# MID TERM COURSE WORK CMO15

# 2020 - 2021

## QUESTION 1

a) $$(abc.de)_x = (ax^2 + bx + c + dx^{-1} + ex^{-2})_{10}$$

IN ORDER TO USE THE EXPANSION METHOD WE SHOULD ASSIGN POSITION NUMBER TO EACH DIGIT OF THE GIVEN NUMBER, IN ADDITION DIGITS TO THE LEFT OF DECIMAL ARE NUMBERED STARTING FROM 0 AND DIGITS TO THE RIGHT ARE NUMBERED STARTING FROM -1. IN ORDER TO HAVE THE DECIMAL DIGIT WE PERFORM THE ADDITION OF ALL TERMS.

THIS METHOD IS VALUE FOR ANY NUMBER OF DIGITS.

b) i) $(723)_8$ TO HEXADECIMAL SYSTEM

IN ORDER TO CONVERT THIS OCTAL IN HEXADECIMAL, FIRSTLY

WE SHOULD CONVERT THIS NUMBER IN BINARY

AND THEN IN HEXADECIMAL.

$(723)_8$     7 IS $(111)_2$

2 $(010)_2$

3 $(011)_2$

$(111)(010)(011) = 111010011_2$

NOW WE NEED TO GROUP EVERY 4 BINARY BITS AND

CALCULATE THE VALUE, FROM LEFT TO RIGHT

$(1)(1101)(0011) = 1\,(D)\,(3) = 1D3_{16} = 723_8$

ii) $(0.ABDF)_{16}$ TO DECIMAL SYSTEM

NOW TO CONVERT THIS NUMBER WE CAN USE THE

EXPANSION METHOD

0 IS EQUAL TO 0

A = 10    B = 11    D = 13    F = 15

$$\frac{10}{16} + \frac{11}{16^2} + \frac{13}{16^3} + \frac{15}{16^4} =$$

$$(0.6713714599609)_{10} = (0.ABDF)_{16}$$

iii) 0.375 TO BINARY SYSTEM

0 IS EQUAL TO 0.

$0.375 \times 2 = 0.75 = 0 + 0,75$ READ

$0,75 \times 2 = 1,5 = 1 + 0,5$

$0,5 \times 2 = 1,0 = 1 + 0$

$0 \to$ STOP

$$(0.375)_{10} = (0.011)_2$$

IV) THE DIGIT NOT ALLOWED IN QUINARY SYSTEM IS 5 BECAUSE THE DIGITS ALLOWED ARE ONLY FIVE FROM 0 TO 4 (0, 1, 2, 3, 4)

V) $(11010.1011)_2$ TO HEXADECIMAL

SPLIT THE BINARY NUMBER FROM LEFT TO RIGHT

EACH GROUP 4 BITS

$$\begin{array}{ccc} (1)(1010) & . & (1011) \\ \downarrow \quad \downarrow & & \downarrow \\ 1 \quad A & . & B \end{array}$$

RESULT $1A.B_{16}$

IN ORDER TO BE MORE CLEAR WE CAN CONVERT THE FOUR DIGIT FIRSTLY IN DECIMAL AND THEN IN HEXADECIMAL.

VI) $(257)_{10}$ TO THE BINARY SYSTEM;

IN ORDER TO CONVERT THIS NUMBER IN BASE 2 WE SHOULD DIVIDE 257 BY 2 AND WRITE THE REMAINDER.

$257 : 2 = 128 \quad R: 1$

$128 : 2 = 64 \quad R: 0$

$64 : 2 = 32 \quad R: 0$

$32 : 2 = 16 \quad R: 0$

$16 : 2 = 8 \quad R: 0$

$8 : 2 = 4 \quad R: 0$

SENSE OF READING

$4 : 2 = 2 \qquad R: 0$

$2 : 2 = 1 \qquad R: 0$

$1 : 2 = 0 \qquad R: 1$

THE RESULT IS: $\underline{100000001}$

c) I) CONVERT 10.0011 IN DECIMAL SYSTEM.

$1 \times 2^1 + 0 \times 2^0 . 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}$

$\downarrow$

$2 + 0 . 0 + 0 + \frac{1}{8} + \frac{1}{16}$

$\underline{2.1875}$

ii) IN THIS CASE THE PLACE VALUE OF 1 IS THOUSANDTH AND TEN-THOUSANDTH

IN CASE WE WANT CONVERT THIS NUMBER IN BASE 10

$0.0011_2 \longrightarrow 0,1875$

THE PLACE VALUE OF 1 IS TENTHS

iii) THE SUM OF 1+1+1+1 IN BINARY IS:

1+
1+
1+
1
___
10+
1+
1
___
011+
1
___
100

THE RESULT IS $100_2$

IN BASE 10 IS: 4

iii) 101 DIVIDED 10 WITH LONG DIVISION

$$\begin{array}{r|l} & 0 \\ \hline 10 & 101 \end{array}$$

THE DIVISOR 10 GOES INTO THE FIRST DIGIT (1) FOR 0 TIMES AND PUT 0 ON TOP

$$\begin{array}{r|l} & 0 \\ \hline 10 & 101 \\ & -0 \\ \cline{2-2} & 1 \end{array}$$

THEN MULTIPLY $10 \times 0 = 0$

AND WRITE THE RESULT BELOW

$$\begin{array}{r|l} & 0 \\ \hline 10 & 101 \\ & -0 \\ \cline{2-2} & 10 \end{array}$$

MOVE DOWN THE SECOND DIGIT AND GO AHEAD WITH THE PROCESS

01

10 | 101
-0
10
-10
0

010

10 | 101
-0
10
-10
01
-0
1

ANSWER 10 R=1

D)

i) 1101 : CORRECT

ii) $(214)_2$ : WRONG, THE ONLY DIGITS ADMITTED ARE 0 AND 1

iii) $(0000)_2$ : CORRECT, IS A WAY TO SAY $(0)_{10}$

iv) $(11)^2$ : WRONG, THIS IS A WAY TO INDICATE A POWER.

## QUESTION 2

a) YES BECAUSE IT REPRESENT A WAY TO WRITE A SPECIFIC ORDERED LIST OF NUMBERS

THE WAY TO FIND A GENERAL TERM IS:

$n$TH TERM $= a + (n-1)d$

$d$: COMMON DIFFERENCE

$a$: FIRST TERM

IN GEOMETRIC PROGRESSION WE CAN USE:

$n$TH TERM: $ar^{n-1}$

$a$: FIRST TERM

$r$: COMMON RATIO

b) TO FIND THE VALUE OF THIS TERM WE CAN USE:

$$\frac{3n-1}{5n+7} = \frac{7}{12}$$

$$\cancel{(5n+7)}\frac{(3n-1)}{\cancel{(5n+7)}} = \frac{7}{12}(5n+7)$$

$$(3n-1) = \frac{7}{12}(5n+7)$$

$$(3n-1) = \frac{35n}{12} + \frac{49}{12}$$

$$(3n-1) = \frac{35n+49}{12}$$

$$12(3n-1) = \frac{35n+49}{\cancel{12}} \cdot \cancel{12}$$

$$36n - 12 = 35n + 49$$

$$36n - 35n = +49 + 12$$

$n = 61$ IF WE SOBSTITE n WITH 61

$$\frac{3(61)-1}{5(61)+7} = \frac{182}{312} = \frac{7}{12}$$

THE TERM IS 61

c)
$$\begin{cases} 18 = a_1 + (4-1)d \\ 50 = a_1 + (12-1)d \end{cases} \quad \begin{cases} 18 = a_1 + 3d \\ 50 = a_1 + 11d \end{cases}$$

$$\begin{cases} 18 = a_1 + 3d \\ 50 = a_1 + 11d \end{cases}$$

MULTIPLY BY $(-1)$

$$\begin{cases} -18 = -a_1 - 3d \\ 50 = a_1 + 11d \end{cases}$$

NOW SUM THE TERMS

$$\frac{32}{8} = \frac{8d}{8}$$

DIVIDE BY 8

$$4 = d$$

NOW I CAN FIND $a_1$

$$a_4 (18) = a_1 + (4-1)4$$

$$18 = a_1 + 12$$

$$18 - 12 = a_1 = 6 = a_1$$

NOW I TEST WITH THE FORMULA $a_n = a_1 (n-1) d$

$$a_{12} = 6 + (12-1)4 = 6 + (11 \times 4) = 50$$

THUS WE CAN ASSUME THAT THE 99th TERM IS

$a_{99} = 6 + (99-1)4 = 6 + 392 = 400$

D) i) THIS SEQUENCE BECAUSE WHEN WE DIVIDE ANY TERMS BY THE PREVIOUS ONE WE OBTAIN THE SAME RESULT

$-3, 3, -3, 3$

SO $\left.\begin{array}{l} \dfrac{3}{-3} = -1 \\[2ex] \dfrac{-3}{3} = -1 \end{array}\right\}$ $-1$ IS THE COMMON RATIO

AND THE NEXT TWO TERMS ARE $-3$ AND $3$

SO = $-3, 3, -3, 3, -3, 3$

ii) THIS SEQUENCE IS NEITHER GEOMETRIC NOR ARITHMETIC, BECAUSE ITS TERMS ARE NOT OBTAINED BY MULTIPLYING OR ADDING A FIXED AMOUNT

iii) THIS SEQUENCE IS NEITHER GEOMETRIC NOR ARITHMETIC, BECAUSE ITS TERMS ARE NOT OBTAINED BY MULTIPLYING OR ADDING A FIXED AMOUNT

E) $b_1 = \frac{1}{9}$ $q = 3$ 243th ?

YES BECAUSE 243 IS A TERM OF A SEQUENCE

$$\underset{①}{\frac{1}{9}} \times 3 = \underset{②}{\frac{1}{3}} \times 3 = \underset{③}{1} \times 3 = \underset{④}{3} \times 3 = \underset{⑤}{9} \times 3 = \underset{⑥}{27} \times 3 = \underset{⑦}{81}$$

⑧

$$81 \times 3 = 243$$

$$a_n^{n-1} = \frac{1}{9} \times 3^{8-1} = \frac{1}{9} \times 3^7 = \frac{1}{9} \times 2187 = 243$$

F) $99^{th} = -52$ $201^{st} = ?$

$4^{th} = -7$

WE USE THE FORMULA OF ARITHMETIC SEQUENCE BECAUSE THE DIFFERENCE IS CONSTANT.

$$\begin{cases} -7 = a_1 + (4-1)d \\ -52 = a_1 + (99-1)d \end{cases} \qquad \begin{cases} -7 = a_1 + 3d \\ -52 = a_1 + 98d \end{cases}$$

MULTIPLY BY $(-1)$ IN ORDER TO AVOID $a_1$

$$\begin{cases} +7 = -a_1 - 3d \\ -52 = a_1 + 98d \end{cases}$$

NOW WE MADE THE SUM

$$\begin{cases} -45 = 95d \end{cases}$$ DIVIDE ALL BY 95

$$\begin{cases} -\frac{45}{95} = d \end{cases} \quad \begin{cases} -\frac{9}{19} = d \end{cases}$$

NOW WE SHOULD FIND THE FIRST TERM

$a_1$ : $\quad -7 = a_1 + (4-1)\left(-\frac{9}{19}\right) =$

$$-7 = a1 - \frac{27}{19}$$

$$a_1 = -7 + \frac{27}{19} = -5{,}57$$

NOW THE $201^{ST}$ TERM

$$201^{ST} = -5{,}57 + (201-1)\left(-\frac{9}{19}\right) =$$

$$-5{,}57 - \left(200 \times \frac{9}{19}\right) = -5{,}57 - 94{,}73 = -100{,}30$$

6) $\lim_{n \to +\infty} \left(\frac{n-1}{n}\right)$ IS CONVERGENT BECAUSE

APROCHES THE VALUE OF 1 , IN FACT WHEN WE

INCREASE THE VALUE OF $n$ WE APROCHE THE VALUE OF 1.

H) $1, -4, 9, -16$ FIND THE FORMULA FOR NTH TERM.

IT REPRESENT A SEQUENCE BECAUSE IT CAN BE ARRANGED IN A DEFINITE MANNER.

$$a_1 = 1 = (-1)^{1-1}(1)^2$$

$$a_2 = -4 = (-1)^{2-1}(2)^2$$

$$a_3 = 9 = (-1)^{3-1}(3)^2$$

$$a_4 = -16 = (-1)^{4-1}(4)^2$$

$$a_n = \underbrace{(-1)^{n-1}(n)^2}_{\text{THE NTH TERM.}}$$

i) $\frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$

WHEN $n$ IS EQUAL TO 1 ?

$$\frac{1}{2^1} = 1 - \frac{1}{2^1} : \quad \frac{1}{2} = \frac{1}{2}$$

WHEN n IS EQUAL TO 2

$$\left.\frac{1}{2} + \frac{1}{2^2} = 1 - \frac{1}{2^2}\right\} \left.\frac{1}{2} + \frac{1}{4} = 1 - \frac{1}{4}\right\} 0{,}75 = 0{,}75$$

SO NOW WE CAN PROVE THAT IS CORRECT FOR ALL POSITIVE INTEGERS.

$$\left(\frac{1}{2} + \frac{1}{2^2} + \dots \frac{1}{2^n}\right) + \left(\frac{1}{2^{n+1}}\right) = \left(1 - \frac{1}{2^n}\right) + \left(\frac{1}{2^{n+1}}\right)$$

$$\downarrow \qquad\qquad \downarrow$$

THIS FACTOR PRODUCE THE SAME RESULT

TO PROVE THAT WE CAN SUBSTITUTE WITH NUMBER

$$\left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3}\right) + \left(\frac{1}{2^{3+1}}\right) = 1 - \frac{1}{2^3} + \frac{1}{2^{3+1}}$$

$$\underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}}_{\frac{15}{16}} = \underbrace{1 - \frac{1}{8} + \frac{1}{16}}_{\frac{15}{16}}$$

$$\frac{15}{16} = \frac{15}{16}$$

PROVED.

i) REMAINDER WHEN $3^{123}$ IS DIVIDED BY 7

BY FERMAT'S LITTLE THEOREM

$$a^{p-1} \equiv 1 \pmod{p}$$

$$3^6 \equiv 1 \pmod{7}$$

$$(3^6)^{20} \equiv 1 \pmod{7}$$

$$3^{120} \equiv 1 \pmod{7}$$

$$3^{123} \equiv 3^{120} \cdot 3^3 \equiv 1 \cdot 3^3 \pmod{7}$$

$$3^{123} \equiv 3^3 \equiv 27 \equiv 6 \pmod{7}$$

REMAINDER IS 6 WHEN $3^{123}$ IS DIVIDED BY 7.

## QUESTION 3

i) THIS PARTICULAR RESULT IS CALLED TRANSIVITY IN CONGRUENCE AND IS OBVIOUSLY TRUE AND IT CAN BE PROOF AS FOLLOWS:

IF $a \equiv b \bmod n$ AND $b \equiv c \bmod n$ THEN $n \mid (b-a)$ AND $n \mid (c-b)$. USING THE LINEAR COMBINATION THEOREM, WE HAVE $n \mid (b-a+c-b)$ OR $n \mid (c-a)$.

THUS, $a \equiv c \mod. n$

ii) $a \pm b \equiv c \pm d \pmod{n}$

WRITE $a = kn + b$ AND $c = ln + d$ FOR SOME $k, l \in \mathbb{Z}$

THEN $a + c = (k+l)n + b + d$. SO, $a + c = b + d + tn$,

$t = k + l \in \mathbb{Z}$

$a + c \equiv b + d \ (\text{mod. } n)$

iii) $7x \equiv 12 \ (\text{mod. } 7)$

THERE IS NO SOLUTION BECAUSE $(7,7) = 7$ IS NOT A DIVISOR OF 12.

B) FIND THE LEAST POSITIVE VALUE OF $x$ SUCH THAT:

$71 \equiv x \ (\text{mod. } 8)$.

$71 - x$ IS DIVISIBLE BY 8

WHEN $x = 1$, $71 - x = 70$ WHICH IS NOT DIVISIBLE BY 8

WHEN $x = 2$, $71 - x = 69$ " "

WHEN $x = 3$, $71 - x = 68$ " "

WHEN $x = 4$, $71 - x = 67$ " "

WHEN $x = 5$, $71 - x = 66$ " "

WHEN $x = 6$, $71 - 6 = 65$ " "

WHEN $x = 7$, $71 - x = 64$ WHICH IS DIVISIBLE BY 8

THUS THE LEAST VALUE OF $x$ IS 7.

c) THE MODULAR MULTIPLICATIVE INVERSE OF AN INTEGER A MODULO M IS AN INTEGER B SUCH THAT

$$ab \equiv 1 \ (\text{mod. } M)$$

IN THIS CASE WE CAN BE WRITE

$$\frac{168b - 1}{83} = n \Rightarrow 168b - 1 = 83n$$

$$168 \times 42 - 1 = 83 \times 85$$

$b = 42$

$n = 85$

THUS THE MULTIPLICATIVE INVERSE IS 42

D) INVERSE OF 4 MOD. 15

$$4 \times 0 \equiv 0 \pmod{15}$$

$$4 \times 1 \equiv 4 \pmod{15}$$

$$4 \times 2 \equiv 8 \pmod{15}$$

$$4 \times 3 \equiv 12 \pmod{15}$$

$$4 \times 4 \equiv 16 \pmod{15} \equiv 1 \pmod{15}$$

THUS THE INVERSE IS 4

OR IN ANOTHER WAY

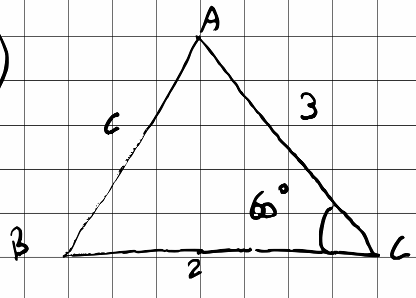
$$x \equiv \frac{1}{4} \text{ MOD. } 15$$

$$\Rightarrow \frac{4x-1}{15} = n \quad \Big\} \quad 4x-1 = 15n \quad \Big] \quad 4 \times 4 - 1 = 15 \times 1$$

$$x = 4, \; n = 1$$

## QUESTION 4

A) i)



$$\cos 60^\circ = \frac{2^2 + 3^2 - c^2}{2 \times 2 \times 3}$$

$$\frac{1}{2} = \frac{13 - c^2}{12}$$

$$6 = 13 - c^2$$

$$c^2 = 7$$

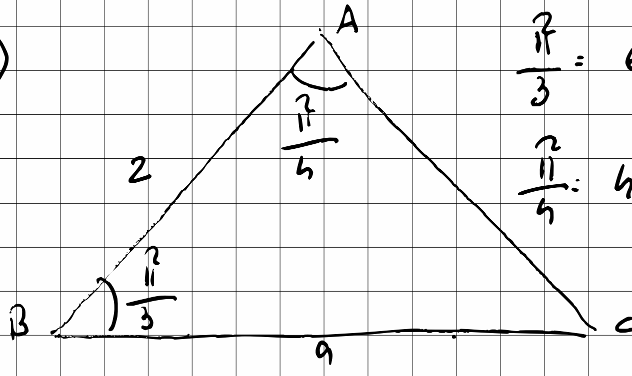
$$c = \sqrt{7}$$

ii) $$\frac{c}{\sin\gamma} = \frac{B}{\sin\beta}$$

$$\frac{\sqrt{7}}{\sin 60} = \frac{3}{\sin\beta}$$

$$\sin\beta = \frac{3 \cdot \sin 60}{\sqrt{7}} = \frac{3}{\sqrt{7}} \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{\sqrt{7}\,2} = \boxed{0{,}98}$$

B)




$$\frac{\pi}{3} = 60^\circ$$

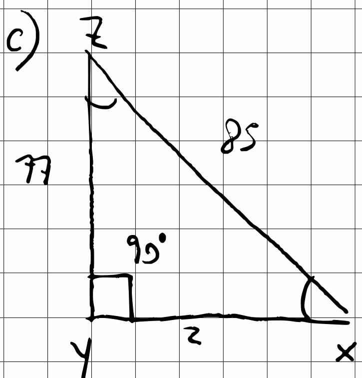
$$\frac{\pi}{4} = 45^\circ$$

$180 - 45 - 60 = 180 - 105 = 75°$

$$\frac{a}{\sin 45} = \frac{2}{\sin 75}$$

$$a = \frac{2 \cdot \sin 45}{\sin 75} = \frac{2 \cdot 0{,}7071}{0{,}96} = \boxed{1{,}46}$$

c)




$$z = \sqrt{85^2 - 77^2} =$$

$$\sqrt{7225 - 5929}$$

$$\sqrt{1296} = 36$$

$$\cos = \frac{77}{85} = 0{,}9998$$

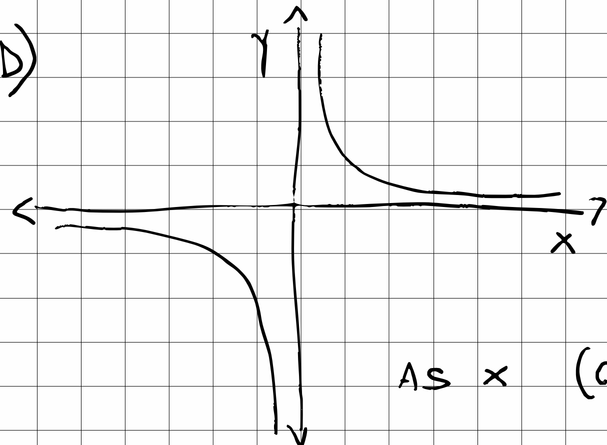
$$Z \text{ angle} = \cos^{-1} \frac{77}{85} = 25{,}0576$$

THE RESULT ARE

$z = 36$ $\qquad$ $Z$ angle $= 25{,}0576$

$\cos Z$ angle $= 0{,}9998$

D)




AS x $(0, \infty)$

ONLY QUADRANT GRAPH WILL

BE CONSIDER

YES g IS CONTINUOS ON ITS GIVEN DOMAIN OF $(0, \infty)$

QUESTION 5

A) LET $f(x_1) = f(x_2)$

$$\begin{cases} x_1 + 1 = x_2 + 1 \\ x_1 = x_2 \end{cases}$$ IF x IS EVEN

$$\begin{cases} x_1 - 3 = x_2 - 3 \\ x_1 = x_2 \end{cases}$$ if x IS ODD

FOR BOTH CASES EVEN AND ODD

$f(x_1) = f(x_2)$ THUS $x_1 = x_2$

FOR THIS REASON THE FUNCTION IS INJECTIVE

ii) $f(x) = \begin{cases} x+1 & \text{IF } x \text{ IS EVEN} \\ x-3 & \text{IF } x \text{ IS ODD} \end{cases}$

INVERSE THE FUNCTION:

$$g(y) = \begin{cases} y-1 & \text{IF } y \text{ IS EVEN} \\ y+3 & \text{IF } y \text{ IS ODD} \end{cases}$$

$$g(y_1) = g(y_2)$$

$$\begin{cases} y_1 - 1 = y_2 - 1 & \text{IF } y \text{ IS EVEN} \\ y_1 = y_2 \end{cases}$$

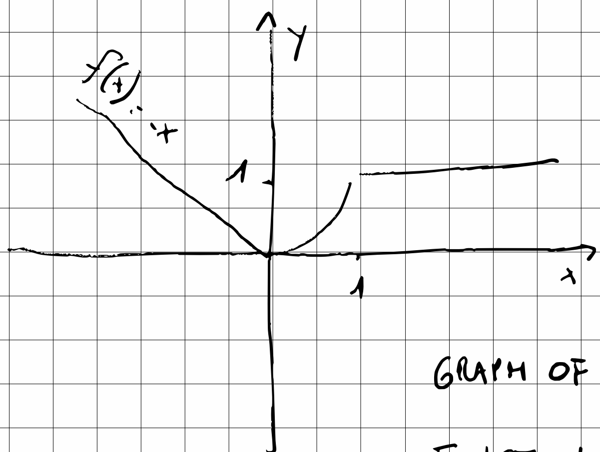
$$\begin{cases} y_1 + 3 = y_2 + 3 & \text{IF } y \text{ IS ODD} \\ y_1 = y_2 \end{cases}$$

FOR BOTH CASES INVERSE FUNCTION $g$ IS INJECTIVE

HENCE THE GIVEN FUNCTION $f$ IS SURJECTIVE.

B) $f : R \to R$

$$f(x) = \begin{cases} -x & \text{IF } x < 0 \\ x^2, & \text{IF } 0 \leq x \leq 1 \\ 1, & \text{IF } x > 1 \end{cases}$$




GRAPH OF THE GIVEN

FUNCTION

FUNCTION IS NOT BIJECTIVE.

$f(2) = f(3)$

BUT $2 \neq 3$

c)

VELOCITY IS $V_1 = 40 - 5t^2$

AT $t = 0 \to V_0 = 40$ M/S

AT $t = 2 \to V_2 = 40 - 5(2)^2$

$V_2 = 40 - 20$

$V_2 = 20\ m/s$

THE AVERAGE ACCELERATION

$$\frac{V_2 - V_0}{t_2 - t_0} = \frac{20 - 40}{2 - 0} = -\frac{20}{2} = -10\ m/s$$

D)

$V_i = 12\ cm/s$

$x_f = -5\ cm$

$x_i = 3\ cm$

$t = 2\ sec$

$$x_f - x_i = V_i t + \frac{1}{2} a t^2$$

$$-5 - 3 = 12(2) + \frac{1}{2} a (2)^2$$

$$-8 = 24 + 2a$$

$$-8 - 24 = 2a$$

$$a = -\frac{32}{2} = -16\ cm/s^2$$