Master Thesis:
Characterizing the fractal dimension of molecular clouds

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Introduction

Rationale:

- Molecular clouds are filamentary and self-similar, forming hierarchies of substructures (Elmegreen & Falgarone 1996).
- Observations show filaments fragment into smaller sub-filaments.

Goals:

- Quantify fractal geometry of the Orion Molecular Cloud.
- Trace changes in fractal dimension with column density.
- Link fractal features to mass–size scaling and star formation modes.

Methods I: Perimeter–Area Relation

Fractal dimension D quantifies the boundary complexity.

Perimeter–Area Relation:

$$P \propto \sqrt{A}^{D} \tag{1}$$

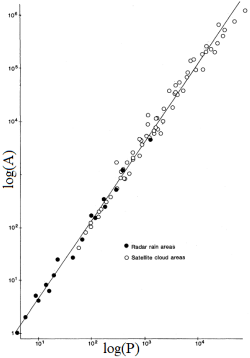
- For a *fixed length*, a smooth perimeter encloses a larger area than a complicated one.
- For a smooth shape, $P \approx \sqrt{A}$ and thus $D = 1$.
- As the perimeter becomes more contorted and doubles back on itself, $P \approx A$ and D approaches 2.

P-A Scaling

Methods II: Proving self-similarity

Global Fractal Dimension

- Different large and small-scale structures follow different perimeter–area (PA) relations.
- A consistent PA scaling suggests **no characteristic scale**.
- $\rightarrow$ Structure is **self-similar**.
- $\rightarrow$ Indicates **fractal geometry**.



P–A Diagram (Lovejoy, 1982)

Methods III: D vs. Column Density

Local Fractal Dimension

$$D(N) = 2 \cdot \frac{\log(P)}{\log(A)} \tag{2}$$

- Explore how D varies with column density N.
- Link changes in D to **physical processes**:
  - Gravitational collapse
  - Mass–size scaling
  - Star formation modes
- Accompanied by **simulations**

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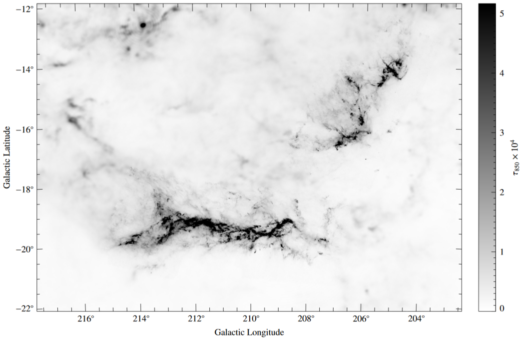
Data: Orion A & B

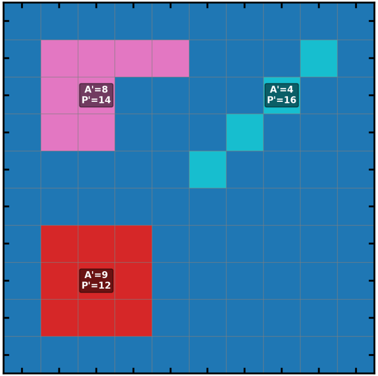
- *ESA's Herschel*:
  - Far-infrared and submillimiter
  - Great dynamic range.
- **Angular resolution**:
  - 36 arcsec
  - $2\times\,10^{20}\;cm^{-2} < N < 5\times\,10^{23}\;cm^{-2}$



Herschel Dust Emission (Lombardi et al. 2014)

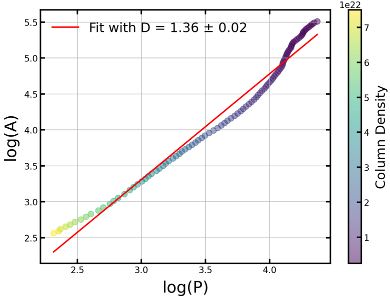
From the Data to the Results

- Apply a column density threshold.
- Identify regions above the threshold.
  - $A = \sum A'$
  - $P = \sum P'$
- Each threshold yields
  - One point in the $\log(P)$ vs $\log(A)$ plot (**Global D**).
  - One point for D by inverting the relation (**Local D**).

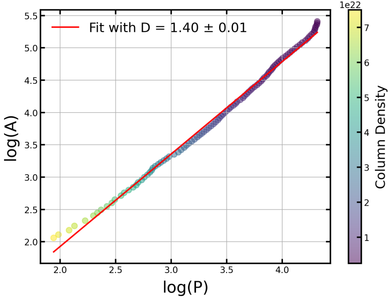


Area and Perimeter for an example threshold.

Results I: Self-similarity



PA Relation for Orion A



PA Relation for Orion B

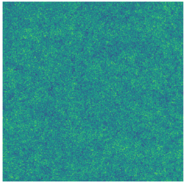
- Good fits of the perimeter–area relation.
- Lack of characteristic length scales.
- Self-similarity across scales.
- Agreeance with literature.

Results II: Simulations

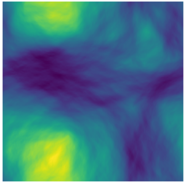
Assessing D across Controlled Structures

- Gaussian Random Fields (GRFs):
  - GRFs with power-law spectra → scale-free
  - GRFs with peaked spectra → characteristic scale
- **Resolution** effects: up to 20% variation.
- **Artifacts**: low pixel counts.

Scale-free GRF, $R^2=0.89$



Peaked GRF, $R^2=0.51$

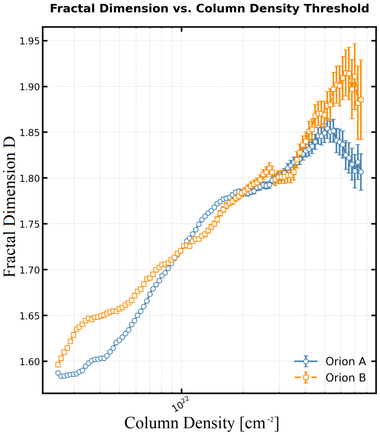


Simulations of GRFs

Results III: D vs. Column Density

Fragmentation at Different Depths

- Varying visualizations:
  - Intercept handling
  - Formula definition
- Simulations reveal a trend in D
- Reflects fragmented networks of hierarchical structures
  - Increase in complexity.
- Local vs. Global D: interpretation is key.

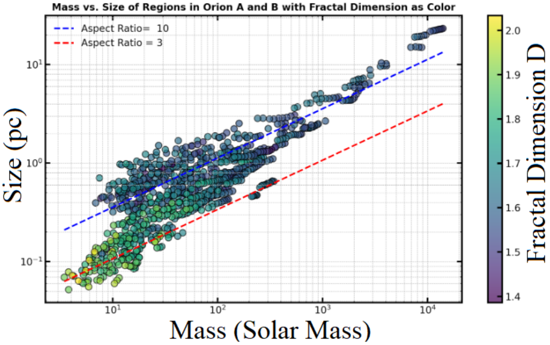


Fractal Dimension Across Column Density

Results IV: Regional Analysis

Mass–Size Relation

- D measured for individual structures at each threshold
- Mass and size extracted per structure
- Compared to expected scaling for:
  - Filamentary: $A = 10$
  - Spheroidal: $A = 3$



Mass–Size–D Diagram

Outlook

So far:

- Evidence points toward self-similar processes shaping cloud structure.
- Consistent trends observed in both simulations and real data.
- Fractal Dimension
    - **Global**: $\mathbf{1.36 \pm 0.02}$ and $\mathbf{1.40 \pm 0.01}$ for Orion A and B.
    - **Local**: Trends capturing complexity of networks.

Next steps:

- Extend simulations and assess robustness.
- Investigate deeper links to physical processes.

References

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