$$MSE = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2 = \frac{1}{N}\sum_{i=1}^{N}\left[y_i - (mx_i + b)\right]^2$$

$\boxed{\text{DERIVATIVE in } b}$

$$\frac{\partial MSE}{\partial b} = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial}{\partial b}\left[y_i - (mx_i+b)\right]^2$$

Pongo $f(b) = \left[y_i - (mx_i + b)\right]$

da cui $\left[y_i - (mx_i + b)\right]^2 = g(f(b))$

quindi

$$\frac{\partial MSE}{\partial b} = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial}{\partial b} g(f(b)) = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial g(f(b))}{\partial f(b)}\cdot\frac{\partial f(b)}{\partial b}$$

$$= \frac{1}{N}\sum_{i=1}^{N}\frac{\partial [f(b)]^2}{\partial f(b)}\cdot\frac{\partial (y_i - mx_i - b)^2}{\partial b} =$$

$$= \frac{1}{N}\sum_{i=1}^{N} 2f(b)\cdot\left[\cancelto{0}{\frac{\partial y_i}{\partial b}} - \cancelto{0}{\frac{\partial mx_i}{\partial b}} - \cancelto{1}{\frac{\partial b}{\partial b}}\right] =$$

$$= \frac{2}{N}\sum_{i=1}^{N} f(b)\cdot\left[0 - 0 - 1\right] = -\frac{2}{N}\sum_{i=1}^{N} f(b) =$$

$$= -\frac{2}{N}\sum_{i=1}^{N}\left[y_i - (mx_i+b)\right] = -\frac{2}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)$$

$\boxed{\text{DERIVATIVE in } M}$

$$\frac{\partial MSE}{\partial m} = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial}{\partial m}\left[y_i - (mx_i+b)\right]^2$$

Pongo $f(m) = \left[y_i - (mx_i + b)\right]$

da cui $\left[y_i - (mx_i + b)\right]^2 = g(f(m))$

quindi

$$\frac{\partial MSE}{\partial m} = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial}{\partial m} g(f(m)) = \frac{1}{N}\sum_{i=1}^{N}\frac{g(f(m))}{\partial f(m)}\cdot\frac{\partial f(m)}{\partial m}$$

$$= \frac{1}{N}\sum_{i=1}^{N}\frac{\partial [f(m)]^2}{\partial f(m)}\cdot\frac{\partial (y_i - mx_i - b)^2}{\partial m} =$$

$$= \frac{1}{N}\sum_{i=1}^{N} 2f(m)\cdot\left[\cancelto{0}{\frac{\partial y_i}{\partial m}} - \cancelto{x_i}{\frac{\partial mx_i}{\partial m}} - \cancelto{0}{\frac{\partial b}{\partial m}}\right] =$$

$$= \frac{2}{N}\sum_{i=1}^{N} f(m)\cdot\left[0 - x_i - 0\right] = -\frac{2}{N}\sum_{i=1}^{N}\left[x_i f(m)\right]$$

$$= -\frac{2}{N}\sum_{i=1}^{N} x_i\left[y_i - (mx_i+b)\right] = -\frac{2}{N}\sum_{i=1}^{N} x_i(y_i - \hat{y}_i)$$