MID TERM ASSIGNMENT

QUESTION 1

a) THE NUMBER SYSTEMS ALLOWS US TO REPRESENT NUMBERS IN DIFFERENT WAY.
FOR EXAMPLE DENARY SYSTEM IS COMMONLY USED IN DAILY LIFE BY HUMANS.
BUT COMPUTER USE THE BINARY SYSTEM TO UNDERSTAND THE INSTRUCTION, WHILE OCTAL AND HEXADECIMAL ARE USED TO COMPACT THE FORM OF BINARY NUMBERS.
ON BALANCE THE NUMBER SYSTEMS ARE USEFUL TO ALLOW THE COMPUTER TO UNDERSTAND INSTRUCTIONS AND, ABOVE ALL, TO COMPUTE.

b) THE MOST IMPORTANT REASON IS THE SIMPLICITY, IN FACT THE USE OF BINARY SYSTEM IS EXTREMELY EASIER THAN THE USE OF DECIMAL SYSTEM, ESPECIALLY IN COMPUTING.

FOR EXAMPLE COMPUTERS USE A BINARY SYSTEM INORDER TO SEND DATA FROM ONE PART TO ANOTHER, AND IT IS DONE BY ELECTRICAL SIGNALS, COMMONLY INDICATED BY "ON" AND "OFF".

THERE ONLY TWO POSSIBILITIES FOR ONE DIGIT.

C) THE CIRCUMFERENCE FORMULA IS $C = 2\pi r$

IN THIS CASE $C = 2\pi 1 = C = 2\pi$ OR $C = 2 \times 3{,}14$

π IS EQUAL TO 3,14 ONLY IN DECIMAL SYSTEM BECAUSE IN ANOTHER NUMBER SYSTEMS MIGHT CHANGE.

D) THE OCTAL NUMER SYSTEM IS BASED ON 8 DIGITS FROM 0 TO 7, SO IN THIS CASE 8 AND 9 ARE NOT ALLOWED

E) FIRST OF ALL WE SHOULD CONVERT 61_8 IN BASE 10, AND WE CAN DO IT IN THIS WAY:

$61_8 = 6 \times 8^1 + 1 \times 8^0 = 48 + 1 = 49$

SO THE RESULT IS $= \sqrt{49} = 7$ IN DECIMAL SYSTEM.

QUESTON 2

a) i) = IF YOU WANT TO CALCULATE THE VALUE OF x WE CAN ASSUME THAT ;

$$\frac{1}{x} - x = 1 - \frac{1}{x} = (x-1)(x+2) = 0$$

$$x^2 + 2x - x - 2 = 0 = x^2 + x - 2$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2(1)} = \frac{-1 \pm 3}{2} \quad \begin{cases} \frac{2}{2} = 1 \\ -\frac{4}{2} = -2 \end{cases}$$

IN THIS CASE $x < 0$ SO $x = -2$

ii) $-2, -\frac{1}{2}, 1$

THE COMMON DIFFERENCE IS $1{,}5$ OR $\frac{3}{2}$

iii) TO FIND THE SUM OF FIRST 20 TERMS WE CAN USE THIS FORMULA:

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right) =$$

SO IN THIS CASE:

$$S_{20} = \frac{20}{2}\left(2(-2) + (20-1)\frac{3}{2}\right) =$$

$$= 10\left(-4 + 30 - \frac{3}{2}\right) = -40 + 300 - 15 = \underline{+245}$$

B)

i) THIS SEQUENCE REPRESENT AN ARITHMETIC SEQUENCE BECAUSE IS CLEAR THAT THE TERMS ARE OBTAINED BY ADDING A FIXED AMOUNT TO THE PREVIOUS ONE. IN THIS CASE THE FIXED AMOUNT IS 19

EX: $u_1 = 500 \quad u_2 = 500 + 19 = 519$

ii) IN ARITHMETIC PROGRESSION THE VALUE OF NTH TERMS IS OBTAINED BY THE FORMULA:

$a+(n-1)d$: nth value

a: FIRST NUMBER d: COMMON DIFFERENCE

n: TERM TO FIND

SO IN THIS CASE: $u_{12} = 500 + (12-1)19 = \underline{709}$

iii) IN THE ARITHMETIC SEQUENCE THE SUM OF NTH TERMS IS OBTAINED BY THIS FORMULA:

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right).$$

SO IN THIS CASE $S_{12} = \frac{12}{2}\left(2 \times 500 + (12-1)19\right) =$

$$6(1000 + 209) = \underline{7254}$$

c) i) THIS IS A GEOMETRIC PROGRESSION BECAUSE IS CLEAR THAT THE TERMS ARE OBTAINED BY MULTIPLYING A FIXED AMOUNT TO THE PREVIOUS ONE.

SO TO OBTAIN A VALUE OF SPECIFIC TERM WE CAN USE:

ar^{n-1} a: FIRST TERM n: TERM TO FIND.

r: COMMON RATIO

$4 \times 2^{8-1} = 4 \times 2^7 = 4 \times 128 = \underline{512}$

ii) THE SUM OF n TERMS IS OBTAINED BY THE FORMULA:

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{10} = \frac{4(1-2^{10})}{1-2} = \frac{4(1-1024)}{1-2} = \frac{-4092}{-1}$$

$$= \underline{4092}$$

QUESTION 3

i) IN THIS CASE THE ARITHMETIC SEQUENCE CAN HELP US TO SOLVE THE QUESTION.

$a + (n-1)d = \text{value} \rightarrow 100 + (n-1)10 = 300$

$$(n-1)10 = 300 - 100$$

$$\frac{(n-1)\cancel{10}}{\cancel{10}} = \frac{200}{10} =$$

$$(n-1) = 20$$

$$n = 20 + 1 = \underline{21}$$

ii) TO ANSWER THIS QUESTION WE CAN USE THE FORMULA FOR SUM IN ARITHMETIC SEQUENCE AND THE AMOUNT OF THE LAST 21 WEEKS.

$$S_{21} = \frac{21}{2}\left(2(100) + (21-1)100\right) = \frac{21}{2}\left(200+200\right) =$$

$$\frac{21}{2} \times 400 = \frac{8400}{2} = 4200$$

$52 - 21 = 21$

EACH WEEK THEY PRODUCE 300 ITEMS SO $21 \times 300 = 6300$

$4200 + 6300 = 10500$ ITEMS PRODUCTED IN 52 WEEKS.

QUESTION 4

i) $\sum_{n=1}^{\infty} \frac{n^3}{n^5+3}$ IS CONVERGENT BECAUSE WHE WE INCREASE n THE RESULT APPROCHES A CONSTANT VALUE

$$\frac{1^3}{1^5+3} = \frac{1}{4}$$

$$\frac{2^3}{2^5+3} = \frac{8}{32+3} = \frac{8}{35}$$

$$\frac{3^3}{3^5+3} = \frac{9}{81+3} = \frac{9}{84}$$

$$\frac{1}{4} = 0,25 \qquad \frac{8}{35} = 0,22 \qquad \frac{9}{84} = 0,10$$

WE CAN ASSUME THAT CONVERGES TO $\underline{0}$

ii) $\sum_{n=1}^{\infty} \frac{3^n}{4^n+4} =$ DIVERGENT, BECAUSE THIS SERIES WHEN INCREASING n IT NEVER REACHES A CONSTANT FINITE VALUE

$$\frac{3^1}{4^1+4} = \frac{3}{8} = 0,375$$

$$\frac{3^2}{4^2+4} = \frac{9}{20} = 0,45$$

$$\frac{3^3}{4^3+4} = \frac{27}{68} = 0,39$$

$$\frac{3^4}{4^4+4} = \frac{81}{260} = 0,31$$

iii) $\frac{1^4}{6(1)} = \frac{4}{6} = 0,66$

$\frac{2^4}{6(2)} = \frac{16}{12} = 1,33$

$\frac{3^4}{6(3)} = \frac{81}{18} = 4,5$

THIS SERIES IS DIVERGENT

iv) IF n IS A POSITIVE NUMBER THIS SEQUENCE IS DIVERGENT, BECAUSE THE VALUE APPROCHES THE INFINITE

$2^2 = 4$, $2^3 = 8$, $2^4 = 16$ ECC...

IF n IS A NEGATIVE NUMBER WHEN INCREASE n THE APPROCHES 0 SO THE SERES IS CONVERGENT

$2^{-1} = \frac{1}{2}$, $2^{-2} = \frac{1}{4}$, ECC...

QUESTION 5

i)

$1+3+5+\ldots+(2n-1)=n^2$ IS CORRECT

$1=1^2=1$

$4=2^2=1+3$

$9=3^2=1+3+5$ ETC...

THE SUM OF THE FIRST TWO ODD NATURAL NUMBERS IS THE SQUARE OF SECOND NATURAL NUMBER, SUM THE OF THE FIRST THREE ODD NATURAL NUMBERS IS THE SQUARE OF THIRD NATURAL NUMBER AND SO ON.

SO WE CAN WRITE

$P(n): 1+3+5+\ldots+(2n-1)\, n^2$

NOW WE SHOULD PROVE THAT $P(n)$ IS TRUE FOR ALL n, SO NOW WE SUPPOSE THAT $P(k)$ IS TRUE FOR SOME POSITIVE INTEGER k AND WE PROVE THAT $P(k+1)$ IS TRUE

SO:

$1+3+5+\ldots+(2k-1)=k^2$

CONSIDER:

$1+3+5+7+\ldots+(2K-1)+\{2(K+1)-1\}=$

$K^2+(2K+1)=(K+1)^2$

THEREFORE, $P(K+1)$ is True.

ii) $1^2+2^2+3^2+\ldots+(2n)^2=\dfrac{n(2n+1)(4n+1)}{3}$

FOR $n=1$

$$1^2+2^2=\frac{(1)(2(1)+1)(4(1)+1)}{3}=\frac{3(5)}{3}=5$$

SO IT'S TRUE, NOW IT REMAINS TO SHOW P_{K+1} IS TRUE

$$1^2+2^2+3^2\ldots+(2(K+1))^2=\frac{(K+1)(2(K+1)+1)(4(K+1)+1)}{3}$$

$$=\frac{(K+1)(2K+2+1)(4K+4+1)}{3}=\frac{(K+1)(2K+3)(4K+5)}{3}$$

$$\frac{(2K^2+5K+3)(4K+5)}{3}=\frac{8K^3+30K^2+37K+15}{3}$$

ON THE OTHER SIDE

$$= 1^2+2^2+3^2+\ldots+(2k+2)^2$$

$$= 1^2+2^2+3^2+\ldots+2k^2+(2k+1)^2+(2k+2)^2$$

$$= \frac{k(2k+1)(4+1)}{3}+(2k+1)^2+(2k+2)^2$$

$$= \frac{k(2k+1)(4+1)}{3}+\frac{3(2k+1)^2+3(2k+2)^2}{3}$$

$$= \frac{k(8k^2+6k+1)+3(4k^2+4k+1)+3(4k^2+8k+4)}{3}$$

$$= \frac{(8k^3+6k^2+k)+(12k^2+12k+3)+(12k^2+24k+12)}{3}$$

$$= \frac{8k^3+30k^2+37k+15}{3}$$

IT IS TRUE

B) $6^1 - 1 = 6 - 1 = 5$

5 IS DIVISIBLE BY 5 SO FOR $n = 1$ IS TRUE

NOW WE CAN DEMONSTRATE THAT IS TRUE FOR ALL INTEGERER NUMBERS.

FIX $K \geq 1$ AND PROCEED TO SHOWS THAT $6^{K+1} - 1$ IS DIVISIBLE BY 5.

$$\begin{aligned} 6^{K+1} - 1 &= 6(6^K) - 1 \\ &= 6(6^K - 1) - 1 + 6 \\ &= 6(6^K - 1) + 5 \end{aligned}$$

THE FIRST TERM IS DIVISIBLE BY 5 AND ALSO THE SECOND.

QUESTION 6

a) i) $31 \equiv 1 \pmod{10}$

$31 - 1 = 30$ AND 30 IS A MULTIPLE OF 10 $\quad \frac{30}{10} = 3$

THUS IT IS CONGRUENT

ii) $43 \equiv 22 \pmod 7$ BECAUSE $\frac{43 - 22}{7} = \frac{21}{7} = 3$

THUS IT IS CONGRUENT

iii) $8 \equiv -8 \pmod{3}$

$8-(-8) = 16$ which is not a multiple of 3

THUS IT IS NOT CONGRUENT

iv) $91 \equiv 18 \pmod{6}$

BECAUSE $\frac{91-18}{6} = \frac{73}{6}$ WHICH IS NOT INTEGER

SO IT IS NOT CONGRUENT.

TWO NUMBERS $\underline{A}$ AND $\underline{B}$ ARE CONGRUENT "MOD n"

IF THEY HAVE REMAINDER WHEN DIVIDED BY n

B)

$-17 \pmod{10}$

IS $-17/10 = -1 \quad R = -7$ BUT IT IS WRONG

SO WE REVERSE THE SIGN

$17 \equiv 7 \pmod{10}$

REVERSE 7 IN -7 SO $-7+10 = 3$

$-17 \equiv 3 \pmod{10}$

c)

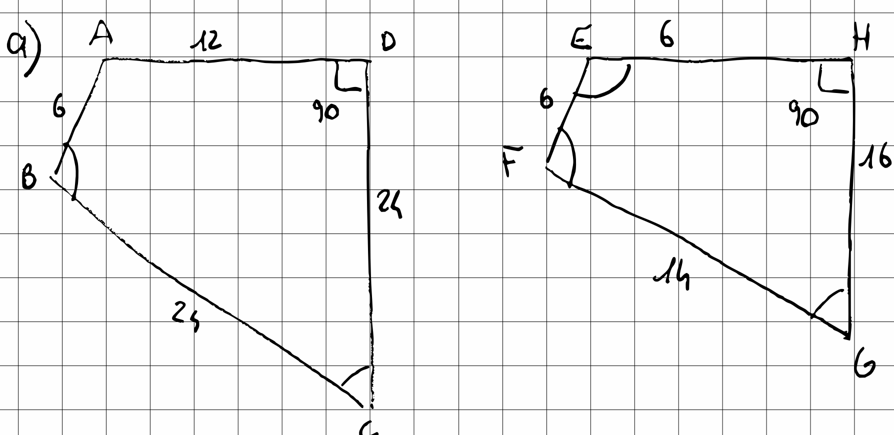
WE COULD DO A NORMAL SUBTRACTION BUT USING THE SUBTRATION RULE WE CAN MAKE THIS:

$$\left.\begin{array}{l} 60002 \equiv 2 \pmod{6} \\ 601 \equiv 1 \pmod{6} \end{array}\right\} 60002 - 601 \equiv 2-1 \equiv 1 \pmod{6}$$

THUS THE REMAINDER IS 1

QUESTION 7

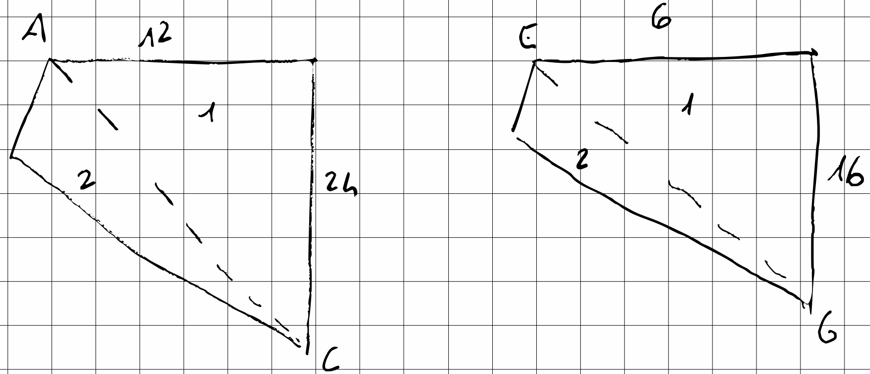
a)



WE CAN DIVIDE THIS TWO FIGURES IN TRIANGLE AND USE PITAGORA TO FIND THE HYPOTENUSE, AFTER THAT WE CAN USE THIS RULE TO CHECK THE SIMILARITY:

1) $AD/EH = AC/EG = DC/HG$

2) AC/EG = AB/EF = BC/FG



$AC = \sqrt{12^2 + 24^2} = \sqrt{144 + 576} = \sqrt{720} = 26{,}83$

$EG \sqrt{6^2 + 16^2} = \sqrt{36 + 256} = \sqrt{292} = 17{,}08$

THUS WE CAN WRITE THIS

1) $12/6 = 24/16 = 26{,}83 / 17{,}08$

$\downarrow \quad \downarrow \quad \downarrow$

$2 = 1{,}5 = 1{,}57$

BUT THEY ARE DIFFERENT

$2 \neq 1{,}5 \neq 1{,}57$

THESE TRIANGLES ARE NOT SIMILAR.

2) $6/6 = 26{,}83/17{,}08 = 24/14$

$\downarrow \quad \downarrow \quad \downarrow$

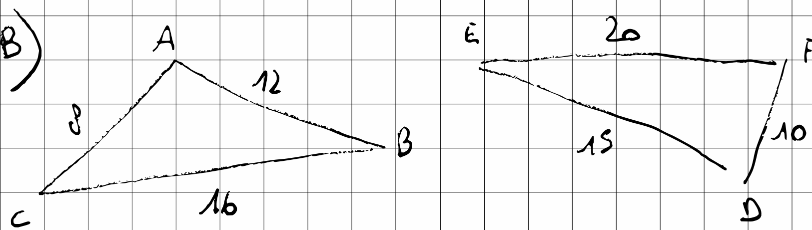
$1 = 1{,}57 = 1{,}71$

BUT THESE ARE DIFFERENT TO EACH OTHER

$1 \neq 1{,}57 \neq 1{,}71$

ON BALANCE WE CAN ASSUME THAT THESE POLYGONS ARE NOT SIMILAR.

B)



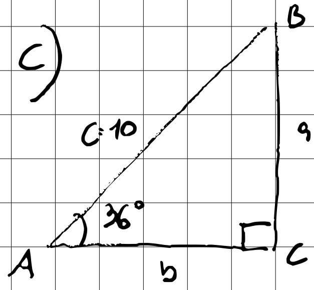
AS THE PREVIOUS EXERCISE THE ONLY WAY TO ESTABLISH IF THESE TWO TRIANGLES ARE SIMILAR IS THAT THE RULE UNDERNEATH IS RESPECTED:

$EF/CB = FD/AC = ED/AB$

$20/16 = 10/8 = 15/12$

$1{,}25 = 1{,}25 = 1{,}25$

THUS THESE TRIANGLES ARE SIMILAR WITH A SCALE FACTOR EQUAL TO 1,25 c)



LOOKING THIS FIGURE WE CAN ASSUME THAT THE

ANGLE γ IS $90°$ SO THE OTHER ANGLES WILL BE

$\alpha = 36°$

$\beta = 180 - 36 - 90 = 54°$

TO SOLVE THE TRIANGLE NOW WE CAN USE THE

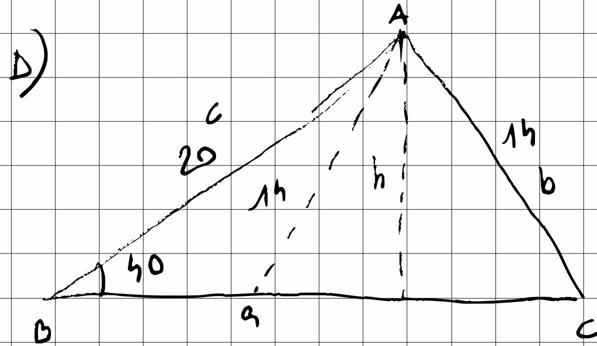
SIN RULE: $a / \sin(\theta) = b / \sin(90-\theta) = h = h / \sin(90)$

$$a = \frac{10 \times \sin 36}{\sin 90} = 5,877$$

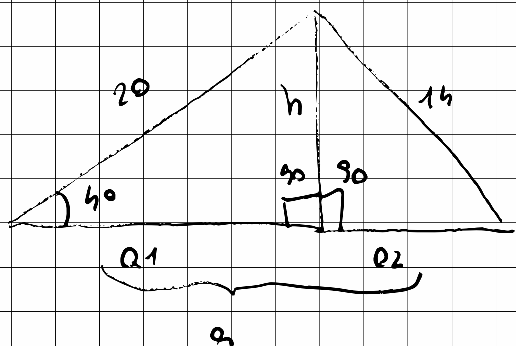
NOW USING PITAGORA WE CAN FIND b

$$b = \sqrt{c^2 - a^2} = \sqrt{100 - 34,53} = \sqrt{65,47} = 8,095$$

D)



TO FIND THE VALUE OF a WE CAN DIVIDE THIS TRIANGLE IN OTHER TWO TRIANGLE



$$h = \frac{20 \times \sin 40}{\sin 90} = \frac{12{,}85}{\sin 90} = 12{,}85$$

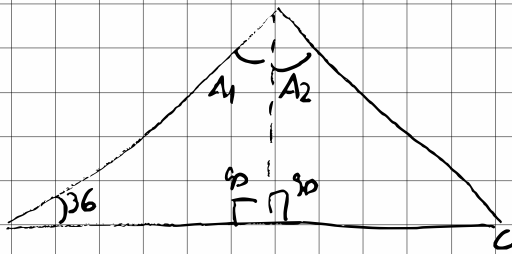
$$a_1 = \sqrt{20^2 - 12{,}85^2} = 15{,}32$$

$$a_2 = \sqrt{14^2 - 12{,}85^2} = 5{,}55$$

$$a = a_1 + a_2 = 15{,}32 + 5{,}55 = 20{,}87$$

TO FIND THE ANGLE A WE CAN DIVIDE

DIVIDE THE ANGLE IN A_1 AND A_2



$A_1 = 180 - 90 - 36 = 54$

$\text{SIN}C = \frac{\text{OPPOSITE}}{\text{HYPOTENUSE}} = \frac{12{,}85}{14} = 0{,}91$

$C = \sin^{-1} 0{,}91 = 66{,}61$

$A_2 = 180 - 90 - 66{,}61 = 23{,}39$

$A = A_1 + A_2 = 54 + 23{,}39 = 77{,}39$

QUESTION 8

a) NO. THERE ARE MANY DIFFERENT CHOICES OF $(p,q) \in \mathbb{Z} \times \mathbb{Z}^+$ WHICH ARE ASSIGNED THE SAME VALUE BY f. FOR EXAMPLE, $f(1,1) = 1 = f(2,2)$

b) YES, EVERY RATIONAL NUMBER IS, BY DEFINITION, A QUOTIENT OF INTEGERS, AND WE CAN ALWAYS ARRANGE FOR THE DENOMINATOR TO BE POSITIVE

c) NO. TO BE A BIJECTION, A FUNCTION MUST BE BOTH AN INJECTION AND SURJECTION.
SINCE f IS NOT AN INJECTION BY PART (A), IT IS NOT A BIJECTION, EITHER.

QUESTION 9

$f(x) = x^3 + x^2 - 10x + 8$; $f(1) = 0$

FOR x INTERCEPT, $F(x) = 0$

$x^3 + x^2 - 10x + 8 = 0$

$x^2(x-1) + 2x(x-1) - 8(x-1) = 0$

$(x^2 + 2x - 8)(x-1) = 0$

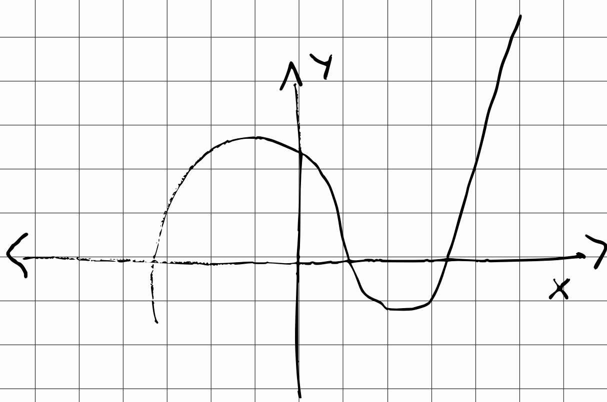
$(x+4)(x-2)(x-1) = 0$

$x = -4, +2, +1$

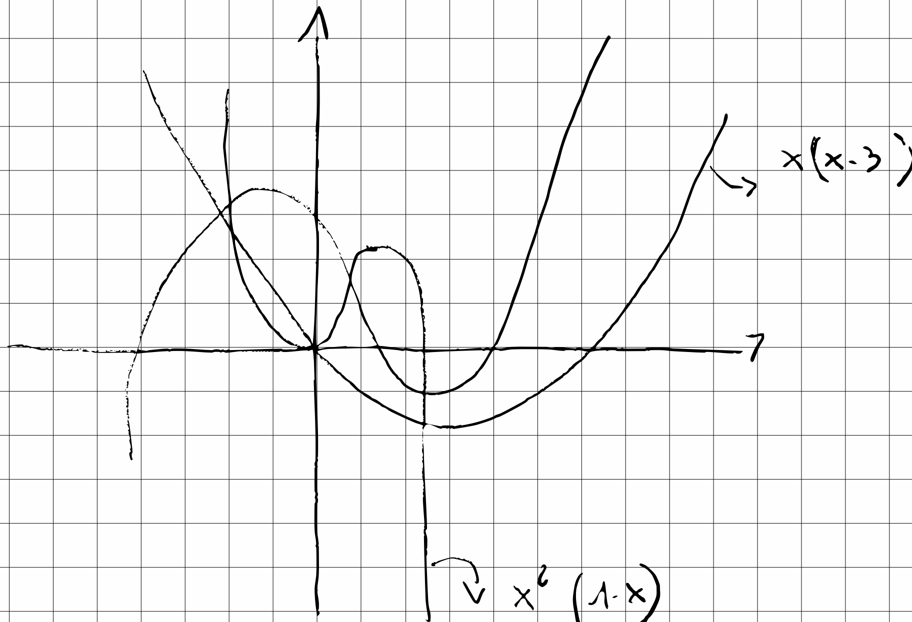
THUS X INTERCEPTS ARE 1, 2, 4

FOR Y INTERCEPT : $f(0) = 8$

ii)



B)



PUT $x(x-3) = x^2(1-x) \Rightarrow x = -\sqrt{3}, 0, \sqrt{3}$

POINT OF INTERSECTION $(-\sqrt{3}, 3+\sqrt{3}), (0,0), (\sqrt{3}, 3-\sqrt{3})$

QUESTION 10

U = initial velocity = 18,5 m/s

V = FINAL VELOCITY = 46,1 m/s

t = Time = 2,47 sec

$V = U + aT$

$46{,}1 = 18{,}5 + 2{,}47a$

$a = 11{,}17 \; m/s^2$

d: distance travelled

$v^2 = u^2 + 2ad$

$(46{,}1)^2 = (18{,}5)^2 + 2 \cdot 11{,}17 \cdot d$

$d = 79{,}8 \; m$