Comprehensive Exam 2021
Biomathematics 202: Structure, Function, and Evolution of Biological Systems

Figures 1 to 3 and Table 1 show how different aspects of health-care spending have increased over the past few decades along with population size and Gross Domestic Product (GDP). The figures plot these changes in terms of semi-log and log-log plots. The questions below ask you to evaluate what these relationships mean in terms of the generic properties of dynamical systems and network structures that we studied in class.

a. (20 points) What simple analytic function describes how healthcare spending increases in time according to Figure 1 (equivalent to the regression-line equations but in a different form)? What ordinary differential equation corresponds to this type of relationship? Explain in words the abstract type of processes that can generate these dynamics. Propose and explain two or more hypotheses for this specific system for how healthcare spending depends on time that and that could generate these dynamics. Interpret what the value of the slope would mean in the context of your hypotheses. Do you think the data—in terms of goodness of fit and distribution of residuals—provide good support for this regression and this relationship?
b. (20 points) What simple analytic function describes how healthcare spending increases with population size or Gross Domestic Product (GDP) in Figures 2 and 3 (equivalent to the regression-line equations but in a different form)? What ordinary differential equation corresponds to this type of relationship? Explain in words the abstract type of processes that can generate these dynamics. Contrast this with your explanation for part **a** and explain in words how these two processes differ and what the consequences mean for absolute versus relative amounts of change? Propose and explain two or more hypotheses for this specific system for how healthcare spending depends on GDP and population size and that could generate these dynamics. Interpret what the value of the slope would mean in the context of your hypotheses. Do you think the data—in terms of goodness of fit and distribution of residuals—provide good support for this regression and this relationship?
c. (20 points) Hospitals are often connected—via being part of the same company or supplied by similar vendors etc.—across cities and states. If you drew these connections as direct pairwise interactions among hospitals, the largest hospitals would likely have the most connections/links, the mid-sized hospitals would have an intermediate amount, and the small hospitals would have the least. What kind of network structure would be well-suited to represent this? On the other hand, if you constructed networks that included the structures of transportation between hospitals—roads, train tracks, etc.—what kind of network structure would be well-suited to represent this?
d. (20 points) For both types of network structures in part **c**, would you expect this to give rise to relationships of the same type as in part **a** or part

b? Which network representation would work better for expressing equations and quantifying each of your hypotheses in parts **a** and **b**? Could you distinguish between the two network types based just on the data? If so, how?

e. (20 points) Are the following examples best described by the types of relationships in part **a**, part **b**, or neither one?
   i. The rate of increase in healthcare spending increases proportionally to the existing amount of healthcare spending but at a rate that is inversely proportional to time. Propose one hypothesis for what might cause this. Is this example similar to the ecological example of logistic growth? If so, is that just a qualitative correspondence or can you show these cases are approximately equal in some limit?
   ii. The rate of healthcare spending increases proportionally with the square of the existing amount of healthcare spending. Propose a hypothesis for what might cause this.

f. (20 points) For any of the hypotheses and differential equations from above, how would you include these in a stochastic differential equation to include effect of randomness and uncertainty? What equation that we covered in class would you start with? Explain why.

Figure 1

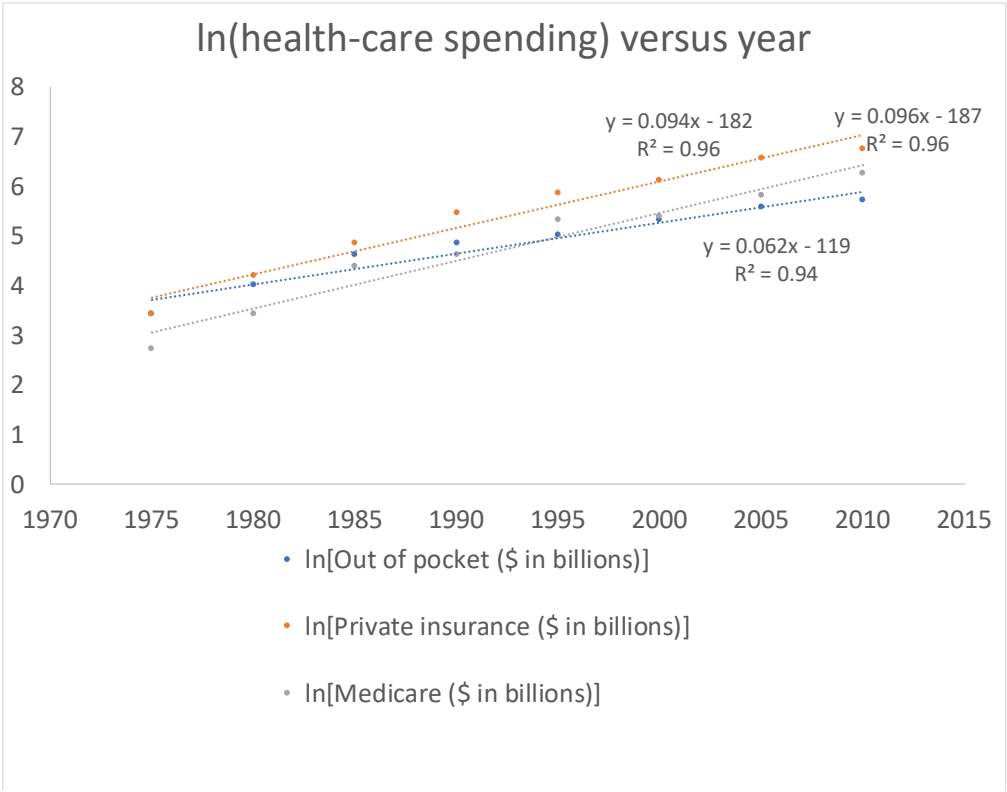

Figure 2

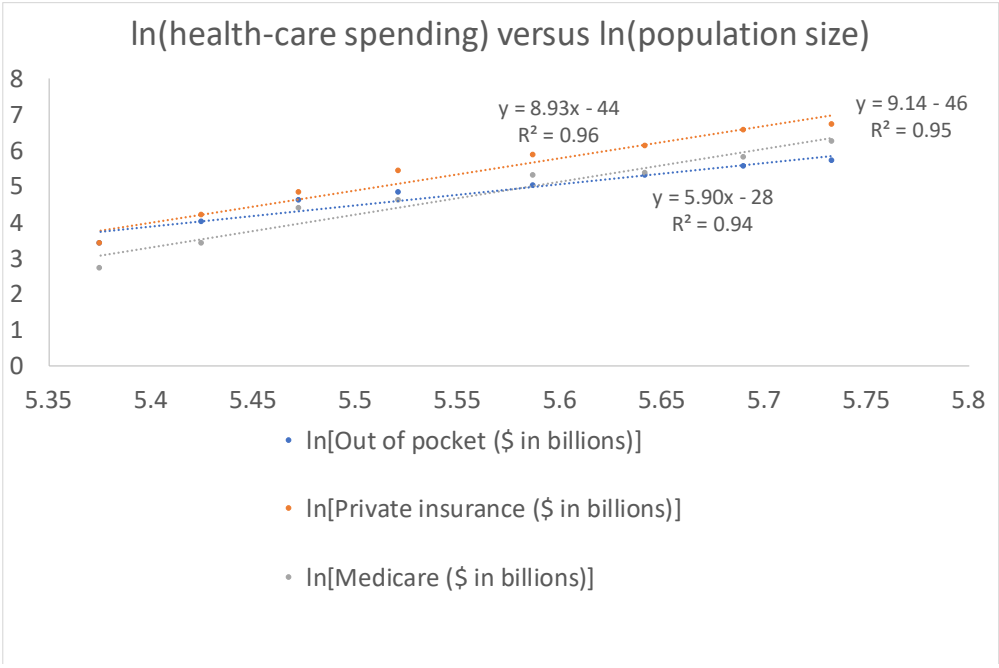

Figure 3

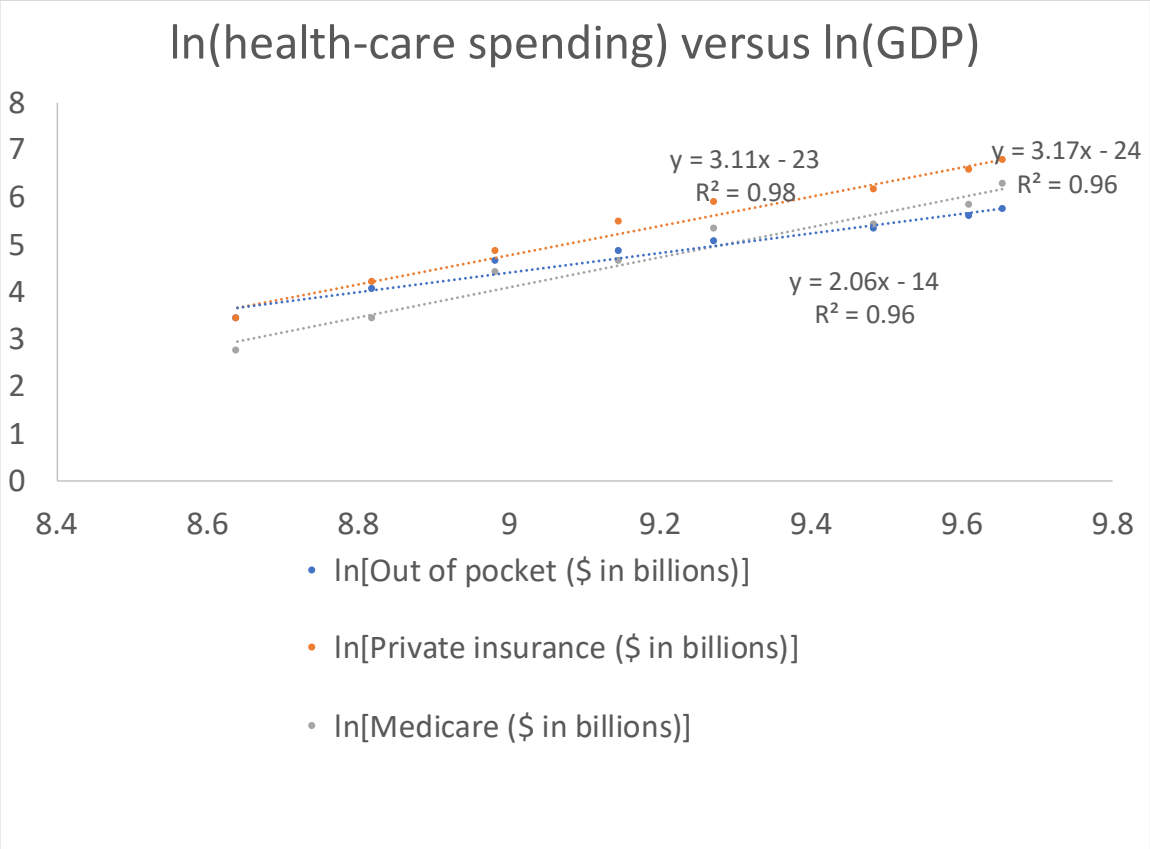

Table 1

Year	Population Size (billions)	GDP ($ in billions)	Out-of-pocket healthcare spending ($ in billions)	Private insurance ($ in billions)	Medicare ($ in billions)
1975	216	5645	30	30	15
1980	227	6759	55	65	30
1985	238	7951	100	125	80
1990	250	9366	125	230	100
1995	267	10630	150	350	200
2000	282	13131	200	450	215
2005	296	14913	260	700	330