Nature-inspired Monte Carlo algorithm for the travelling salesman problem

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AGENDA

- Travelling salesman problem
  - Computational complexity
- Techniques used
  - Simulated Annealing
  - Metropolis Algorithm
  - Genetic algorithm
- Dataset
  - Distance measure
- Application
- Results
- Conclusions
- Extra: Analysis of Metropolis algorithm

TRAVELLING SALESMAN PROBLEM (TSP)

Find the shortest route a travelling salesman has to take to visit all the cities only once and to return to the starting point

Find the hamiltonian cycle a travelling salesman has to take, given G = (N, E)

1) N is the set of cities to visit
2) E = N × N is the weights matrix

COMPUTATIONAL COMPLEXITY

The problem is NP-HARD

1. The number of possible cycles of n cities is $(n-1)!/2$
2. The problem cannot be solved exactly in polynomial time
   $2^n = o(n!)$ for large n
3. Only approximate solutions are feasible for large n

APPROXIMATE SOLUTIONS

Top-down explanation

1. SIMULATED ANNEALING
2. METROPOLIS ALGORITHM
3. GENETIC ALGORITHM

METAHEURISTIC AND SIMULATED ANNEALING

Approximate solution can be computed by a metaheuristic algorithm that guides the search

It is based on two principles

1. Intensification -> intensify the search near optima
2. Diversification -> spread the search near many optima

Simulated Annealing becomes a metaheuristic when it is applied on a population (diversification) since it alone intensifies the search near the optimum

SIMULATED ANNEALING

Mimic the behaviour of heated metals slowly cooled down: reach of minimal cost states

Assumption: states follow the Boltzmann distribution

$$e^{-\beta(cost(state))}$$

- β is the inverse of the temperature
- Cost is the distance

Most unbiased distribution

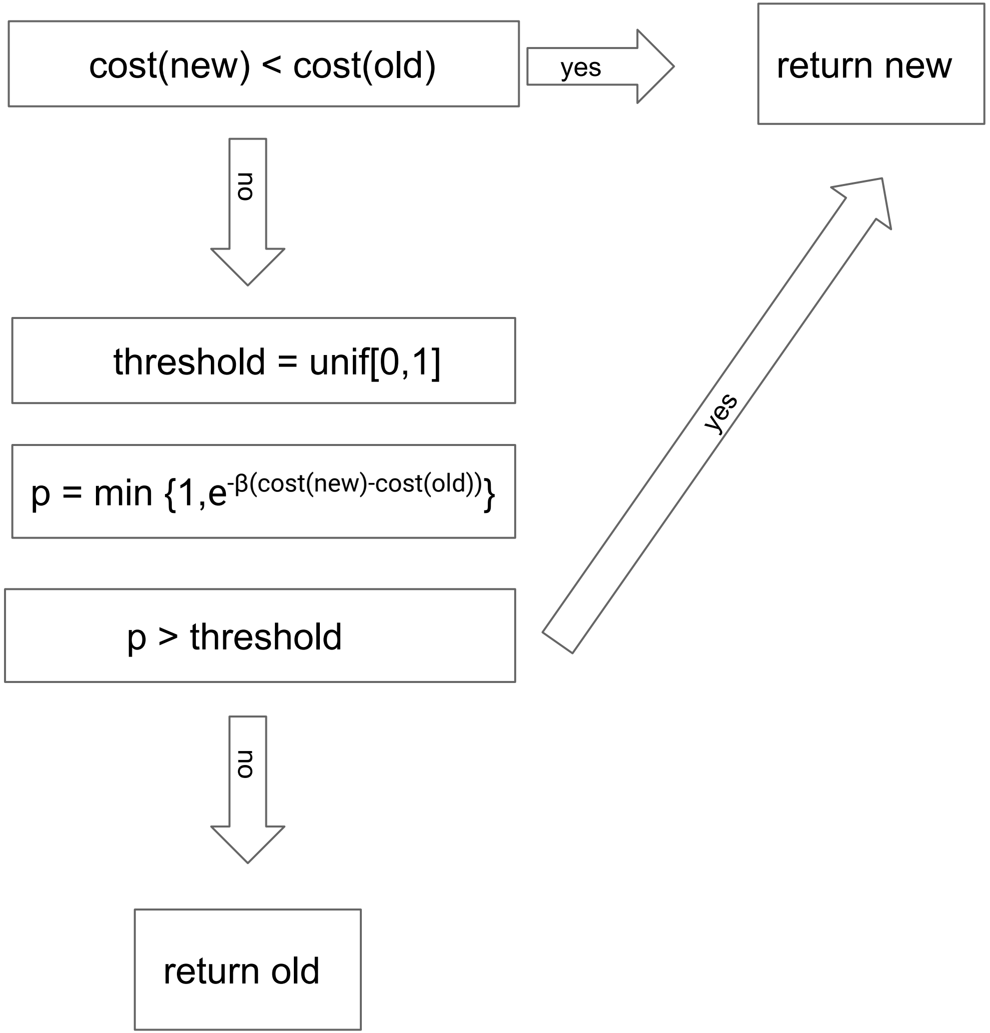
It requires no prior information

METROPOLIS ALGORITHM

Sample hamiltonian cycles from the Boltzmann distribution

Consider to have two states: new, old

Move toward higher probability states



GENETIC ALGORITHM

It is a broad category of algorithms mimicking natural evolution

SELECTION

CROSSBREEDING

MUTATION

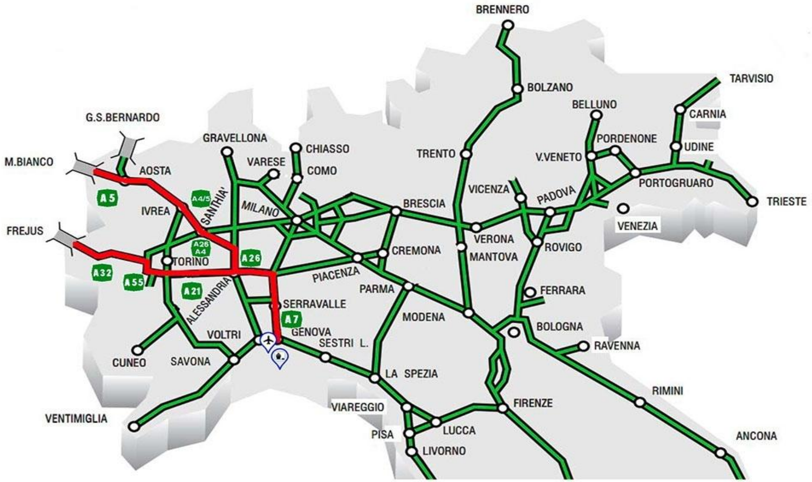
DATASET

Latitude and longitude of cities of northern Italy

Varese, Novara, Torino, Milano, Como, Bergamo, Brescia, Cremona, Lecco, Lodi, Mantova, Monza, Pavia, Sondrio, Verona, Vicenza, Padova, Venezia

WEIGHTS = DISTANCES

Euclidean distance Is the most suitable measure to approximate the highway system



APPLICATION

Find the best hamiltonian cycle starting from Varese

Compute the solution with Simulated Annealing

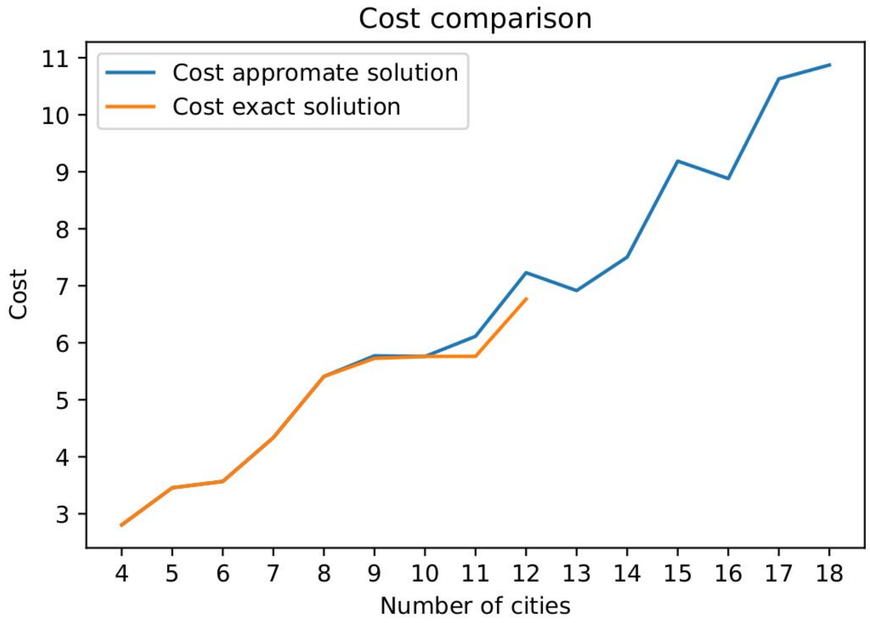
Compute the exact solution, when feasible

Compare the solutions considering:

- Optimality of the solutions
- Execution time

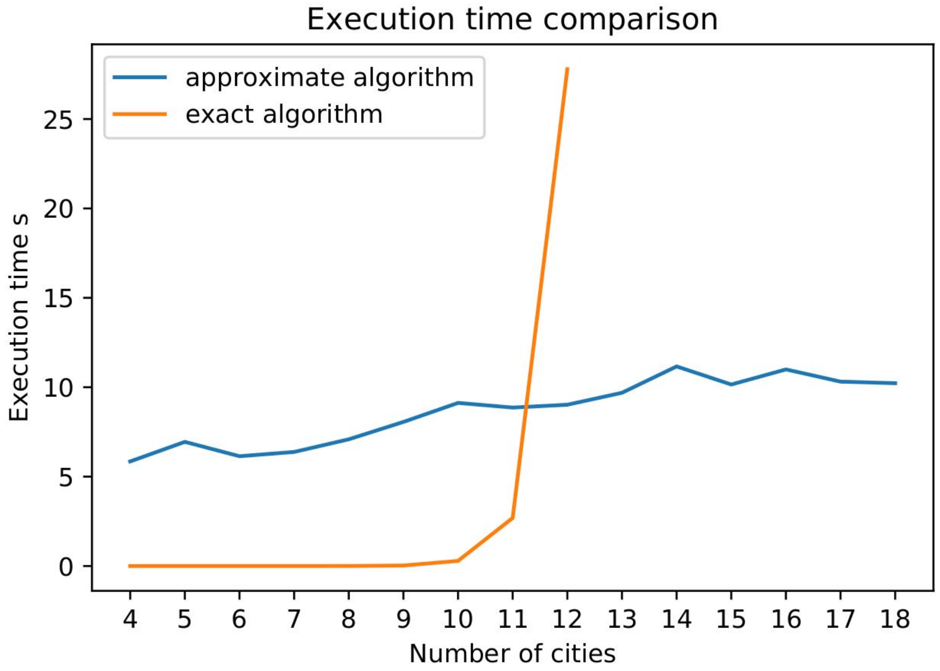
RESULTS

Simulated Annealing find solutions close or equal to the optimal one



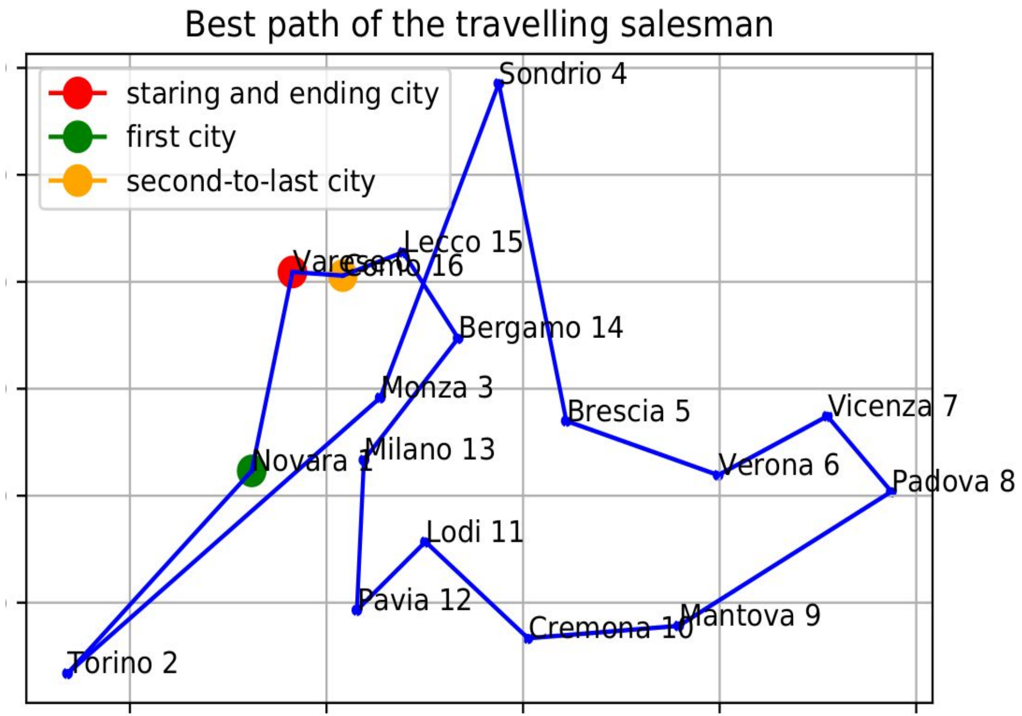
RESULTS

The execution time of Simulated Annealing is very low



RESULTS

Optimal solution taking into account all the cities



CONCLUSIONS

- Simulated Annealing execution time is very low
- The number of cities has a small effect on the execution time
- Simulated Annealing reaches optimal solutions
- Suboptimal solution are more likely to be found as the number of cities increase

Q&A

ANALYSIS OF METROPOLIS ALGORITHM

GRAPH MODELLING

Graph that represent the possible cycles

Undirected and connected graph G=(S,E)

- S set of hamiltonian cycle
- E set of transition probabilities

TRANSITION MATRIX

Transition matrix P is

- aperiodic
- irreducible

therefore P is primitive

π is the target distribution

$$p(i,j) = \begin{cases} 0 & \text{if } \{i,j\} \notin E \\ \frac{1}{d_i} \min\{1, \frac{\pi(j)}{\pi(i)}\} & \text{if } \{i,j\} \in E \text{ , } i \neq j \\ 1 - \frac{1}{d_i} \sum_{l \in adj(i)} \min\{1, \frac{\pi(l)}{\pi(i)}\} & \text{if } i = j \end{cases}$$

STATIONARY DISTRIBUTION

It defines a reversible distribution π

A reversible distribution is stationary

Therefore π is a stationary distribution

Theorem 1. *If a probability distribution π is a reversible distribution for a markov chain $\{X_n\}$, then it is also a stationary distribution.*

Proof. $(\pi' P)_j = \sum_{i \in S} \pi(i) p(i,j) = \sum_{i \in S} \pi(j) p(j,i) = \pi(j)$ $\square$

UNIQUENESS STATIONARY DISTRIBUTION

π is unique since P is primitive

π is the limit distribution

Theorem 2. *Let* $\{X_n\}$ *be a markov chain with a primitive (i.e. irreducible and aperiodic) transition matrix on a finite set, then there exist only one stationary distribution and it as also the limit distribution* $\lim_{n\to\infty} p^n(i,j) = \pi(j)\, \forall i \in S$

APPROXIMATION OF THE LIMIT DISTRIBUTION

The limit distribution π can be approximated using any starting distribution in logarithmic time

The approximation can be computed exploiting the ergodic coefficient, having fixed the approximation ε

THE END