Yield Spreads and Interest Rate Movements: A Bird's Eye View

John Y. Campbell, Robert J. Shiller

Yield Spreads and Interest Rate Movements: A Bird's Eye View

JOHN Y. CAMPBELL
Princeton University and NBER

and

ROBERT J. SHILLER
Yale University and NBER

This paper examines postwar U.S. term structure data and finds that for almost any combination of maturities between one month and ten years, a high yield spread between a longer-term and a shorter-term interest rate forecasts rising shorter-term interest rates over the long term, but a declining yield on the longer-term bond over the short term. This pattern is inconsistent with the expectations theory of the term structure, but is consistent with a model in which the spread is proportional to the value implied by the expectations theory.

1. INTRODUCTION

Does the slope of the term structure—the yield spread between longer-term and shorter-term interest rates—predict future changes in interest rates? And if so, is the predictive power of the yield spread in accordance with the expectations theory of the term structure?

These questions are important, both for forecasting interest rates and for interpreting shifts in the yield curve. If the expectations theory is an adequate description of the term structure, then rational expectations of future interest rates are the dominant force determining current long-term interest rates. On the other hand, if the expectations theory is very far from accurate, then predictable changes in excess returns must be the main influence moving the term structure. It makes sense to thoroughly explore the validity of the simple expectations theory before undertaking a detailed study of the sources of predictable time-variation in excess returns.[1]

The literature on the term structure contains a bewildering variety of answers to these questions. Almost all studies statistically reject the expectations theory of the term structure; but some studies suggest that the yield spread does predict interest rate movements in roughly the way one would expect if the expectations theory is true, while other studies reach the opposite conclusion. Different studies use different econometric methods, test different implications of the expectations theory, and look at different interest rate maturities.

---

1. We refer to "the" expectations theory of the term structure. One problem that has hampered empirical work is that in fact there are many different versions of the expectations theory, as emphasized by Cox, Ingersoll, and Ross (1981) and others. We have however argued elsewhere that these different expectations theories are in important respects very similar, and are all closely approximated by a single linear expectations theory (Shiller (1979), Shiller, Campbell, and Schoenholtz (1983), Campbell (1986)). Note also that we are including the hypothesis of rational expectations in our definition of the expectations theory. This contrasts with the usage of some authors (e.g. Froot (1989)).

In this paper we show that certain statements can be made quite generally. For almost any pair of maturities between one month and ten years, the following is true: When the yield spread between the longer-term interest rate and the shorter-term interest rate is relatively high, the yield on the longer-term bond tends to fall over the life of the shorter-term bond. This runs counter to the expectations theory. At the same time, shorter-term rates tend to rise over the life of the longer-term bond, in accordance with the expectations theory. In a nutshell, when the spread is high the long rate tends to fall and the short rate tends to rise.

The data set used here consists of continuously compounded yields on riskless pure discount bonds. These yields were calculated by McCulloch (1990) from raw data on U.S. Treasury bill, note and bond prices, measured over the period 1952:1–1987:2 at the end of each month.[2] We will present results for all possible pairs of maturities in the range 1, 2, 3, 4, 6, and 9 months and 1, 2, 3, 4, 5, and 10 years. Thus, the results in this paper are two-dimensional tables of evaluations of the linear expectations theory. While the main findings hold over all possible sets of maturities, there are some interesting differences between the behaviour of the short end of the term structure (maturities less than one year) and the long end of the term structure (maturities greater than one year). We discuss these further below.

The organization of the paper is as follows. In Section 2 we derive the implications of the expectations theory of the term structure for the relation between yield spreads and subsequent interest rate movements. We discuss several ways in which these implications can be tested, including regression methods and a modification of the vector autoregressive approach of Campbell and Shiller (1987). In Section 3 we apply these methods to the McCulloch term-structure data. We also use a Monte Carlo study to check the finite-sample properties of our procedures. In Section 4 we try to interpret our results further. We argue that one simple alternative, in which the yield spread equals its value under the expectations theory plus orthogonal noise, is not consistent with the data. We suggest another alternative, which makes the yield spread a constant multiple of its value under the expectations theory. This model could be generated by time-varying risk premia which are correlated with expected increases in short-term interest rates, or by a failure of rational expectations in our sample period.

2. THE EXPECTATIONS THEORY OF THE TERM STRUCTURE

The expectations theory of the term structure of interest rates is a relationship between a longer-term n-period interest rate $R_t^{(n)}$ and a shorter-term m-period interest rate $R_t^{(m)}$, where n/m is an integer. In the case of pure discount bonds, as with our data here, this is:

$$R_t^{(n)} = (1/k) \sum_{i=0}^{k-1} E_t R_{t+mi}^{(m)} + c, \qquad k = n/m. \tag{1}$$

Equation (1) states that the n-period rate is a constant, plus a simple average of the current and expected future m-period rates up to $n - m = (k-1)m$ periods in the future. Note that the sum of the coefficients of the m-period interest rates is one. The parameter c reflects a term premium, that is a predictable excess return on the n-period bond over the m-period bond. The term premium may vary with m and n but is assumed to be constant through time.

---

2. McCulloch's data actually begin in 1946:12, but we drop the data from the period before the Treasury Accord of 1951.

Equation (1) can be obtained directly if one assumes that expected continuously compounded yields to maturity on all discount bonds are equal, up to a constant; this is the approach taken by Fama (1984). Equation (1) can also be derived as a linear approximation to any of several different nonlinear expectations theories of the term structure. The approximation is quite adequate for most purposes (Shiller, Campbell, and Schoenholtz (1983), Campbell (1986)). For example, if one assumes that the expected total return over m periods on buying an n-period bond and selling it m periods later equals the return on holding an m-period bond to maturity plus a constant, then one finds that the expectation of a nonlinear expression in $R_t^{(n)}$ and $R_{t+m}^{(n-m)}$ equals $R_t^{(m)}$ plus a constant. Linearizing this expression around $R_t^{(n)} = R_{t+m}^{(n-m)} = 0$, one gets a rational expectations model that if solved forward yields (1).

It should be noted that (1) is a time-consistent model. If the model holds for $m = 1$ and all n, then it holds for all $m \geqq 1$ and all n. This is an important property which many time-series models do not possess. An AR (2) model with monthly data is not consistent with an AR (2) model with quarterly data, for example. There is no such problem here.

The spread as a forecast of changes in interest rates

Our purpose is to state as simply as possible what elements of truth can be found in the expectations theory of the term structure. We therefore concentrate our attention on the behaviour through time of a simple measure of the shape of the term structure: the spread between the n-period rate and the m-period rate, $S_t^{(n,m)} = R_t^{(n)} - R_t^{(m)}$. The spread is of course proportional to the slope of the term structure between m and n.

The expectations theory of the term structure implies that the spread is a constant risk premium, plus an optimal forecast of changes in future interest rates. We can test the model by regressing the appropriate changes onto the spread and testing whether the coefficient equals one. And we can, apart from testing the model, evaluate its usefulness by checking to what extent the spread resembles an optimal forecast of the changes in interest rates.

There are several ways to write the spread as a forecast of future changes in interest rates. First, the spread predicts the m-period change in yield on the longer-term bond. This bond has n periods to maturity at time t, so it has $n - m$ periods to maturity at time $t + m$. According to the expectations theory,

$$s_t^{(n,m)} \equiv (m/(n-m))S_t^{(n,m)} = E_t R_{t+m}^{(n-m)} - R_t^{(n)}, \tag{2}$$

where we use $s_t^{(n,m)}$ to denote a maturity-specific multiple of the yield spread, and we are suppressing constant terms for simplicity. The intuition behind equation (2) is that if the yield on the n-period bond is expected to rise over the next m periods, this will give a capital loss to holders of the bond. To equate expected returns over m periods, the n-period bond has to have a higher current yield than the m-period instrument. One may test (2) by regressing $R_{t+m}^{(n-m)} - R_t^{(n)}$ onto a constant and its predicted value $s_t^{(n,m)}$ defined in (2) as: $s_t^{(n,m)} \equiv (m/(n-m))S_t^{(n,m)}$. Since we are assuming that expectations are rational, the slope coefficient should be one.

Second, by subtracting $R_t^{(m)}$ from both sides of equation (1) and rearranging terms, one can show that the spread forecasts a weighted average $S_t^{(n,m)*}$ of changes in shorter-term (m-period) interest rates over n periods:

$$S_t^{(n,m)} = E_t S_t^{(n,m)*},$$
$$S_t^{(n,m)*} \equiv (1/k) \sum_{i=1}^{k-1}\left(\sum_{j=1}^{i}\Delta^m R_{t+jm}^{(m)}\right) = \sum_{i=1}^{k-1}(1 - i/k)\Delta^m R_{t+im}^{(m)}. \tag{3}$$

The notation Δ^m indicates that a change is measured over m periods, so for example $\Delta^m R_{t+m}^{(m)} = R_{t+m}^{(m)} - R_t^{(m)}$.

The variable $S_t^{(n,m)*}$ may be called the "perfect-foresight spread", since it is the spread that would obtain, given the model, if there were perfect foresight about future interest rates. With perfect foresight, if m-period rates are going to rise over the life of the n-period bond, then the n-period yield needs to be higher than the current m-period yield to equate the returns on the n-period bond, held to maturity, and a sequence of m-period bonds. Below we shall regress the perfect-foresight spread onto a constant and the actual spread to evaluate the model. Again, because of the rational expectations assumption, the slope coefficient should be one.

Equations (2) and (3) are complete characterizations of the expectations theory of the term structure; if (2) holds for all m and n, then (3) holds for all m and n, and vice versa. However it is important to note that for any *particular* values of m and n, the validity of (2) does not generally imply the validity of (3) or vice versa.

An exception to this statement occurs when $n = 2m$, a case that is often studied in empirical work.[3] In this case equations (2) and (3) take particularly simple forms because $R_{t+m}^{(n-m)}$ equals $R_{t+m}^{(m)}$ in equation (2), while the weighted sum on the right hand side of (3) has only one element. Equation (2) becomes

$$S_t^{(n,m)} = E_t R_{t+m}^{(m)} - R_t^{(n)},\tag{4}$$

and (3) becomes

$$S_t^{(n,m)} = (1/2)E_t \Delta^m R_{t+m}^{(m)} = (1/2)E_t[R_{t+m}^{(m)} - R_t^{(m)}].\tag{5}$$

Equation (4) says that the spread equals the difference between the optimal forecast of the m-period rate m periods from now and the n-period rate today, while equation (5) says that the spread equals one-half the optimal forecast of the change in the m-period rate over the next m periods. If (4) holds for a particular m and n, then (5) must also hold, and vice versa. (To see this, just subtract $R_t^{(m)}$ from both sides of (4) and rearrange.)

Since (4) and (5) are special cases of (2) and (3), they can be tested using the same regression approach. If the regression coefficient for the test of (4) is written b, the regression coefficient for the test of (5) will be $(1+b)/2$.

The theoretical spread

Regression tests of the expectations theory have the great merit of simplicity. But they also have some serious disadvantages. First, the regression of the perfect-foresight spread onto the actual spread involves n-period overlapping errors. One only has an entirely independent observation of the forecast power of the term structure every n periods. While econometric methods are available to correct regression standard errors for overlap, they do not work well when the degree of overlap is large relative to the sample size. (See for example Stock and Richardson (1989) and Hodrick (1990).) Since n can be as large as 10 years, and we have only 35 years of data, this is a worrisome problem.

Second, regression tests do not tell us how similar are the movements of the actual spread to the movements implied by the expectations theory. We would like to evaluate the ability of the expectations theory to explain the shape of the term structure, and regression tests are not well suited for this purpose.

---

3. See for example Shiller, Campbell, and Schoenholtz (1983), who looked at 3- and 6-month Treasury bills.

In earlier work (Campbell and Shiller (1987)), we proposed a vector-autoregressive (VAR) approach for evaluating present value models. That paper dealt with the case in which n is infinite, but the approach can easily be modified to handle a finite value of n. The VAR approach avoids the need to estimate regressions with overlapping errors. The VAR includes the 1-period change in the m-period interest rate, and the actual yield spread. From the estimated VAR coefficients, one can compute the optimal forecast of m-period interest rate changes over any horizon; the long-run behaviour of interest rates is inferred from their short-run behaviour in the sample period, rather than being estimated directly. The appropriate weighted average of forecast interest rate changes (the "theoretical spread") can be calculated, and compared with the actual spread. If the expectations theory is true, the two variables should be the same.

The details of the VAR approach are given in Campbell and Shiller (1987); here we merely summarize the method and show how we can apply it to the expectations theory with a finite horizon n. The first step in the analysis is to note that equation (3) can be projected onto a subset of the information set used by market participants. This is necessary because of course the true information set of the market is unobservable. Provided that we use an information subset that includes the yield spread $S_t^{(n,m)}$ itself, equation (3) takes the same form when projected onto the subset. It says that the observed yield spread should equal the optimal forecast of future increases in short-term rates, conditional on the information subset used in our test.

Next we specify the form of the information subset more precisely. We assume that $\Delta R_t^{(m)}$ is a stationary stochastic process, and that the expected excess return on an n-period bond over an m-period bond is stationary. These assumptions imply that $x_t \equiv [\Delta R_t^{(m)}, S_t^{(n,m)}]$ is a stationary vector-stochastic process. We shall use the history of x_t as our information subset.

We assume that x_t can be represented as a p-th-order VAR. This system can be rewritten as a first-order VAR in the companion form $z_t = A z_{t-1} + u_t$, where z_t has $2p$ elements, first $\Delta R_t^{(m)}$ and $p-1$ lags and then $S_t^{(n,m)}$ and $p-1$ lags. The vector z_t summarizes the whole history of x_t. Multi-period interest rate forecasts are easily computed from the companion form, since $E[z_{t+k} | x_t, x_{t-1}, \dots] = E[z_{t+k} | z_t] = A^k z_t$.

We next define vectors g and h such that $g'z_t = S_t^{(n,m)}$ and $h'z_t = \Delta R_t^{(m)}$. (These vectors have $2p$ elements, all of which are zero except for the first element of h and the $p+1$-st element of g, which equal one.) Then we can use (3) to compute the VAR forecast of the perfect-foresight spread; we write this forecast $S_t'^{(n,m)}$. Tedious algebra shows that $S_t'^{(n,m)}$ can be expressed as[4]

$$S_t'^{(n,m)} \equiv h'A[I - (m/n)(I - A^n)(I - A^m)^{-1}](I - A)^{-1} z_t. \tag{6}$$

We call $S_t'^{(n,m)}$ the "theoretical spread", since it is the spread which would obtain if the expectations theory were true. Equation (3), projected onto the information subset z_t, implies that

$$S_t^{(n,m)} = g'z_t = S_t'^{(n,m)}. \tag{7}$$

Note that if the expectations theory of the term structure is true, (7) should hold whatever information set economic agents are using. The intuitive explanation is that if term premia are constant, all the relevant information of market participants is embodied in the yield spread $S_t^{(n,m)}$, which is included in the VAR system. Of course, if the expectations theory

---

4. Note that this expression reduces to the much simpler formula in Campbell and Shiller (1987) when n is infinite.

is not true, then the VAR system may not adequately summarize the information available to the market.

The equality of the actual spread and the theoretical spread puts a set of nonlinear restrictions on the coefficients of the estimated VAR. These can be tested formally using a Wald test (Campbell and Shiller (1987)). We do not pursue this approach here, since the regression methods discussed above are simpler if one's purpose is merely to test the model. Instead, we use the VAR to compare the historical behaviour of $S_t^{(n,m)}$ and $S_t'^{(n,m)}$. We compute the correlation of $S_t^{(n,m)}$ and $S_t'^{(n,m)}$, and the ratio of their standard deviations. If $S_t^{(n,m)}$ equals $S_t'^{(n,m)}$, of course, the correlation and the standard deviation ratio should both equal one.

Some previous results

We found in our earlier work (Shiller (1979), Shiller, Campbell and Schoenholtz (1983)) that for very large n and small m when $R_{t+m}^{(n-m)} - R_t^{(n)}$ is regressed on $s_t^{(n,m)}$ then the coefficient of $s_t^{(n,m)}$ tends often to be significantly different from one. In fact, the point estimate is negative. We concluded that the theory was very far off-track; that the expectations theory of the term structure is just wrong.[5]

In later work (Campbell and Shiller (1987)), we used a different metric to evaluate the expectations theory for large n and small m, and we found that the expectations theory seemed to work fairly well. We computed the correlation between $S_t^{(n,m)}$ and $S_t'^{(n,m)}$ and found it to be quite high. This suggests that the actual spread behaves much as it should according to the expectations model.[6] The correlation between $S_t^{(n,m)}$ and $S_t'^{(n,m)}$ can be high and still we may get a wrong sign in a regression test of equation (2). This can happen, for example, if $S_t^{(n,m)}$ equals $S_t'^{(n,m)}$ plus a small serially uncorrelated noise term, because the noise term appears on the left- and right-hand sides of (2) with opposite signs.

Other recent work has also emphasized the ability of the yield spread to forecast short rate changes over long horizons. Fama and Bliss (1987) set $m = 1$ year and $n = 2$, 3, and 4 years. They worked with "forward premia", which are linear combinations of two different yield spreads. According to the expectations theory, forward premia should forecast unweighted averages of changes in short rates, as opposed to the weighted averages forecast by yield spreads. Fama and Bliss regressed the appropriate short rate changes onto forward premia, and found that the forecasting power of the term structure improves as the horizon n increases. They attributed this to a slowly mean-reverting interest rate process that is more easily forecast over long time periods than over short intervals.

A largely unrelated literature has looked at the short end of the term structure, with n up to 12 months. Shiller, Campbell, and Schoenholtz (1983) found that the yield spread between 3- and 6-month Treasury bill rates helps to forecast the change in the 3-month bill rate, but not as strongly as the expectations theory requires. Fama (1984) found some evidence that the slope of the term structure predicts interest rate changes over a few months, but the predictive power seemed to decay rapidly with the horizon.[7]

5. See also Mankiw and Summers (1984) and Froot (1989).
6. Shiller (1989) has used plots of the actual spread and the perfect foresight spread to make the same point.
7. Mishkin (1988a) has updated the results in Fama (1984). Fama (1990) and Mishkin (1988b, 1990) have extended the analysis to look at the forecast power of the yield curve, at both short and long horizons, for future inflation rates. See Shiller (1990) for a survey of other research on the term structure.

3. DATA AND EMPIRICAL RESULTS

The McCulloch monthly term structure data (1990) give pure discount (zero coupon) bond yields for U.S. Government securities over the period 1946:12–1987:2. We use the sample 1952:1–1987:2, to allow for lags and to exclude data from before the Treasury Accord of 1951. The data include maturities of 0, 1, 2, 3, 4, 5, 6, and 9 months, and 1, 2, 3, 4, 5, and 10 years. Longer maturities are available only for part of the sample. These data are continuously compounded yields to maturity.

Of course, pure discount government bonds of long maturity do not exist. But existing government bonds may be regarded as portfolios of pure discount bonds, bonds maturing on all coupon dates and the maturity date. One may suppose that they are priced, except for some tax considerations, as the sum of the values of the constituent bonds in the portfolio. One can therefore infer the prices (and thus the yields) of the constituent bonds. There are problems in making this inference: there may be more than one way to infer the yield of a certain discount bond (giving possibly different answers in practice) and no way to infer the yield of other discount bonds (since coupon and maturity dates may not be evenly spaced through time). Thus, McCulloch (1990) inferred the discount yields by an interpolation method using cubic splines.[8]

McCulloch's data are very clean in the sense that they are unaffected by differing coupons, coupon dates, or differing compounding conventions across maturities. They are ideally suited to the kind of analysis we wish to do here over a broad range of maturities. The data are not interval averaged, but are observed at the end of each month.

Forecasting the change in the longer-term yield

Table 1(a) confirms that the slope of the term structure between almost *any two* maturities gives the wrong direction of forecast for the change in yield of the longer-term (n-period) bond over the life of the shorter-term (m-period) bond.[9] Asymptotic standard errors, which have been corrected for heteroskedasticity and equation error overlap in the manner of Hansen and Hodrick (1980) and White (1984), show that almost all the coefficients are significantly different from one at conventional significance levels. Thus, earlier conclusions in Shiller (1979) and Shiller, Campbell and Schoenholtz (1983) for some maturities are found to extend to just about all maturity pairs m and n. By this metric, the expectations theory of the term structure is a resounding failure.

The general tendency for wrong signs to appear throughout the table is robust to the sample period. In Table 1(b) we report results for the case m = 1 over a variety of subsamples, including the 1952–1978 period (the longest possible subsample which avoids the 1979 monetary policy regime shift) and the shorter periods 1952–1959, 1960–1969, 1970–1978 and 1979–1987. We find a predominance of wrong signs in every period except 1952–1959.

As a check on the McCulloch data, these results can be compared with earlier results using different monthly data on U.S. Treasury obligations. Shiller, Campbell, and Schoenholtz (1983) found that when the three-month change in the three-month rate is regressed on the predicted change implicit in the spread between six-month and three-month rates, with data January 1959 to June 1979, the slope coefficient was 0·285. Here, the slope

---

8. Fama and Bliss (1987) inferred yields on pure discount bonds using the assumption that instantaneous forward rates are step functions of maturity. This is an alternative to McCulloch's procedure. Our results below are similar to those of Fama and Bliss for the maturities they consider.

9. The lower left-hand part of the table, where n is large and m is small, uses the approximation that $R_{t+m}^{(n-m)} = R_{t+m}^{(n)}$. In the rest of the table we give results only where we observe $R_{t+m}^{(n-m)}$ directly.

TABLE 1(a)

Regression of $R_{t+m}^{(n-m)} - R_t^{(n)}$ on predicted change $s_t^{(n,m)}$

	m							
n	1	2	3	4	6	12	24	60
2	0·002 (0·238) 0·000 0·000	—	—	—	—	—	—	—
3	−0·176 (0·362) 0·001 0·001	−0·361 (0·502)	—	—	—	—	—	—
4	−0·437 (0·469) 0·000 0·002	−0·611 (0·562) 0·011 0·005	−0·452 (0·366)	—	—	—	—	—
6	−1·029 (0·537) 0·000 0·000	−1·276 (0·557) 0·000 0·000	−1·294 (0·400) 0·001 0·000	−1·203 (0·309)	—	—	—	—
9	−1·219[a] (0·598) 0·019 0·000	—	−1·682 (0·486) 0·004 0·000	−1·482 (0·311)	−0·654 (0·508)	—	—	—
12	−1·381[a] (0·683) 0·038 0·000	−1·592[a] (0·712) 0·064 0·000	−1·967 (0·601) 0·003 0·000	—	−0·913 (0·657) 0·056 0·013	—	—	—
24	−1·815[a] (1·151) 0·122 0·017	−1·919[a] (1·142) 0·099 0·011	−1·694[a] (0·939) 0·114 0·007	−1·482[a] (0·842) 0·150 0·010	−0·893[a] (0·743) 0·134 0·041	−1·034 (0·620) 0·047 0·042	—	—
36	−2·239[a] (1·444) 0·122 0·037	−2·164[a] (1·462) 0·104 0·032	−1·922[a] (1·210) 0·109 0·022	−1·692[a] (1·065) 0·128 0·035	−1·184[a] (0·877) 0·128 0·034	−1·396 (0·883) 0·034 0·034	−0·465 (1·086)	—
48	−2·665[a] (1·634) 0·087 0·023	−2·561[a] (1·662) 0·055 0·046	−2·208[a] (1·392) 0·091 0·028	−1·960[a] (1·206) 0·093 0·032	−1·447[a] (0·991) 0·129 0·033	−1·736 (1·027) 0·019 0·033	−0·725 (1·233) 0·115 0·353	—
60	−3·099[a] (1·749) 0·079 0·019	−2·941[a] (1·795) 0·070 0·043	−2·525[a] (1·523) 0·080 0·030	−2·276[a] (1·318) 0·088 0·028	−1·750[a] (1·096) 0·099 0·038	−2·022 (1·205) 0·024 0·049	−0·811 (1·369)	—
120	−5·024[a] (2·316) 0·029 0·009	−4·695[a] (2·424) 0·032 0·024	−4·298[a] (2·107) 0·028 0·015	−3·944[a] (1·851) 0·047 0·019	−3·198[a] (1·673) 0·060 0·027	—	—	4·575 (1·926) 0·146 0·331

Notes. The first number in any set is the estimated regression slope coefficient of $R_{t+m}^{(n-m)} - R_t^{(n)}$ onto $s_t^{(n,m)} \equiv (m/(n-m))S_t^{(n,m)}$. According to the expectations theory, this coefficient should equal one. Constant terms (not shown) are included in all regressions. Hansen-Hodrick standard errors are below estimated coefficients, in parentheses. For each regression, the sample is the longest possible using data from 1952:1 through 1987:2. Where n/m is an integer the table also gives, below the standard errors, two numbers from a Monte Carlo experiment. The first is the fraction of 1000 runs which produced an estimated regression coefficient which was further away from one in the same direction than the coefficient obtained in the data. The second number is the fraction of 1000 runs in which a 2-sided t-test of the hypothesis that the coefficient equals one rejected the null more strongly than the t-test on the actual data.

[a] Uses the approximation that $R_{t+m}^{(n-m)} = R_{t+m}^{(n)}$.

TABLE 1(b)

Regression of $R_{t+1}^{(n-1)} - R_t^{(n)}$ on predicted change $s_t^{(n,1)}$: subsamples

	Sample period (number of observations)					
n	1952-1987 (421)	1952-1978 (323)	1952-1959 (95)	1960-1969 (119)	1970-1978 (107)	1979-1987 (97)
2	0·002 (0·238)	−0·267 (0·151)	−0·261 (0·193)	−0·089 (0·253)	−0·436 (0·419)	0·347 (0·342)
3	−0·176 (0·362)	−0·471 (0·223)	−0·194 (0·284)	−0·381 (0·311)	−0·855 (0·691)	0·167 (0·577)
4	−0·437 (0·469)	−0·509 (0·283)	−0·101 (0·388)	−0·481 (0·333)	−0·850 (0·893)	−0·287 (0·789)
6	−1·029 (0·537)	−0·537 (0·340)	−0·121 (0·542)	−0·567 (0·393)	−0·981 (0·982)	−1·345 (0·902)
9	−1·219[a] (0·598)	−0·394[a] (0·458)	0·590[a] (0·633)	−0·598[a] (0·544)	−0·907[a] (1·124)	−1·826[a] (1·066)
12	−1·381[a] (0·683)	−0·672[a] (0·598)	0·771[a] (0·752)	−1·044[a] (0·661)	−1·216[a] (1·307)	−1·778[a] (1·237)
24	−1·815[a] (1·151)	−1·031[a] (0·986)	1·796[a] (1·557)	−2·876[a] (1·284)	−1·063[a] (1·594)	−2·218[a] (2·052)
36	−2·239[a] (1·444)	−1·210[a] (1·187)	3·021[a] (2·379)	−3·840[a] (1·891)	−1·245[a] (1·774)	−2·791[a] (2·431)
48	−2·665[a] (1·634)	−1·272[a] (1·326)	3·807[a] (2·852)	−4·373[a] (2·254)	−1·293[a] (1·936)	−3·468[a] (2·714)
60	−3·099[a] (1·749)	−1·483[a] (1·442)	4·138[a] (3·264)	−4·886[a] (2·535)	−1·424[a] (2·083)	−4·052[a] (2·894)
120	−5·024[a] (2·316)	−2·263[a] (1·869)	3·099[a] (4·801)	−6·029[a] (3·796)	−2·103[a] (2·650)	−6·830[a] (3·817)

Notes. This table gives the same regression coefficients and standard errors as Table 1(a), except that $m = 1$ throughout the table and results are reported for subsamples. No Monte Carlo results are reported.
[a] See notes to Table 1(a).

coefficient in Table 1(a) for $m = 3$ and $n = 6$ is $b = -1·279$. The implied slope coefficient for the regression run by Shiller, Campbell, and Schoenholtz is $(1 + b)/2 = -0·140$. If we use the January 1959 to June 1979 sample our estimated b is $-0·302$ so that $(1 + b)/2 = 0·348$, which compares reasonably well with the earlier result. Fama (1984) found that when the actual change over one month in the one-month rate is regressed using data 1959-1982 on the predicted change the slope coefficient is $0·46$. Our $(1 + b)/2$ from Table 1(a) is $0·51$, and if b is reestimated using their sample period $b = -0·302$ and $(1 + b)/2 = 0·54$. Fama and Bliss (1987) found that when the actual change in the one-year rate over one year is regressed 1964-1984 on the predicted change the slope coefficient is $0·09$. Using our Table 1(a), $(1 + b)/2 = -0·14$, using their sample period $b = -0·882$ and $(1 + b)/2 = 0·06$. These comparisons show that the McCulloch data give similar results to results with other data sets.

Forecasting changes in short rates

Table 2 shows the results of regressing $S_t^{(n,m)*}$ on $S_t^{(n,m)}$ for combinations of n and m that are represented in the McCulloch data, for which n is an integer multiple of m. The

TABLE 2

Slope coefficients in regression of $S_t^{(n,m)*}$ on $S_t^{(n,m)}$

n	m=1	2	3	4	6	12	24	60
2	0·501 (0·119) 0·000 0·000	—	—	—	—	—	—	—
3	0·446 (0·190) 0·000 0·006	—	—	—	—	—	—	—
4	0·436 (0·238) 0·001 0·022	0·195 (0·281) 0·011 0·005	—	—	—	—	—	—
6	0·237 (0·167) 0·000 0·000	0·021 (0·163) 0·010 0·001	−0·147 (0·200) 0·001 0·000	—	—	—	—	—
9	0·151 (0·165) 0·001 0·000	—	−0·008 (0·147) 0·015 0·000	—	—	—	—	—
12	0·161 (0·228) 0·006 0·019	0·078 (0·192) 0·033 0·004	0·044 (0·189) 0·023 0·005	−0·056 (0·185) 0·022 0·000	0·044 (0·329) 0·056 0·013	—	—	—
24	0·302 (0·212) 0·031 0·067	0·287 (0·210) 0·096 0·063	0·269 (0·230) 0·116 0·086	0·229 (0·262) 0·117 0·098	0·186 (0·324) 0·073 0·100	−0·017 (0·372) 0·047 0·042	—	—
36	0·614 (0·230) 0·151 0·318	0·642 (0·243)* 0·211 0·368	0·610 (0·101) 0·225 0·070	0·568 (0·174) 0·210 0·194	0·526 (0·225) 0·193 0·223	0·257 (0·408) 0·119 0·250	—	—
48	0·873 (0·291) 0·256 0·817	0·929 (0·271) 0·289 0·868	0·951 (0·240) 0·299 0·898	0·959 (0·205) 0·321 0·913	0·942 (0·088) 0·299 0·697	0·720 (0·335) 0·230 0·613	0·137 (0·617) 0·115 0·353	—
60	1·232 (0·182) 0·581 0·501	1·289 (0·168) 0·574 0·418	1·292 (0·161) 0·576 0·370	1·297 (0·141) 0·594 0·334	1·242 (0·161) 0·630 0·474	1·130 (0·193) 0·677 0·698	—	—
120	1·157 (0·094) 0·713 0·583	1·207 (0·093) 0·685 0·463	1·223 (0·095) 0·667 0·437	1·227 (0·097) 0·681 0·443	1·228 (0·101) 0·690 0·429	1·274 (0·113) 0·673 0·423	1·345 (0·169) 0·618 0·429	2·788 (0·963) 0·146 0·331

Notes. $S_t^{(n,m)*}$, the perfect-foresight spread, is defined in equation (3) in the text. The first element in each set is the slope coefficient in a regression with a constant term, and that in parentheses is the Hansen–Hodrick standard error. By the expectations theory, the slope coefficient should be one. The sample period for each element is the longest possible sample using data from 1952:1 to 1987:2. Since computation of $S_t^{(n,m)*}$ requires data extending $n - m$ periods into the future, the sample in the regression ends $n - m$ months before 1987:2. The table also gives two numbers from a Monte Carlo experiment. These are constructed in the same way as the Monte Carlo results reported in Table 1(*a*).

* Newey–West (1987) correction used because Hansen–Hodrick procedure gave a negative standard error on constant or spread.

regression coefficients should be one if the expectations hypothesis is valid. In fact, the coefficients are almost always positive but also deviate substantially from one when the maturity n of the longer-term bond is below 3 or 4 years. At this short end of the term structure, asymptotic standard errors imply rejection of the expectations theory at conventional significance levels. At the long end of the term structure, the regression coefficients are very close to one and the expectations theory is not rejected.

We thus see an apparent paradox: the slope of the term structure almost always gives a forecast in the wrong direction for the short-term change in the yield on the longer bond, but gives a forecast in the right direction for long-term changes in short rates. We next use our VAR procedures in order to judge how the spread moves through time in relation to an unrestricted forecast of changes in short rates.

Table 3(a) reports correlations between the actual spread $S_t^{(n,m)}$ and the estimated theoretical spread $S_t^{\prime(n,m)}$. For each estimated correlation we first ran a fourth-order vector autoregression for the vector $[\Delta R_t^{(m)}, S_t^{(n,m)}]'$ using monthly data from January 1952 to February 1987 and used the result to form the A matrix for the companion form. This estimate of A was then substituted into equation (7) to arrive at $S_t^{\prime(n,m)}$, and the correlation with $S_t^{(n,m)}$ computed. An asymptotic standard error was calculated in the way described in Campbell and Shiller (1987). The correlation is almost always positive and often very high. This result also holds up in subsamples, as shown in Table 3(b).

Table 4(a) shows the standard deviation of $S_t^{\prime(n,m)}$ divided by that of $S_t^{(n,m)}$. We find that the coefficient is typically around one-half, regardless of m and n. Thus, the spread is too variable to accord with the simple expectations model.[10] Similar results hold in all subsamples (Table 4(b)), except for large n in the 1952–1959 period.

These results can be compared to results in Table 2 in the following way. The product of an element in Table 3(a) (the correlation of $S_t^{\prime(n,m)}$ and $S_t^{(n,m)}$) and the corresponding element in Table 4(a) (the standard deviation of $S_t^{\prime(n,m)}$ divided by the standard deviation of $S_t^{(n,m)}$) is the regression coefficient when $S_t^{\prime(n,m)}$ is regressed on $S_t^{(n,m)}$. This regression coefficient should be (except for sampling error) the same as that in Table 2. We find that they are about the same when n is small, but diverge substantially when n is large.

There are several possible explanations for this discrepancy. First, in order to regress $S_t^{(n,m)*}$ on $S_t^{(n,m)}$ we must truncate the sample enough to allow computation of $S_t^{(n,m)*}$ which requires data $n - m$ periods into the future. (For this reason we do not try to apply the approach of Table 2 in subsamples.) To do the calculations in Tables 3 and 4 there is no such need to truncate the sample. Thus, the discrepancy in results might just be due to different sample periods, and our estimates with shorter samples in Tables 3(b) and 4(b) show that this is a large part of the explanation. In Tables 3(b) and 4(b), the expectations theory performs noticeably better in the shorter sample period 1952–1978 than in the full sample period 1952–1987; but at the bottom of Table 2 the full sample has been truncated by almost ten years, so the effective sample period is roughly 1952–1978.[11]

A second possible reason for the discrepancy is that perhaps our vector autoregression truncated after four months is too short, and thus our estimate of $S_t^{\prime(n,m)}$ is subject to error. In our previous work we have suggested that long lags may be useful in forecasting interest rates (Campbell and Shiller (1984)). When we increase the lag length to eight

10. This statement is subject to a caveat. As discussed above, if the expectations theory of the term structure is false, then the VAR system is not guaranteed to summarize all the relevant information available to market participants. For more on this point, see Beltratti and Shiller (1990).

11. Campbell and Shiller (1987) also found that the expectations theory performed better in the pre-1979 period.

TABLE 3(a)

Correlation of $S_t^{\prime(n,m)}$ and $S_t^{(n,m)}$

n	m							
	1	2	3	4	6	12	24	60
2	0·736	—	—	—	—	—	—	—
	(0·148)							
	0·000							
	0·094							
3	0·761	—	—	—	—	—	—	—
	(0·190)							
	0·002							
	0·437							
4	0·720	0·502	—	—	—	—	—	—
	(0·213)	(0·451)						
	0·001	0·063						
	0·285	0·545						
6	0·486	0·058	−0·355	—	—	—	—	—
	(0·373)	(0·566)	(0·556)					
	0·003	0·012	0·000					
	0·210	0·118	0·015					
9	0·374	—	−0·156	—	—	—	—	—
	(0·421)		(0·904)					
	0·005		0·008					
	0·181		0·269					
12	0·391	0·282	0·126	−0·072	−0·111	—	—	—
	(0·468)	(0·909)	(1·211)	(1·309)	(1·494)			
	0·007	0·049	0·017	0·017	0·015			
	0·237	0·603	0·663	0·576	0·514			
24	0·543	0·629	0·612	0·512	0·409	0·212	—	—
	(0·764)	(0·941)	(1·148)	(1·565)	(1·960)	(1·865)		
	0·021	0·088	0·102	0·108	0·062	0·023		
	0·749	0·920	0·953	0·964	0·980	0·930		
36	0·770	0·851	0·860	0·833	0·776	0·645	—	—
	(0·531)	(0·431)	(0·442)	(0·594)	(0·917)	(1·311)		
	0·021	0·088	0·102	0·108	0·062	0·023		
	0·749	0·920	0·953	0·964	0·980	0·930		
48	0·867	0·920	0·930	0·923	0·900	0·839	0·896	—
	(0·328)	(0·210)	(0·189)	(0·231)	(0·363)	(0·580)	(0·348)	
	0·101	0·192	0·232	0·249	0·224	0·182	0·301	
	0·861	0·801	0·763	0·804	0·885	0·908	0·852	
60	0·912	0·948	0·956	0·954	0·939	0·893	—	—
	(0·218)	(0·128)	(0·109)	(0·128)	(0·209)	(0·375)		
	0·115	0·213	0·271	0·270	0·232	0·206		
	0·810	0·737	0·731	0·766	0·863	0·869		
120	0·979	0·986	0·988	0·988	0·984	0·975	0·983	0·990
	(0·045)	(0·027)	(0·021)	(0·023)	(0·038)	(0·062)	(0·041)	(0·020)
	0·159	0·250	0·290	0·315	0·284	0·235	0·312	0·624
	0·667	0·642	0·568	0·593	0·698	0·680	0·708	0·623

Notes. This table gives the correlation coefficients of $S_t^{\prime(n,m)}$ with $S_t^{(n,m)}$ and estimated standard errors (in parentheses). $S_t^{\prime(n,m)}$ is computed from equation (6) in the text based on a vector autoregression starting in 1952:1 and ending in 1987:2. The vector autoregression had four lags. The table also gives two numbers from a Monte Carlo experiment. The first number is the fraction of 1000 runs in which the difference between the estimated correlation and one was larger than in the actual data. The second number is the fraction of 1000 runs in which the difference between the estimated correlation and one, divided by its standard error, was larger than in the actual data.

TABLE 3(b)

Correlation of $S_t^{'(n,1)}$ and $S_t^{(n,1)}$: subsamples

	Sample period (number of observations)					
n	1952–1987 (421)	1952–1978 (323)	1952–1959 (95)	1960–1969 (119)	1970–1978 (107)	1979–1987 (97)
2	0·738 (0·148)	0·626 (0·115)	0·668 (0·134)	0·785 (0·170)	0·303 (0·169)	0·696 (0·138)
3	0·763 (0·189)	0·585 (0·172)	0·717 (0·192)	0·687 (0·218)	0·256 (0·378)	0·707 (0·184)
4	0·723 (0·212)	0·604 (0·204)	0·729 (0·254)	0·589 (0·283)	0·538 (0·406)	0·675 (0·228)
6	0·493 (0·372)	0·583 (0·279)	0·712 (0·353)	0·513 (0·421)	0·657 (0·495)	0·395 (0·348)
9	0·386 (0·419)	0·561 (0·394)	0·742 (0·382)	0·417 (0·646)	0·628 (0·638)	0·282 (0·383)
12	0·404 (0·463)	0·587 (0·468)	0·801 (0·342)	0·240 (0·839)	0·695 (0·653)	0·301 (0·504)
24	0·558 (0·743)	0·829 (0·349)	0·963 (0·088)	−0·145 (0·986)	0·925 (0·241)	0·260 (0·984)
36	0·779 (0·509)	0·922 (0·180)	0·987 (0·021)	−0·129 (1·254)	0·971 (0·103)	0·388 (1·203)
48	0·872 (0·313)	0·968 (0·079)	0·991 (0·009)	−0·087 (1·623)	0·991 (0·035)	0·513 (1·181)
60	0·914 (0·210)	0·986 (0·037)	0·993 (0·007)	0·116 (2·087)	0·997 (0·011)	0·626 (1·006)
120	0·979 (0·044)	0·997 (0·007)	0·996 (0·004)	0·800 (1·048)	0·999 (0·003)	0·867 (0·356)

Notes. This table gives the same statistics as Table 3(a), except that $m = 1$ throughout the table and results are reported for subsamples. No Monte Carlo results are reported.

months, we find that the estimated standard deviation ratio is roughly unchanged; the estimated correlation tends to fall, but is less precisely estimated. Overall, the increase in lag length does not reduce the discrepancy between the results in Table 2 and those in Tables 3 and 4.

A third explanation for the discrepancy is that the asymptotic standard errors reported in Table 2 may understate the possible effects of sampling error on the regression slope coefficients in this table. Below we use a Monte Carlo experiment to confirm that this point is also important.

The effect of the forecast horizon

Fama and Bliss (1987) have emphasized that the forecast power of the term structure for changes in short rates improves as the forecast horizon increases from 2 years to 5 years. Our study of the full range of maturities confirms this result, but shows that in fact below 1 year the forecast power deteriorates with the horizon. The forecast power reaches its minimum at 9 to 12 months, and then starts to improve. This effect can be seen in the "U-shaped" pattern of coefficients as one moves down the first column of Table 2, Table 3(a), or Table 4(a). It is evident also in most of the subsamples in Tables 3(b) and 4(b).

REVIEW OF ECONOMIC STUDIES

TABLE 4(a)

$$\sigma(S_t^{\prime(n,m)})/\sigma(S_t^{(n,m)})$$

				m				
n	1	2	3	4	6	12	24	60
2	0·681 (0·136) 0·000 0·022	—	—	—	—	—	—	—
3	0·586 (0·145) 0·000 0·008	—	—	—	—	—	—	—
4	0·607 (0·162) 0·000 0·024	0·388 (0·216) 0·000 0·009	—	—	—	—	—	—
6	0·501 (0·145) 0·000 0·004	0·357 (0·196) 0·000 0·002	0·404 (0·225) 0·001 0·024	—	—	—	—	—
9	0·424 (0·129) 0·000 0·000	—	0·317 (0·168) 0·000 0·001	—	—	—	—	—
12	0·382 (0·119) 0·000 0·000	0·280 (0·158) 0·000 0·000	0·263 (0·153) 0·000 0·000	0·285 (0·151) 0·000 0·000	0·332 (0·155) 0·000 0·000	—	—	—
24	0·303 (0·135) 0·000 0·000	0·255 (0·249) 0·000 0·017	0·236 (0·283) 0·000 0·020	0·224 (0·262) 0·000 0·013	0·238 (0·217) 0·000 0·004	0·272 (0·135) 0·000 0·000	—	—
36	0·308 (0·225) 0·000 0·020	0·289 (0·353) 0·000 0·116	0·279 (0·398) 0·001 0·130	0·264 (0·417) 0·000 0·185	0·266 (0·427) 0·000 0·141	0·273 (0·346) 0·000 0·091	—	—
48	0·334 (0·274) 0·000 0·070	0·336 (0·382) 0·001 0·188	0·335 (0·422) 0·006 0·212	0·325 (0·449) 0·004 0·217	0·323 (0·481) 0·002 0·239	0·321 (0·444) 0·001 0·188	0·376 (0·526) 0·008 0·286	—
60	0·357 (0·291) 0·000 0·093	0·365 (0·381) 0·017 0·168	0·367 (0·415) 0·036 0·219	0·358 (0·441) 0·018 0·250	0·353 (0·476) 0·011 0·261	0·340 (0·450) 0·003 0·215	—	—
120	0·474 (0·285) 0·043 0·129	0·481 (0·337) 0·053 0·205	0·488 (0·356) 0·070 0·228	0·487 (0·372) 0·068 0·236	0·485 (0·398) 0·079 0·281	0·478 (0·383) 0·070 0·232	0·523 (0·379) 0·081 0·255	0·552 (0·385) 0·136 0·275

Notes. This table gives first the standard deviation of $S_t^{\prime(n,m)}$ divided by the standard deviation of $S_t^{(n,m)}$ and then estimated standard errors of this ratio (in parentheses). $S_t^{\prime(n,m)}$ is computed from equation (6) in the text based on a vector autoregression starting in 1952:1 and ending in 1987:2. The vector autoregression had four lags. The table also gives two numbers from a Monte Carlo experiment. The first is the fraction of 1000 runs which produced an estimated standard deviation ratio which was further away from one in the same direction than the ratio obtained in the data. The second number is the fraction of 1000 runs in which a 2-sided t test of the hypothesis that the ratio equals one rejected the null more strongly than the t test on the actual data.

TABLE 4(b)

$\sigma(S_t'^{(n,1)})/\sigma(S_t^{(n,1)})$: *Subsamples*

	Sample period (number of observations)					
n	1952–1987 (421)	1952–1978 (323)	1952–1959 (95)	1960–1969 (119)	1970–1978 (107)	1979–1987 (97)
2	0·681 (0·136)	0·585 (0·128)	0·545 (0·123)	0·581 (0·112)	0·931 (0·365)	0·986 (0·148)
3	0·586 (0·144)	0·503 (0·132)	0·483 (0·117)	0·582 (0·112)	0·674 (0·389)	0·922 (0·144)
4	0·607 (0·162)	0·508 (0·125)	0·472 (0·142)	0·551 (0·128)	0·679 (0·376)	0·948 (0·170)
6	0·500 (0·144)	0·461 (0·099)	0·489 (0·180)	0·429 (0·102)	0·599 (0·277)	0·824 (0·200)
9	0·424 (0·129)	0·427 (0·101)	0·554 (0·217)	0·329 (0·085)	0·508 (0·234)	0·714 (0·181)
12	0·382 (0·119)	0·400 (0·100)	0·649 (0·275)	0·309 (0·103)	0·441 (0·235)	0·627 (0·165)
24	0·304 (0·138)	0·416 (0·219)	1·170 (0·566)	0·328 (0·222)	0·459 (0·408)	0·470 (0·123)
36	0·311 (0·227)	0·473 (0·272)	1·510 (0·575)	0·316 (0·238)	0·504 (0·411)	0·392 (0·142)
48	0·338 (0·274)	0·541 (0·295)	1·578 (0·471)	0·277 (0·218)	0·579 (0·401)	0·370 (0·202)
60	0·360 (0·290)	0·589 (0·293)	1·539 (0·406)	0·235 (0·115)	0·637 (0·378)	0·365 (0·260)
120	0·476 (0·284)	0·717 (0·231)	1·288 (0·238)	0·266 (0·351)	0·743 (0·282)	0·428 (0·354)

Notes. This table gives the same statistics as Table 4(*a*), except that $m = 1$ throughout the table and results are reported for subsamples.

The movements of longer-term yields, however, do not display this U-shaped pattern. In Tables 1(*a*) and 1(*b*) the coefficients become increasingly negative as the horizon increases. The tendency of the long rate to fall when the spread is high is quite robust to the maturity of the longer-term instrument.

Monte Carlo results

Our VAR approach can also be used to generate artificial data for Monte Carlo simulations. It is known that there can be small-sample bias in standard errors of regressions with predetermined but not exogenous variables. (See Mankiw and Shapiro (1986) and Stambaugh (1986) for a discussion of this in the context of rational expectations models.) The problem is particularly serious when asymptotic corrections are used to handle equation-error overlap which is large relative to the sample size (Stock and Richardson (1989), Hodrick (1990)). Monte Carlo simulations are an appealing way to avoid excessive reliance on asymptotic distribution theory.

To generate data which match the moments of the actual data while obeying the restrictions of the expectations theory, we start by drawing normal random errors and feeding them through the VAR system estimated on the actual data. This gives us artificial

time series for $\Delta R_t^{(m)}$ and $S_t^{(n,m)}$. We then replace $S_t^{(n,m)}$ with $S_t'^{(n,m)}$, calculated using the true VAR coefficients (those estimated on the actual data and which generate the artificial data). This procedure can be used whenever n/m is an integer. It gives us artificial data which obey the expectations theory in population, but the estimated VAR coefficients do not necessarily obey the restrictions of the theory in each artificial sample.

Tables $1(a)$, $2(a)$, $3(a)$, and 4 give two numbers for each entry which summarize the results of the Monte Carlo experiment. The first number is the fraction of 1000 runs which produced an estimated regression coefficient (or correlation, or standard deviation ratio) which was further away from one in the same direction (that is, usually, smaller) than the coefficient obtained in the data. This is a "one-sided" empirical significance level for the coefficient. The second number is the fraction of 1000 runs in which a t-test of the expectations theory, computed using the asymptotic standard error on the coefficient, rejected the null more strongly than the t-test on the actual data. This is a "two-sided" empirical significance level for the expectations theory t-test. One can compare the empirical significance level with the significance level from the normal distribution to evaluate the quality of the asymptotic standard errors in the tables.

The Monte Carlo results show that there is some finite-sample bias in asymptotically valid regression tests of the expectations theory. The bias is particularly noticeable when the order of equation error overlap is large (that is, for large m on the right hand side of Table 1 and for large n at the bottom of Table 2). In these parts of the tables, the asymptotic standard errors greatly understate the true uncertainty about the regression coefficients.

However this bias is not enough to overturn our conclusions about the expectations theory. In Table 1 the expectations theory is rejected very strongly at the short end, and comfortably at the long end. In Table 2 the theory is rejected at the short end but not the long end. The VAR procedures generally give weak evidence that the actual and theoretical spreads are imperfectly correlated, and stronger evidence that the actual spread has higher variability than the theoretical spread.

One interesting feature of the Monte Carlo results is that the two empirical significance levels are sometimes quite different from one another. For example, in Table $3(a)$ with $m = 1$ and $n = 24$, only 21 out of 1000 runs delivered a correlation between $S_t^{(n,m)}$ and $S_t'^{(n,m)}$ that was lower than the one estimated in the data. But 749 out of 1000 runs delivered a correlation which was further from one when normalized by its asymptotic standard error. In this table, it seems that the artificial data sets tended to deliver higher correlations between $S_t^{(n,m)}$ and $S_t'^{(n,m)}$ and smaller standard errors than are found in the actual data. We conjecture that this is due to heteroskedasticity in the actual data which is not captured by our Monte Carlo experiment.

4. FURTHER INTERPRETATION

We have documented the fact that for any pair of maturities the yield spread fails to correctly predict subsequent movements in the yield on the longer-term bond, yet it does forecast short rate movements in roughly the way implied by the expectations theory. The purpose of this section is to explore some possible explanations for this fact. The explanations we will consider are not finance-theoretic models of time-varying risk premia, but simply econometric descriptions of ways in which the expectations theory might fail.

One obvious alternative to the expectations theory is a model of the form

$$S_t^{(n,m)} = E_t S_t^{(n,m)*} + c + v_t, \tag{8}$$

where v_t is an error term which is orthogonal to $E_t S_t^{(n,m)^*}$. Equation (8) can generate a negative coefficient when $R_{t+m}^{(n-m)} - R_t^{(n)}$ is regressed on $s_t^{(n,m)}$, since the error term v_t appears positively in the independent variable and negatively in the dependent variable of this regression. The numerator of the regression coefficient, the covariance between the right-hand and left-hand side variables, has a term $\text{Cov}(v_t, v_{t+m}) - \text{Var}(v_t)$ added to it. This can make the regression coefficient negative if v_t is variable and not too highly serially correlated.

Equation (8) also implies that the regression coefficient when $S_t^{(n,m)^*}$ is regressed on $S_t^{(n,m)}$ is $1/(1 + VR)$, where VR is the ratio of the variance of v_t to the variance of $S_t^{(n,m)^*}$. The correlation coefficient between $E_t S_t^{(n,m)^*}$ and $S_t^{(n,m)}$ and the ratio of the standard deviation of $E_t S_t^{(n,m)^*}$ to the standard deviation of $S_t^{(n,m)}$ both equal the square root of $1/(1 + VR)$.

The VAR results in Tables 3 and 4 do not suggest that this model fits the data, since we found that the correlation coefficient was very close to one while the ratio of standard deviations was roughly one half. It is also possible to test (8) more directly, if we impose extra orthogonality conditions on the error term v_t. For example, if we assume that v_t is white noise, then any variables dated $t-1$ or earlier can be used as instruments in an instrumental variables regression of $R_{t+m}^{(n-m)} - R_t^{(n)}$ onto $s_t^{(n,m)}$. These lagged variables will be orthogonal to v_t, so the IV coefficient should be one as implied by the expectations theory. Alternatively, if we assume that v_t follows an MA (q) process, then any variables dated $t - q - 1$ or earlier can be used as instruments.

In Table 5 we report the results of instrumental variables regressions of $R_{t+m}^{(n-m)} - R_t^{(n)}$ onto $s_t^{(n,m)}$. We set $m = 1$ and use the full 1952-1987 sample. For comparison, the first column reports the OLS regression results from Table 1(a). The next three columns give IV regression coefficients with standard errors, where the instruments are the spread lagged 1 month, 6 months, and 12 months respectively. The results are not encouraging for the model (8); the IV coefficients are always negative and often more so than the OLS coefficients. Standard errors increase with the lag length of the instrument, but this does not provide any positive evidence in favour of (8).[12]

An alternative orthogonality condition would be that v_t is orthogonal to current and past short-term interest rates. If this is so, then the projection of the yield spread onto the history of short rates has the form implied by the expectations theory; the failure of the expectations theory is caused by the behaviour of the error term in this projection. We can test this idea by using current and lagged short rates as instruments in the IV regression of $R_{t+m}^{(n-m)} - R_t^{(n)}$ onto $s_t^{(n,m)}$. In the last two columns of Table 5 we use the level of $R_t^{(1)}$, less a 12-month or 60-month backwards moving average of $R_t^{(1)}$, as an instrument. We write these variables as $X_{t,12}$ and $X_{t,60}$, respectively. $X_{t,12}$ is a better instrument for shorter-term yield spreads, while $X_{t,60}$ is a better instrument for longer-term yield spreads. But neither set of results is very encouraging for the model. At least over the period 1952-1987, it appears that the component of the yield spread which is correlated with past short rates is at least partly responsible for the failure of the expectations theory.[13]

---

12. These results are consistent with those reported in Campbell and Shiller (1987) using a VAR approach to test (8).

13. We analyzed this question earlier in Campbell and Shiller (1984). Using a different approach, we reached a similar conclusion for the period 1959-1982. Over the period 1959-1979, however, we found no evidence against equation (9) with an error term orthogonal to short rates. Indeed an IV regression of the type reported in the last column of Table 5 does give some positive coefficients over the shorter sample period, but the standard errors are quite large.

TABLE 5

Instrumental variables regression of $R_{t+1}^{(n-1)} - R_t^{(n)}$ on predicted change $s_t^{(n,1)}$

			Instruments			
n	None (OLS)	$s_{t-1}^{(n,1)}$	$s_{t-6}^{(n,1)}$	$s_{t-12}^{(n,1)}$	$X_{t,12}$	$X_{t,60}$
2	0·002 (0·238)	−1·687 (1·134)	−1·997 (1·484)	−0·106 (0·563)	0·309 (1·561)	−4·578 (4·921)
3	−0·176 (0·362)	−1·373 (1·160)	−3·491 (2·166)	−0·363 (0·747)	−0·164 (1·374)	— (—)
4	−0·437 (0·469)	−1·223 (1·046)	−4·490 (2·179)	−0·733 (1·017)	−0·309 (1·380)	4·610 (10·931)
6	−1·029 (0·537)	−1·678 (0·953)	−5·071 (2·461)	−1·664 (1·574)	−0·142 (1·772)	4·199 (10·071)
9	−1·219[a] (0·598)	−2·089[a] (1·027)	−6·937[a] (3·453)	−2·395[a] (2·283)	0·060[a] (2·089)	2·200[a] (5·364)
12	−1·381[a] (0·683)	−2·146[a] (0·959)	−6·000[a] (2·901)	−2·997[a] (3·144)	−0·061[a] (1·919)	0·990[a] (3·155)
24	−1·815[a] (1·151)	−2·262[a] (1·127)	−4·392[a] (2·552)	−4·219[a] (3·692)	−0·269[a] (2·049)	0·078[a] (2·367)
36	−2·239[a] (1·444)	−2·328[a] (1·398)	−4·850[a] (2·588)	−4·586[a] (4·106)	−0·611[a] (2·153)	−0·559[a] (2·255)
48	−2·665[a] (1·634)	−2·719[a] (1·598)	−5·767[a] (2·865)	−4·477[a] (4·251)	−1·067[a] (2·376)	−1·095[a] (2·313)
60	−3·099[a] (1·749)	−3·058[a] (1·728)	−6·205[a] (3·039)	−4·513[a] (4·433)	−1·518[a] (2·539)	−1·632[a] (2·384)
120	−5·024[a] (2·316)	−4·908[a] (2·405)	−8·063[a] (3·968)	−4·909[a] (6·256)	−3·524[a] (3·394)	−3·839[a] (2·992)

Notes. Table 5 gives estimated regression slope coefficients of $R_{t+1}^{(n-1)} - R_t^{(n)}$ onto $s_t^{(n,m)} \equiv (m/(n-m))S_t^{(n,m)}$. Regressions were estimated using instrumental variables, which are identified at the top of each column. The first three columns use lagged spreads, the last two use the difference between the current short rate and a 12-month backwards moving average $(X_{t,12})$ and the difference between the current short rate and a 60-month backwards moving average $(X_{t,60})$. According to the model given in equation (8), the slope coefficients should equal one. Constant terms (not shown) are included in all regressions. Hansen-Hodrick standard errors are below estimated coefficients, in parentheses. For each regression, the sample is the longest possible using data from 1952:1 through 1987:2
[a] Uses approximation that $R_{t+m}^{(n-m)} = R_{t+m}^{(n)}$.

These results suggest that equation (8) may be too restrictive to explain the data. A more promising model may be

$$S_t^{(n,m)} = kE_t S_t^{(n,m)*} + c,\tag{9}$$

where the coefficient k is greater than one. This model can also generate a negative coefficient when $R_{t+m}^{(n-m)} - R_t^{(n)}$ is regressed on $s_t^{(n,m)}$. Now the covariance between the left-hand and right-hand side variables of the regression has a term

$$k(k-1) \operatorname{Cov}(E_{t+m}S_{t+m}^{(n,m)*}, E_t S_t^{(n,m)*}) - k(k-1) \operatorname{Var}(E_t S_t^{(n,m)*})$$

added to it. This can make the regression coefficient negative if $E_t S_t^{(n,m)*}$ is not too serially correlated and k is large enough.

The model (9) also implies that the regression of $S_t^{(n,m)*}$ on $S_t^{(n,m)}$ will give a coefficient of $1/k$. The correlation of $E_t S_t^{(n,m)*}$ and $S_t^{(n,m)}$ will be one, but the ratio of their standard

deviations will be $1/k$. This model roughly fits the results from Tables 2, 3, and 4, except that the Table 2 regression coefficients tend to exceed the Table 4 standard deviation ratios for long horizons.[14]

Equation (9) could be described as an *overreaction* model of the yield spread. It says that the long rate differs from the short rate in the direction implied by the expectations theory; however, the spread between the two rates is larger than can be justified by rational expectations of future short rate changes.

An alternative way to describe equation (9) is to say that long rates *underreact* to short-term interest rates. This interpretation was offered in Campbell and Shiller (1984), although the empirical results in that paper were not as comprehensive as those here. We argued there that long rates are fairly well described as a distributed lag on short rates, where the distributed lag weights are all positive and their sum is very close to one. (It should be exactly one if predictable excess returns are stationary and the short-term interest rate follows a stochastic process with a unit root.) Relative to the predictions of the expectations theory, the estimated distributed lag gives too little weight to the current short rate, and too much weight to lagged short rates. In this sense the long rate underreacts to the current level of the short rate.[15]

To see that the Campbell and Shiller (1984) model is consistent with our results here, consider its implications for the spread between the long rate and the short rate. The spread will be a distributed lag on past short rates with negative weight on the current short rate and positive weight on the distributed lag of past short rates, where all weights sum to zero. The Campbell and Shiller (1984) model implies that the spread's distributed lag representation would conform more closely to the expectations theory if the absolute value of all weights were scaled down towards zero. This also follows from equation (9).[16]

The deviation from the expectations theory described by equation (9) could be caused by time-varying risk premia which are correlated with expected increases in short-term interest rates. Alternatively, it is possible that in our sample period the bond market underestimated the persistence of movements in short rates (and thus overestimated the predictability of future short rate changes). Variations in the long-short spread were due primarily to sudden movements in short rates, and in this sample period long rates reacted too sluggishly to these sudden movements, so that the consequential movements in the spread were too large to be in accordance with the expectations theory.

REFERENCES

BELTRATTI, A. and SHILLER, R. J. (1990), "Actual and Warranted Relations Between Asset Prices" (Unpublished paper, University of Turin and Yale University).
CAMPBELL, J. Y. (1986) "A Defense of Traditional Hypotheses about the Term Structure of Interest Rates", *Journal of Finance*, **41**, 183–193.

14. As discussed above, this discrepancy is largely due to the difference in sample periods caused by the truncation of the sample in Table 2.

15. Mankiw and Summers (1984) described a similar underreaction model in which the long rate gives too much weight to *expected future* short rates, as opposed to lagged short rates. The underreaction story is not what one would expect given the evidence for "excess volatility" of long rates presented by one of the authors (Shiller (1979)). However that evidence depended on the assumption that the short-term interest rate is stationary, which is probably inappropriate for the postwar period in the U.S. See Shiller (1989) for further discussion.

16. Equation (9) is stronger in that it implies that the component of the spread which is orthogonal to current and lagged short rates should also be scaled down.

CAMPBELL, J. Y. and SHILLER, R. J. (1984), "A Simple Account of the Behavior of Long-Term Interest Rates", *American Economic Review, Papers and Proceedings*, 74, 44-48.

CAMPBELL, J. Y. and SHILLER, R. J. (1987), "Cointegration and Tests of Present Value Models", *Journal of Political Economy*, 95, 1062-1088.

COX, J. C., INGERSOLL, J. E. and ROSS, S. A. (1981), "A Reexamination of Traditional Hypotheses about the Term Structure of Interest Rates", *Journal of Finance*, 36, 769-799.

FAMA, E. F. (1984), "The Information in the Term Structure", *Journal of Financial Economics*, 13, 509-528.

FAMA, E. F. (1990), "Term Structure Forecasts of Interest Rates, Inflation, and Real Returns", *Journal of Monetary Economics*, 25, 59-76.

FAMA, E. F. and BLISS, R. R. (1987), "The Information in Long-Maturity Forward Rates", *American Economic Review*, 77, 680-692.

FROOT, K. A. (1989), "New Hope for the Expectations Hypothesis of the Term Structure of Interest Rates", *Journal of Finance*, 44, 283-305.

HANSEN, L. P. and HODRICK, R. J. (1980), "Forward Rates as Optimal Predictors of Future Spot Rates", *Journal of Political Economy*, 88, 829-853.

HODRICK, R. J. (1990), "Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement" (Unpublished paper, Northwestern University).

MCCULLOCH, J. (1990), "U.S. Government Term Structure Data", in Friedman, B. and Hahn, F. (eds.), *The Handbook of Monetary Economics* (Amsterdam: North Holland).

MANKIW, N. G. and SHAPIRO, M. D. (1986), "Do We Reject Too Often? Small Sample Properties of Tests of Rational Expectations Models", *Economics Letters*, 20, 139-145.

MANKIW, N. G. and SUMMERS, L. H. (1984), "Do Long-Term Interest Rates Overreact to Short-Term Interest Rates?", *Brookings Papers on Economic Activity*, 1, 223-242.

MISHKIN, F. S. (1988a), "The Information in the Term Structure: Some Further Results", *Journal of Applied Econometrics*, 3, 307-314.

MISHKIN, F. S. (1988b), "The Information in the Longer-Maturity Term Structure About Future Inflation" (Unpublished paper, Columbia University).

MISHKIN, F. S. (1990), "What Does the Term Structure Tell Us About Future Inflation?", *Journal of Monetary Economics*, 25, 77-95.

NEWEY, W. K. and WEST, K. D. (1987), "A Simple, Positive Definite, Heteroscedasticity and Autocorrelation Consistent Covariance Matrix", *Econometrica*, 55, 703-708.

SHILLER, R. J. (1979), "The Volatility of Long-Term Interest Rates and Expectations Theories of the Term Structure", *Journal of Political Economy*, 87, 1190-1219.

SHILLER, R. J. (1989) *Market Volatility* (Cambridge, Mass.: MIT Press).

SHILLER, R. J. (1990), "The Term Structure of Interest Rates", in Friedman, B. and Hahn, F. (eds.), *The Handbook of Monetary Economics* (Amsterdam: North Holland).

SHILLER, R. J., CAMPBELL, J. Y. and SCHOENHOLTZ, K. L. (1983), "Forward Rates and Future Policy: Interpreting the Term Structure of Interest Rates", *Brookings Papers on Economic Activity*, 1, 173-217.

STAMBAUGH, R. F. (1986), "Bias in Regressions with Lagged Stochastic Regressors" (Unpublished paper, Graduate School of Business, University of Chicago).

STOCK, J. H. and RICHARDSON, M. (1989), "Drawing Inferences from Statistics Based on Multi-Year Asset Returns" (Unpublished paper, Harvard University and Stanford University).

WHITE, H. (1984) *Asymptotic Theory for Econometricians* (New York: Academic Press).