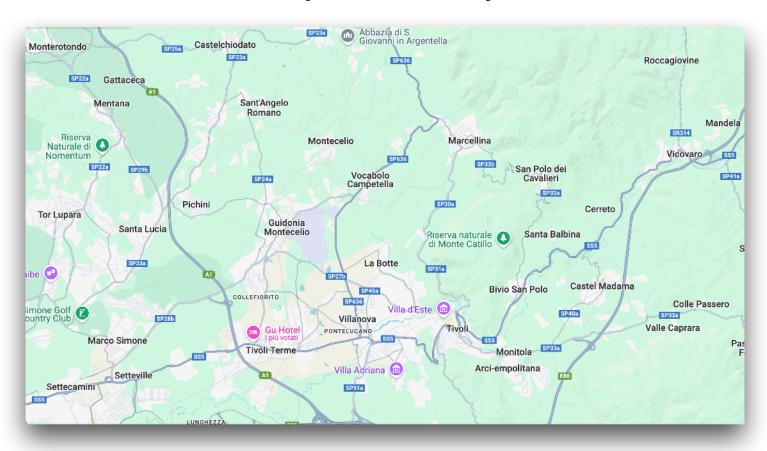
Maximum Path Search in a Directed Acyclic Graph (DAG).

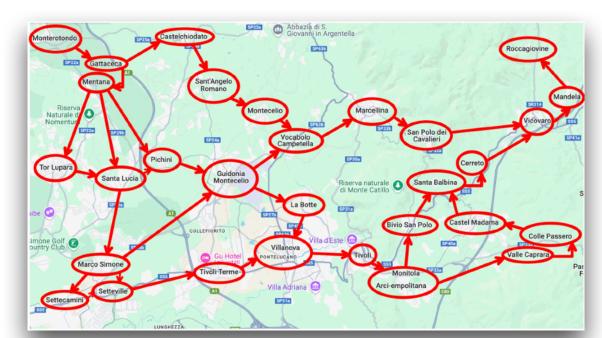
Document prepared by Simone Remoli.

Consider the following Directed Acyclic Graph (DAG). Determine the length of the maximum path.

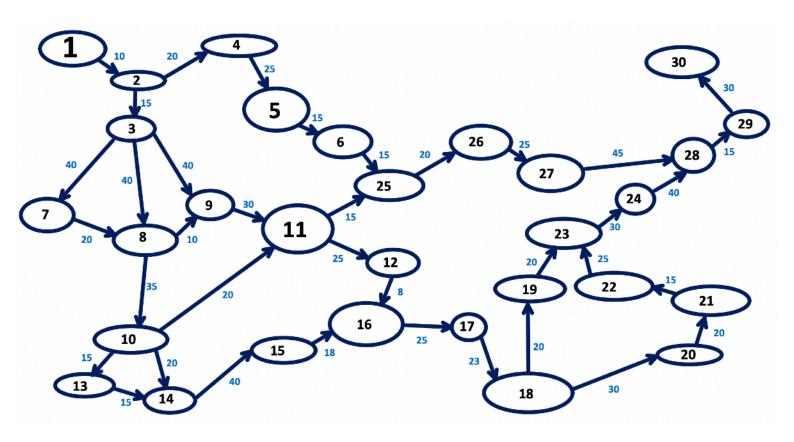


Let us first consider a road network in the northeastern area of Rome, Italy.

We aim to find the maximum path to travel from one point to another within the network.



Let us construct the graph.



Our objective is to find the maximum path from node 1 to node 30.

Additionally, the nodes are assigned values corresponding to their pre-determined **topological numbering**.

Below is the table presenting the updated cost values.

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Now, let us proceed to backtrack and derive the maximum path.

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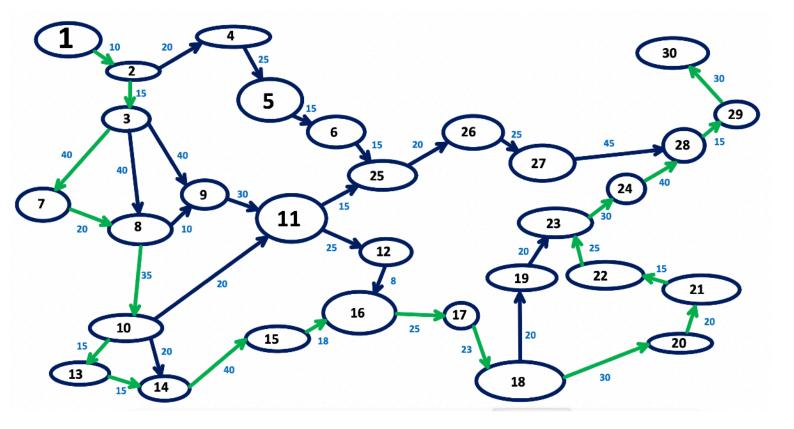
The algorithm proceeds in reverse, and we now move forward to outline the maximum path.

The maximum path is as follows:

The maximum path has a value of 461.

The path is graphically shown on the graph below.

To travel from "Monterotondo" to "Roccagiovine," I need to pass through twenty nodes, with a maximum total of 461 minutes.



461 minutes correspond to 7 hours and about 30 minutes .

The values I assigned to the edges of the graph are random, but if we were able to input the exact values corresponding to the travel time in minutes between nodes, we could achieve a precise measurement.

End.