Four objectives

	NSGAII	NSGAIII
IGD Values	Median: 1.04668 Max: 2.14757 Min: 0.21317	Median: 0.05946 Max: 2.00214 Min: 0.00334
HyperVolume	Min IGD: 448683.96822 Pareto: 448684.16435	Min IGD: 0.55568 Pareto: 0.56169

In this case the NSGAII algorithm seems to fail to converge properly, in particular the extremely high hypervolume metric suggests the population is not concentrated in the proximity of the

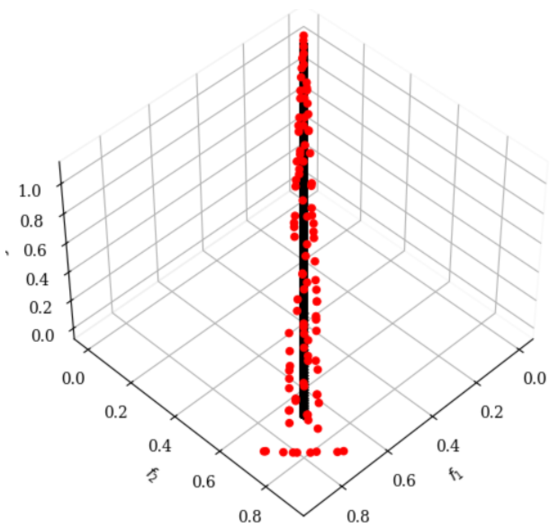
Pareto front. NSGAIII instead converges properly, with low IGD and hypervolume values. Again we can notice a significant variance in IGD values.

3 DTLZ6 problem

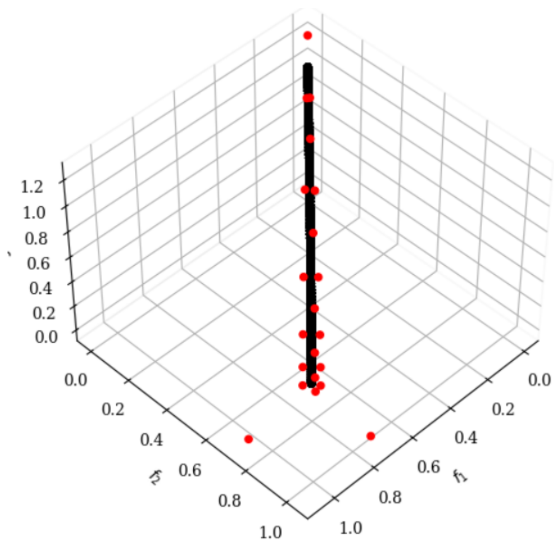
This problem is not really defined in the case with 2 objectives, therefore we consider only the cases with 3 and 4 objectives.

3.1 Three Objectives

	NSGAII	NSGAIII
IGD Values	Median: 0.10183 Max: 0.1636 Min: 0.06277	Median: 0.11539 Max: 0.1535 Min: 0.06725
HyperVolume	Min IGD: 0.1369 Pareto: 0.19142	Min IGD: 0.39826 Pareto: 0.52387



NSGAII



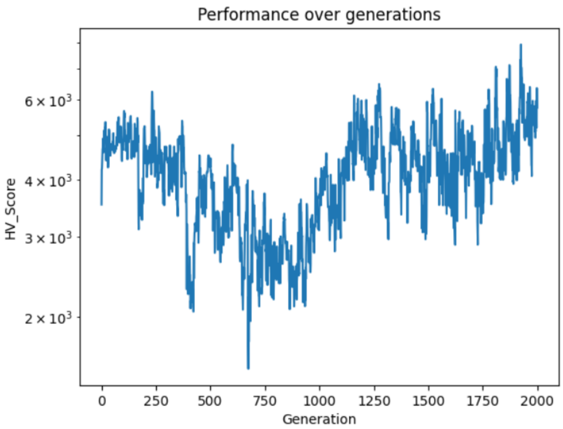
NSGAIII

Figure 5: Non-dominated front for the two algorithms in red and Pareto front in black.

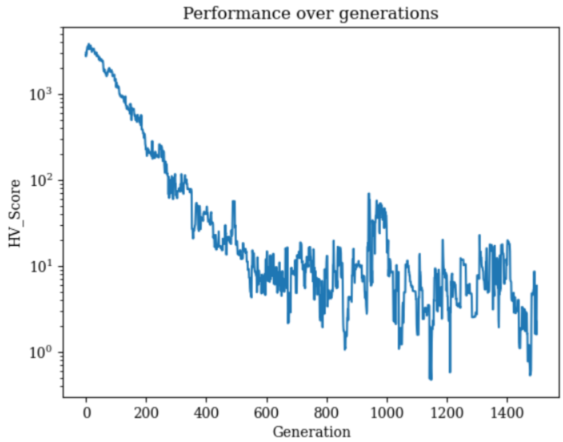
The IGD values are similar for the two algorithms, but the hypervolume analysis favors NSGAII. Also from the plots we see that NSGAIII has only few non dominated points out of the 100 individuals of the population. in this case NSGAII seems to converge better.

3.2 Four objectives

	NSGAIII
HyperVolume	Median: 9.08415 Max: 34.64039 Min: 0.64847



NSGAII



NSGAIII

Figure 6: Hypervolume value as a function of the number of generations.

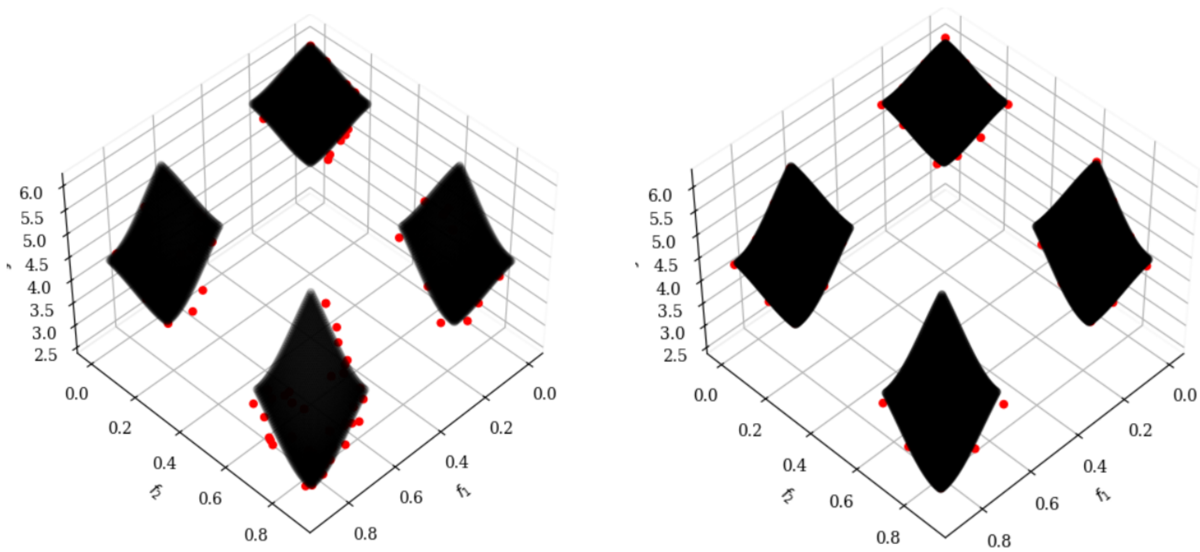
The Pareto front is not implemented yet for this problem in pymoo, so we cannot compute IGD and we have to rely on the hypervolume only. We see that for NSGAII the hypervolume does not decrease monotonically during the optimization, indicating that the algorithm is not converging properly. The situation is much better for NSGAIII and therefore we consider median, max and min hypervolume over 10 runs. NSGAIII seems to converge well, even though the hypervolume sequence has a high variance as we can see from the table.

4 DTLZ7 problem

Also in this case we can only consider the cases with 3 and 4 objectives.

4.1 Three objectives

	NSGAII	NSGAIII
IGD Values	Median: 0.071 Max: 0.076 Min: 0.06791	Median: 0.0969 Max: 0.35901 Min: 0.09425
HyperVolume	Min IGD: 0.8573 Pareto: 0.98315	Min IGD: 0.93267 Pareto: 1.07048



NSGAII NSGAIII

Figure 7: Non-dominated front for the two algorithms in red and Pareto front in black.

In this case NSGAII seems to perform better: it has better metrics and more non dominated points close to the true Pareto front.

4.2 Four objectives

	NSGAII	NSGAIII
HyperVolume	Median: 1.26855 Max: 1.47005 Min: 1.12044	Median: 0.94228 Max: 1.14404 Min: 0.84756

Also in this case we cannot compute the IGD value and therefore we rely on the hypervolume only. Both algorithms converge but he metric is better for NSGAIII.