SECTION B

QUESTION 2

a) i) $\dfrac{6+7n^2}{5-3n^2}$ $\qquad \lim_{n \to \infty} \left(\dfrac{1}{n^2}\right) = 0$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{6+7n^2}{5-3n^2}$$

$$= \lim \frac{n^2\left(7+6/n^2\right)}{n^2\left(5/n^2-3\right)} = -\frac{7}{3}$$

This sequence is convergent and converge to $-\frac{7}{3}$

ii) Given that,

let $a_n = \sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{1}{n}\right)$

The series is an alternating series

and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^n \cos\left(\frac{1}{n}\right)$

$$\lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = \cos(0) = 1$$

So assuming that the series will be $-1, 1, -1$

and so the series is divergent.

The series have two limits -1 and 1.

b) $P(\text{Odd and Head}) = P(\text{Odd})\,P(\text{Head}) = 0.5 \times 0.5 = 0{,}25$

$= \frac{1}{4}$

$\text{Probability} = \frac{\text{favourable outcome}}{\text{Total n. of outcome.}}$

c) 1) $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) =$

$\cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

ii) $\log_{x^2} x^3$

using the property $\log_{a^m} b^n = \frac{n}{m} \log_a b^n$

$\log_a a = 1$

$= \frac{3}{2} \log_x x$

$= \frac{3}{2}$

D) Given that

The position of an object is given by an equation

$s(t) = 2te^t$

Here, $s(t) = 2te^t$

Now derivating the above position equation with respect to time to get velocity time equation.

$\frac{d}{dt}(s(t)) = \frac{d}{dt}(2te^t)$

$$\frac{d}{dt}(s(t)) = e^t \frac{d}{dt}(2t) + (2t)\frac{d}{dt}(e^t)$$

$$v(t) = e^t(2) + 2t(e^t)$$

$$v(t) = 2e^t + 2te^t$$

$$v(t) = 2e^t(1+t)$$

Now we have the velocity - time equation of object

velocity $v(t) = 2e^t(1+t)$

As we know,

this object stop moving at moment when the velocity of this object will zero

Now, equating the velocity time equation to zero to get the time when the velocity will equal to zero

$$v(t) = 0$$

$$2e^t(1+t) = 0$$

$$e^t(1+t) = 0$$

$$t = -1$$

There is no value of t where e^t equal to zero

So $e^t(1+t)$ is zero only when $t = -1$

Negative value of time is not possible.

So, the object will never stop moving.

e) Both triangle are similar so,

$FED \simeq ABC$

$$\frac{FE}{AB} = \frac{FD}{AC}$$

$$\frac{6}{15} = \frac{m}{i}$$

$$m = \sqrt{6^2 + 8^2} = 10$$

$$\frac{6}{15} = \frac{10}{i}$$

$$i = \frac{10 \times 15}{6} = 25$$

These triangles are similar because two angles are equal for both.

QUESTION 3 a) i)

if $\theta \in (\pi, \frac{3}{2}\pi)$ that means θ lies in III quadrant

sine (-) Tang (+) cos (-)

$\cos\theta = -\frac{1}{3}$

$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{1}{9}} = -\sqrt{\frac{8}{9}}$

$\sin\theta = -\frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-1/3}{-2\sqrt{3}/3}$

$\tan\theta = \frac{1}{2\sqrt{3}}$

ii) here $f(x) = \frac{x^2-x}{x^2-1} = \frac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$

$f(x) = \frac{x}{x+1}$

since, $f(x)$ is not defined for $x = -1, 1$

And $f(x)$ is defined for all $\mathbb{R} \setminus \{-1,1\}$

since $f(x) = \dfrac{p(x)}{q(x)}$ is rational function.

and rational function are continuous on their domain of definition.

$F(x)$ is continuous on $\mathbb{R} \setminus \{-1,1\}$.

different

B)

If a die is thrown two times then $n(S) = 36$

So the probability of getting 5 when two numbers are added is $(1,4)\,(2,3)\,(3,2)\,(4,1)$

$$P(\text{getting } 5) = \frac{n(E)}{n(S)} = \frac{4}{36}$$

The probability of getting 10 when two numbers are added is $(4,6),\,(5,5),\,(6,4)$

$$P(\text{getting } 10) = \frac{n(E)}{n(S)} = \frac{3}{36}$$

Now the probability of getting a number divisible by five is

$P\begin{pmatrix}\text{getting 5 when} \\ \text{two numbers added}\end{pmatrix} + P\begin{pmatrix}\text{getting 10 when two} \\ \text{number added}\end{pmatrix}$

$= \frac{4}{36} + \frac{3}{36}$

$= \frac{7}{36}$

c) i) $A\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = (1)(1) - (-1)(1) =$

$1+1 = 2$

$|A| = 2$

adj $A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}}{2} = A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} =$

$\begin{pmatrix} 0.5 & 0.5 \\ -0.5 & 0.5 \end{pmatrix}$

ii) $3x + 8y = 5$

$4x + 11y = 7$

$A = \begin{pmatrix} 3 & 8 \\ 4 & 11 \end{pmatrix}$; $B = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

$|A| = (3)(11) - (4)(8)$

$|A| = 33 - 32$

$|A| = 1$

$\text{adj } A = \begin{pmatrix} 11 & -8 \\ -4 & 3 \end{pmatrix}$

$A^{-1} = \dfrac{\text{adj } A}{|A|} = \dfrac{\begin{pmatrix} 11 & -8 \\ -4 & 3 \end{pmatrix}}{1} = \begin{pmatrix} 11 & -8 \\ -4 & 3 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} B$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 & -8 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \times 5 & + (-8) \times 7 \\ -4 \times 5 & (3) \times 7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 55 - 56 \\ -20 + 21 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$x = -1 \qquad y = 1$

D) Given

$f: \{1, 2, \ldots 8\} \to \{1, 2, \ldots 8\}$

we know that,

for $f: A \to B$,

if $n(A) \neq n(B)$, there are no bijective functions.

if $n(A) = n(B) = m$ then there are $m!$

Bijective functions

Here $n(A) = n(B) = 8$

so, that's $8!$ bijective functions

So the number of bijective functions are $8!$

or 40320

E) From the figure

$x_0 = -2.0\ m$

But, $x - x_0 = v_0 t + \frac{1}{2} a t^2$

Now, with $t = 1.0\ s$, $x = 0\ m$

$\to 2.0\ m = v_0 + \frac{1}{2} a$ ... equation (i)

Again, with $T = 2.0\,s$, $x = 6.0\,m$

$\rightarrow (6.0 + 2.0)\,m = 2V_0 + 2a$

$\rightarrow (4.0\,m) = V_0 + a$ ... equation (ii)

If you solving the equations i and ii you obtain

$i = 0\,m/s$

$ii = a = 4.0\,m/s^2$

<u>QUESTION 4</u> a)

i) $x = 2t^3 - 15t^2 + 24t$

$V = \frac{dx}{dt} = 3 \cdot 2t^2 - 2 \cdot 15t + 24 = 6t^2 - 30t + 24$

$a = \frac{dv}{dt} = 2 \cdot 6t - 30 = 12t - 30$

ii) In order to calculate the direction, we need to find the slope.

$6(4) - 30(2) + 24 = 48 - 60 = -12$

$\tan(\theta) = -12$

$\theta = \tan^{-1}(-12)$ in anti clockwise direction.

$\theta = \tan^{-1}(12)$ in clockwise direction.

velocity of x at $t = 2s = v(2) = -12$ (calculated above)

acceleration at $t = 2s = a(2) = 12(2) - 30 = -6$

as acceleration is negative.

iii) $v = 6t^2 - 30t + 24 = 0$

$6(t^2 - 5t + 4) = 0$

$t^2 - 5t + 4 = 0$

$t^2 - t - 4t + 4 = 0$

$t(t-1) - 4(t-1) = 0$

$(t-4)(t-1) = 0$

Point P will rest at $t = 1s$ and $t = 4s$.

B) You can invite 7 friends out of 15 friends in

$^{15}C_7$ ways $= \dfrac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9}{7 \times 6 \times 5 \times 4 \times 3 \times 2} =$

and we get 6435 ways

ii) you can invite 7 friends in ${}^{15}C_7$ ways.

after that we can make them sit in 7 chairs in 7! ways.

So total number of ways = ${}^{15}C_7 \times 7!$

$= 32,342,400$ ways.

c) $3\log(x+5) = 2\log(7-x)$

$a\log x = \log(x^a)$

$\log(x+5)^3 = \log(7-x)^2$

$\log(x+5)^3 - \log(7-x)^2 = 0$

$\log\left[\frac{(x+5)^3}{(7-x)^2}\right] = 0$

$\frac{(x+5)^3}{(7-x)^2} = 1$

$(x+5)^3 = (7-x)^2$

$(x+1)(x^2+13x+76) = 0$

$\boxed{x=-1}$

D) i) $y = e^{x^3-3x^2}$

$\frac{dy}{dx} = y' = \frac{d}{dx}\left\{ e^{(x^3-3x^2)} \right\}$

$= e^{(x^3-3x^2)} \frac{d}{dx}(x^3-3x^2)$

$y' = e^{(x^3-3x^2)} \times (3x^2-6x)$

ii) $y = 10^{5x}$

$\frac{dy}{dx} = \frac{d}{dx}(10^{5x})$

$= 10^{5x} \log(10) \frac{d}{dx}(5x)$

$$\frac{dy}{dx} = 10^{5x} \log(10)(5)$$

$$y' = 5\ln(10)\, 10^{(5x)}$$

E) Critical points:

$$(x, f(x)) = (0,0)$$

$$(x, f(x)) = \left(2, \frac{4}{e^2}\right) \approx (2, 0.5413411)$$

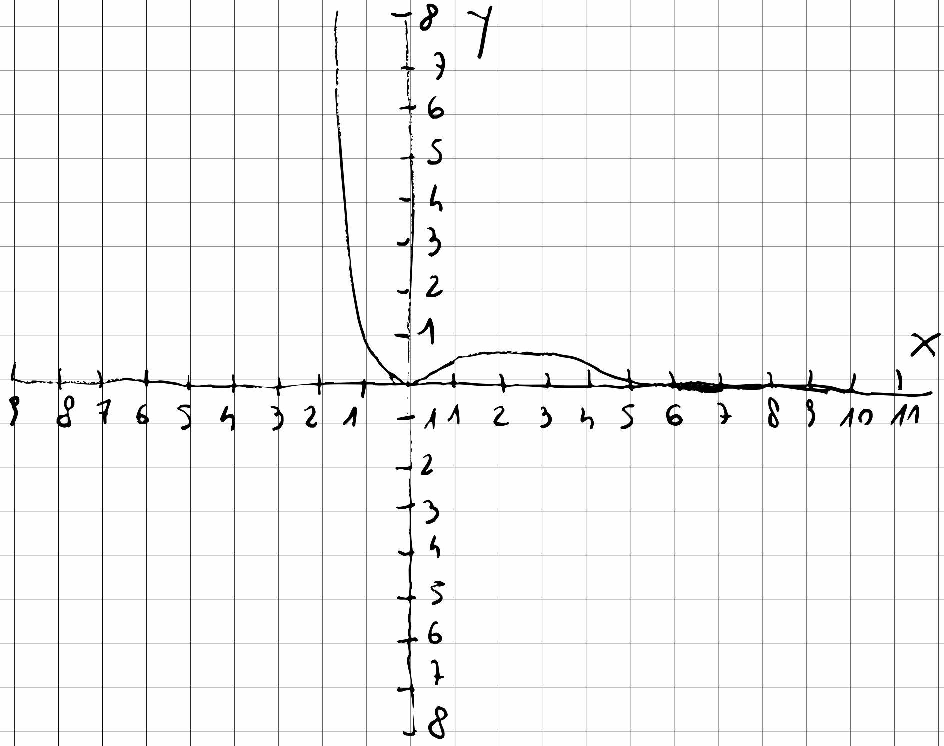
Minimum value:

$$(x, f(x)) = (0,0)$$

Maximum value:

No point of maxima

GRAPH:



y
x
9 8 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 10 11
8 7 6 5 4 3 2 1
-1 2 3 4 5 6 7 8