PSET 2

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These are some of the libraries/modules you will require for this homework.

In [ ]:
```
%load_ext autoreload
%autoreload 2
%matplotlib inline
import numpy as np
import scipy
from PIL import Image
import skimage
from skimage import data
from skimage.transform import warp, AffineTransform
import matplotlib.pyplot as plt
import copy
import os
```

These are some functions which will be useful throught the homework to (1) display a single grayscale image, (2) display multiple images using subplots.

In [ ]:
```
def display_gray(x: np.array, normalized:bool = False):
    plt.figure(figsize=(10,10))
    if not normalized:
        plt.imshow(x,cmap='gray',vmin=0,vmax=1)
    else:
        plt.imshow(x/x.max(),cmap='gray',vmin=0,vmax=1)
```

In [ ]:
```
def display_axis(ax: plt.axis, x: np.array, title: str, normalized:bool = Fals
    if not normalized:
        ax.imshow(x,cmap='gray',vmin=0,vmax=1)
    else:
        ax.imshow(x/x.max(),cmap='gray',vmin=0,vmax=1)
    ax.set_title(title,size=18)
```

In [ ]:
```
def display_axis_bw(ax: plt.axis, x: np.array, title: str, normalized:bool = Fa
    if not normalized:
        imax = ax.imshow(x,cmap='gray',vmin=0,vmax=1)
    else:
        imax = ax.imshow(x/x.max(),cmap='gray',vmin=0,vmax=1)
    ax.set_title(title,size=18)
    fig.colorbar(imax,ax=ax)
```

In [ ]:
```
def display_axis_color(ax: plt.axis, x: np.array, title: str, normalized:bool =
    if not normalized:
        imax = ax.imshow(x,cmap='coolwarm',vmin=0,vmax=1)
    else:
        imax = ax.imshow(x/x.max(),cmap='coolwarm',vmin=0,vmax=1)
    ax.set_title(title,size=18)
    fig.colorbar(imax,ax=ax)
```

Question 2

Blob Detection

In this question, you will be using the Laplacian of Gaussian Filter to perform blob detection. Using the previous parts of the question you should have an analytical expression for the Laplacian of Gaussian. You will be using that result to design a LoG filter. You will then be using that filter to detect blobs of different scales in the image by varying the standard deviation parameter.

In [ ]:

```
# Copy paste your conv2D function from the previous homework here.
def conv2D(image: np.array, kernel: np.array = None):
    """
    Perform 2D convolution on an image ensuring
    the output has the same size as the input.

    Author: Simon Lee
    """
    try:
        # If no kernel is provided, raise an error
        if kernel is None:
            raise ValueError("Kernel cannot be None. Please \
            provide a valid kernel.")

        # Like performing a convolution graphically,
        # we begin by flipping the kernel for convolution
        kernel = np.flipud(np.fliplr(kernel))

        # We define the padding sizes around all the sides of the image
        pad_height = (kernel.shape[0] - 1) // 2
        pad_width = (kernel.shape[1] - 1) // 2

        # Pad the input image with zeros on all sides
        padded_image = np.pad(image,
                              ((pad_height, pad_height),
                              (pad_width, pad_width)),
                              mode='constant',
                              constant_values=0
                              )

        # We need to initialize the output by creating a
        # np.array of size
        image
        convolved_image = np.zeros_like(image)

        # Convolution loop (From Lecture)
        for x in range(image.shape[0]):
            for y in range(image.shape[1]):
                # Extract the region to perform convolution
                region = padded_image[x:x+kernel.shape[0],
                                      y:y+kernel.shape[1]]
                # Perform element-wise multiplication and sum the result
                convolved_image[x, y] = np.sum(region * kernel)
```

```
        return convolved_image

    # except blocks for ValueError and Other Errors
    except ValueError as ve:
        print(f"Value Error: {ve}")
    except Exception as e:
        print(f"An error occurred: {e}")
```

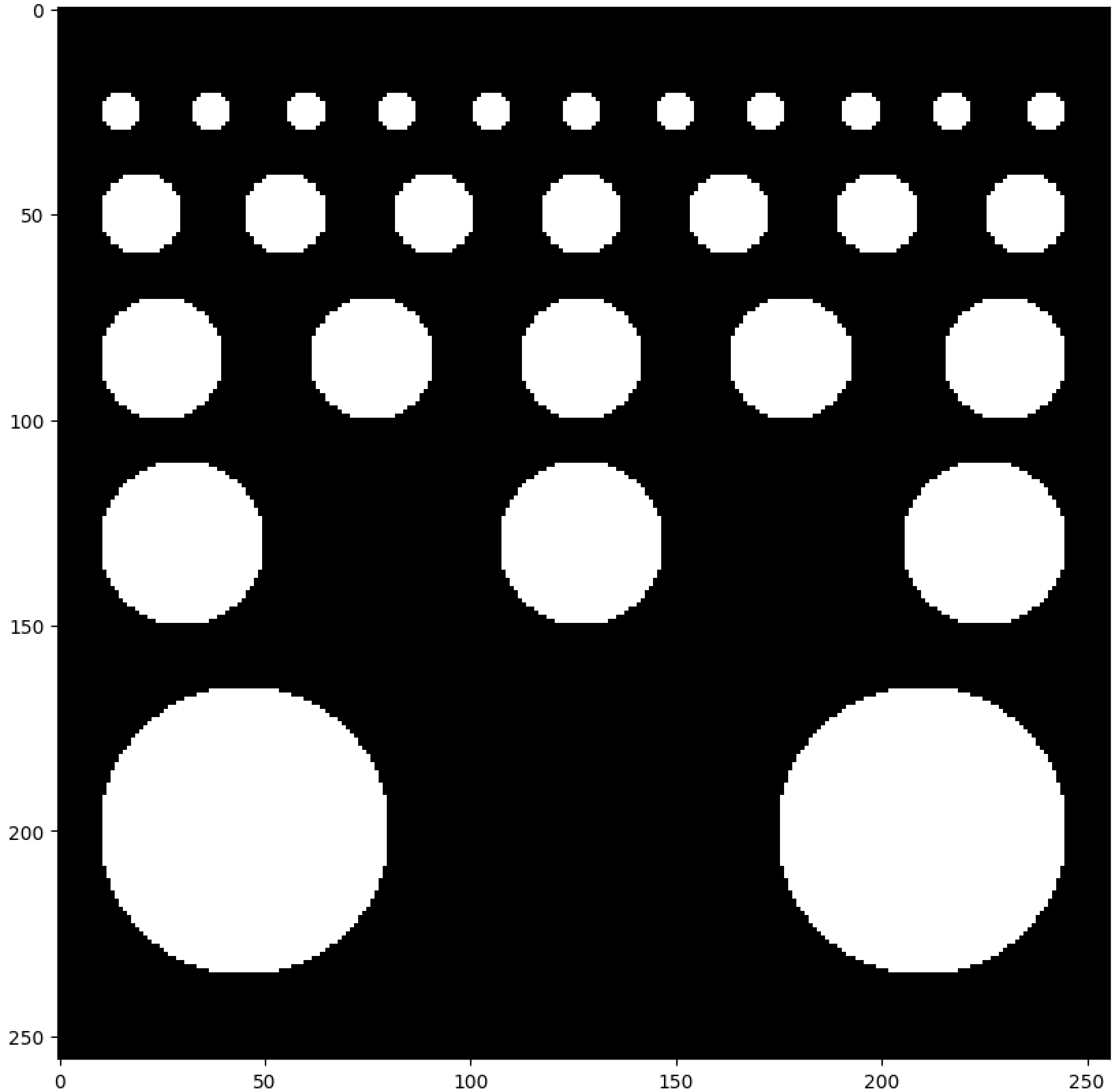
We use the two functions below to create blobs of different sizes. You will have 5 different sizes of blobs in the image below. You will have to tune the standard deviation of your LoG filter such that you get the maximum response.

In [ ]:
```
def make_circle(img: np.array, x: int, y: int, radius: int):
    for i in range(img.shape[0]):
        for j in range(img.shape[1]):
            if np.sqrt((x-i)**2 + (y-j)**2) < 1.0*radius:
                img[i,j]=1
    return img
```

In [ ]:
```
def draw_circle(img, y, radius):
    rad_buffer = radius + 5
    start = rad_buffer+5
    end = 255 - rad_buffer - 5
    centers = np.linspace(start, end, int((end-start)/(2*rad_buffer)))
    for c in centers:
        c = int(c)
        make_circle(img, y, c, radius)
    return img
```

In [ ]:
```
blob_img = np.zeros((256,256))
blob_img = draw_circle(blob_img, 25, 5)
blob_img = draw_circle(blob_img, 50, 10)
blob_img = draw_circle(blob_img, 85, 15)
blob_img = draw_circle(blob_img, 130, 20)
blob_img = draw_circle(blob_img, 200, 35)
```

In [ ]:
```
display_gray(blob_img)
```



In [ ]:
```
img = copy.deepcopy(blob_img)
```

Answer 2.5

For this sub-part, you will be writing a function `log_filter(size, sigma)`, which takes as input the size of the LoG filter and the sigma, and returns a scale-normalized LoG filter.

Copy paste your solution in the cell below on Overleaf for Question 2.5.

In [ ]:
```
# Write your answer in this cell.
def log_filter(size: int, sigma: float):
    """
    Computes the Laplacian of Gaussian (LoG) filter.

    Author: Simon Lee
    """
    try:
        # We need an odd size to have a center pixel
        if not isinstance(size, int) or size <= 0:
```

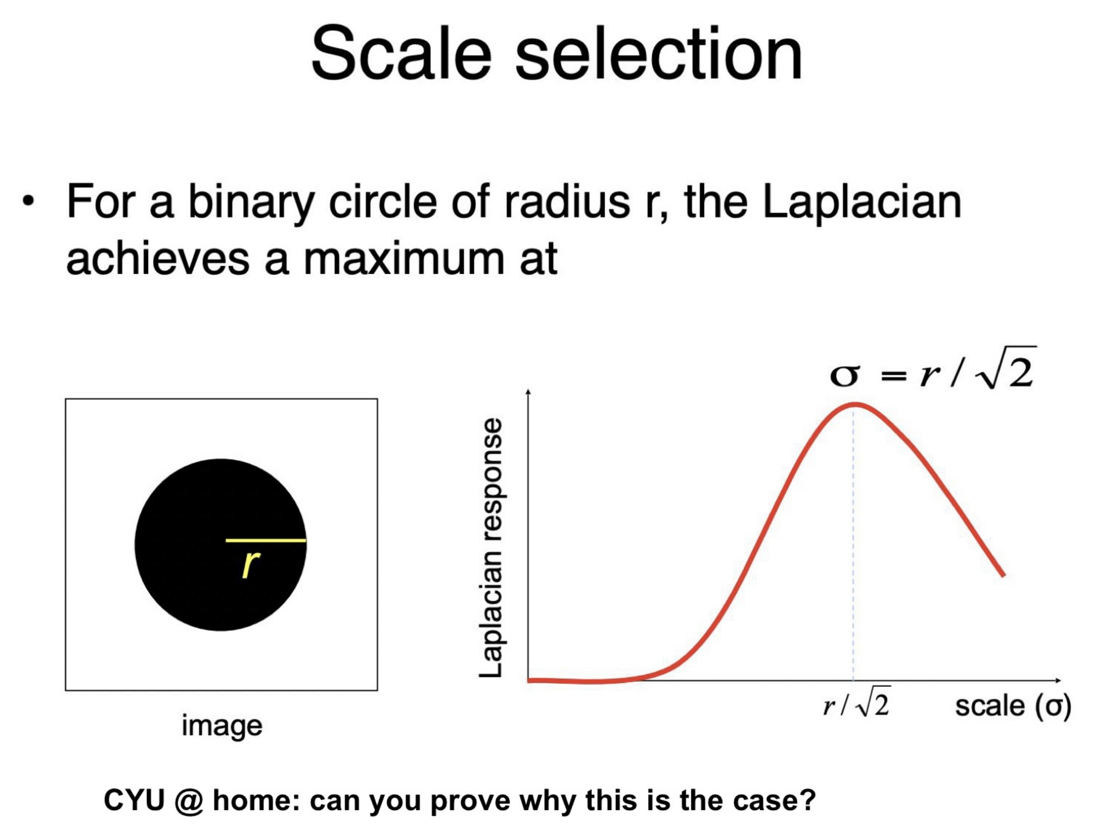
```
            raise ValueError("Size must be a positive integer.")
        size = size + 1 if size % 2 == 0 else size
        radius = size // 2

        # error handling for division by zero
        if sigma <= 0:
            raise ValueError("Sigma must be greater than zero.")

        y, x = np.ogrid[-radius:radius+1, -radius:radius+1]
        g = (1 / (2 * np.pi * sigma**2)) * np.exp(-(x**2 + y**2) / \
         (2 * sigma**2)) # Gaussain
        LoG = ((x**2 + y**2 - 2*sigma**2) / (sigma**4)) * g # LoG
        LoG_normalized = sigma**2 * LoG # Normalization

        # Ensure the sum of the filter is 0 (for edge detection property)
        LoG_normalized -= LoG_normalized.mean()

        return LoG_normalized

    except ValueError as ve:
        print(f"ValueError: {ve}")
    except Exception as e:
        print(f"An error occurred: {e}")
```

As you might have seen above, the image blobs have 5 different scales. Hence you have to find 5 `sigma` values which will give maximum response when the LoG filter is convolved with the image. You may want to use the results from the class lectures to find the values for `sigma`. To visualize maximum response we will be plotting the filtered images using a color map where blue color would correspond to smaller values and red color would correspond to higher values.

Answer 2.6

Tune the values of the 5 `sigmas`, so that you get the maximum response for each scale. So, `sigma_1` should be such that you get the maximum response for the smallest blobs, `sigma_5` should give you the maximum response for the largest blobs.

Write the values of the 5 `sigmas` you get here on Overleaf.



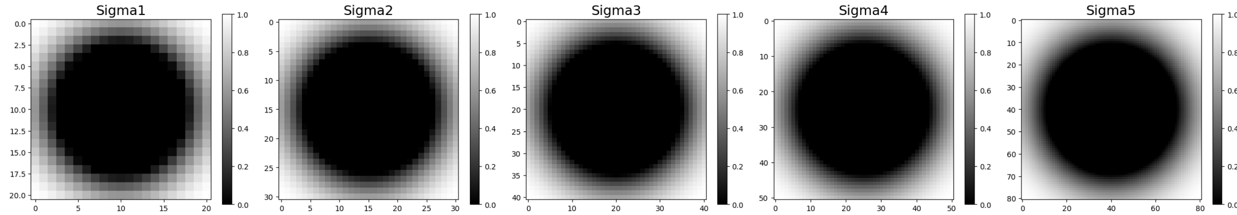
In [ ]:
```
# from class notes
import math
sigma_1 = (21/2)/math.sqrt(2)
log_1 = log_filter(21, sigma_1)
sigma_2 =(31/2)/math.sqrt(2)
log_2 = log_filter(31, sigma_2)
sigma_3 =(41/2)/math.sqrt(2)
log_3 = log_filter(41, sigma_3)
sigma_4 =(51/2)/math.sqrt(2)
log_4 = log_filter(51, sigma_4)
sigma_5 =(81/2)/math.sqrt(2)
log_5 = log_filter(81, sigma_5)
```

Answer 2.7

In this sub-part, you will visualize the LoG filters.

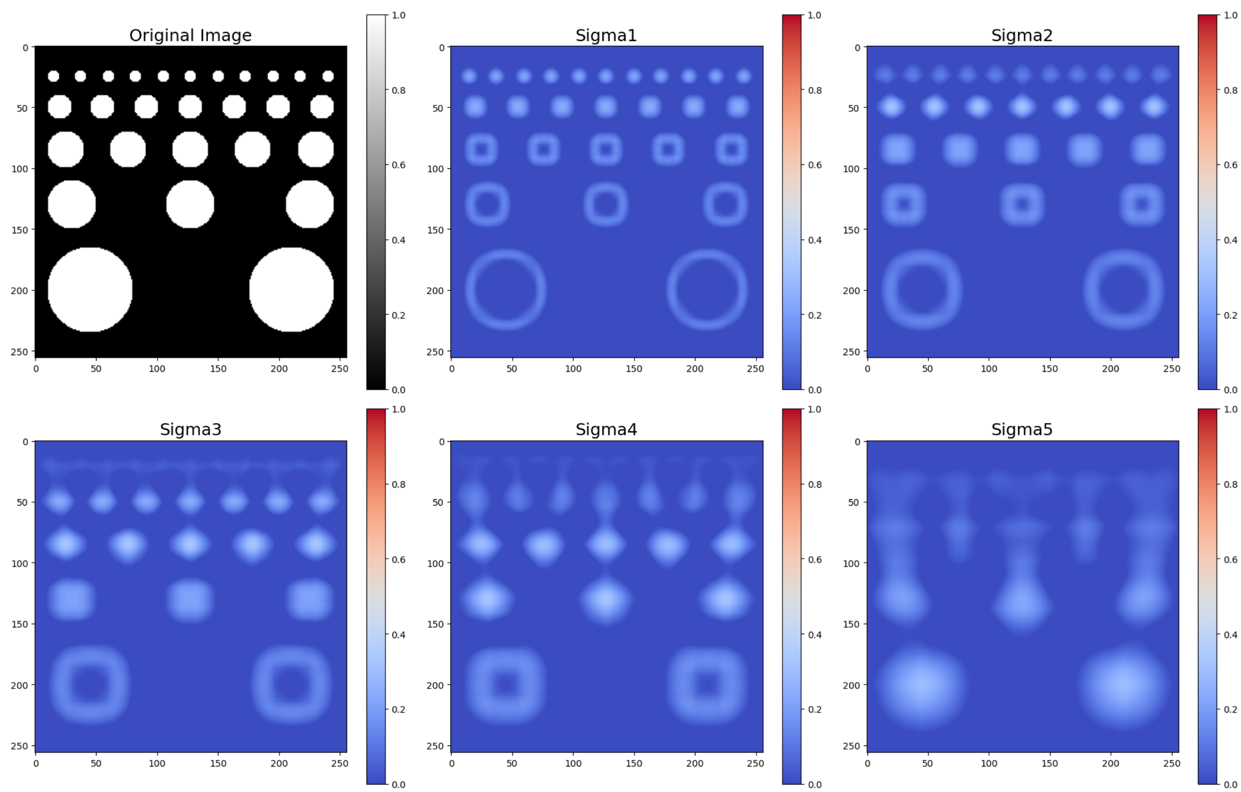
Upload the saved image on Overleaf for Question 2.7.

In [ ]:
```
fig, ax = plt.subplots(1,5,figsize=(1 + 5*4.5,4))
display_axis_bw(ax[0],log_1,'Sigma1',normalized=True,fig=fig)
display_axis_bw(ax[1],log_2,'Sigma2',normalized=True,fig=fig)
display_axis_bw(ax[2],log_3,'Sigma3',normalized=True,fig=fig)
display_axis_bw(ax[3],log_4,'Sigma4',normalized=True,fig=fig)
display_axis_bw(ax[4],log_5,'Sigma5',normalized=True,fig=fig)
fig.tight_layout()
os.makedirs('Data/Solutions', exist_ok=True)
fig.savefig('Data/Solutions/question_2_7.pdf', format='pdf', bbox_inches='tigh
```



In [ ]:
```
log_conv_1 = conv2D(img, -log_1)
log_conv_2 = conv2D(img, -log_2)
log_conv_3 = conv2D(img, -log_3)
log_conv_4 = conv2D(img, -log_4)
log_conv_5 = conv2D(img, -log_5)
```

In [ ]:
```
# with sigma r/sqtt(2)
fig, ax = plt.subplots(2,3,figsize=(1 + 3*6,2*6))
display_axis_bw(ax[0,0],img,'Original Image',fig=fig)
display_axis_color(ax[0,1],log_conv_1,'Sigma1',fig=fig)
display_axis_color(ax[0,2],log_conv_2,'Sigma2',fig=fig)
display_axis_color(ax[1,0],log_conv_3,'Sigma3',fig=fig)
display_axis_color(ax[1,1],log_conv_4,'Sigma4',fig=fig)
display_axis_color(ax[1,2],log_conv_5,'Sigma5',fig=fig)
fig.tight_layout()
fig.savefig('Data/Solutions/question_2_8.pdf', format='pdf', bbox_inches='tigh
```

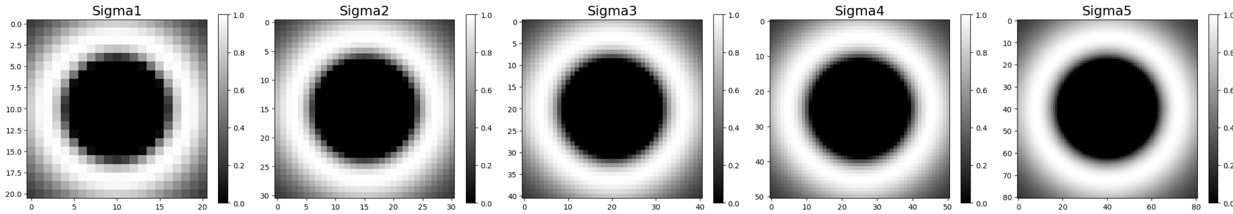


Better Estimators

In [ ]:
```
sigma_1 = 21/5
log_1 = log_filter(21, sigma_1)
sigma_2 =31/5
log_2 = log_filter(31, sigma_2)
sigma_3 =41/5
log_3 = log_filter(41, sigma_3)
sigma_4 =51/5
```

```
log_4 = log_filter(51, sigma_4)
sigma_5 =81/5
log_5 = log_filter(81, sigma_5)
```

In [ ]:
```
fig, ax = plt.subplots(1,5,figsize=(1 + 5*4.5,4))
display_axis_bw(ax[0],log_1,'Sigma1',normalized=True,fig=fig)
display_axis_bw(ax[1],log_2,'Sigma2',normalized=True,fig=fig)
display_axis_bw(ax[2],log_3,'Sigma3',normalized=True,fig=fig)
display_axis_bw(ax[3],log_4,'Sigma4',normalized=True,fig=fig)
display_axis_bw(ax[4],log_5,'Sigma5',normalized=True,fig=fig)
fig.tight_layout()
os.makedirs('Data/Solutions', exist_ok=True)
fig.savefig('Data/Solutions/question_2_7.pdf', format='pdf', bbox_inches='tigh
```



Convolve the image with the 5 filters. Note that we multiply the filters with -1 so that the maximum response is positive.

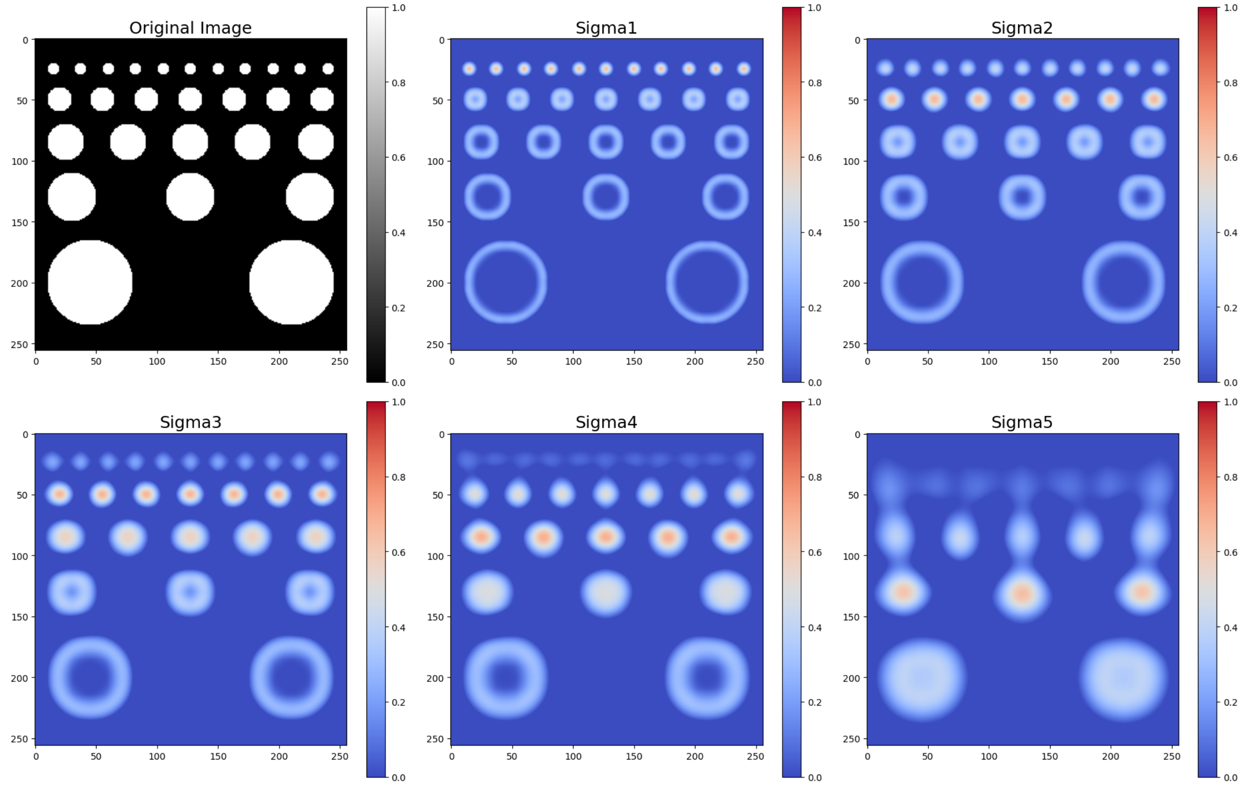
In [ ]:
```
log_conv_1 = conv2D(img, -log_1)
log_conv_2 = conv2D(img, -log_2)
log_conv_3 = conv2D(img, -log_3)
log_conv_4 = conv2D(img, -log_4)
log_conv_5 = conv2D(img, -log_5)
```

Answer 2.8

In this sub-part, you will visualize the blob detection results. We also plot the colorbar with each image. You should use that to tune the values for `sigma`.

Upload the saved image on Overleaf for Question 2.8.

In [ ]:
```
fig, ax = plt.subplots(2,3,figsize=(1 + 3*6,2*6))
display_axis_bw(ax[0,0],img,'Original Image',fig=fig)
display_axis_color(ax[0,1],log_conv_1,'Sigma1',fig=fig)
display_axis_color(ax[0,2],log_conv_2,'Sigma2',fig=fig)
display_axis_color(ax[1,0],log_conv_3,'Sigma3',fig=fig)
display_axis_color(ax[1,1],log_conv_4,'Sigma4',fig=fig)
display_axis_color(ax[1,2],log_conv_5,'Sigma5',fig=fig)
fig.tight_layout()
fig.savefig('Data/Solutions/question_2_8.pdf', format='pdf', bbox_inches='tigh
```



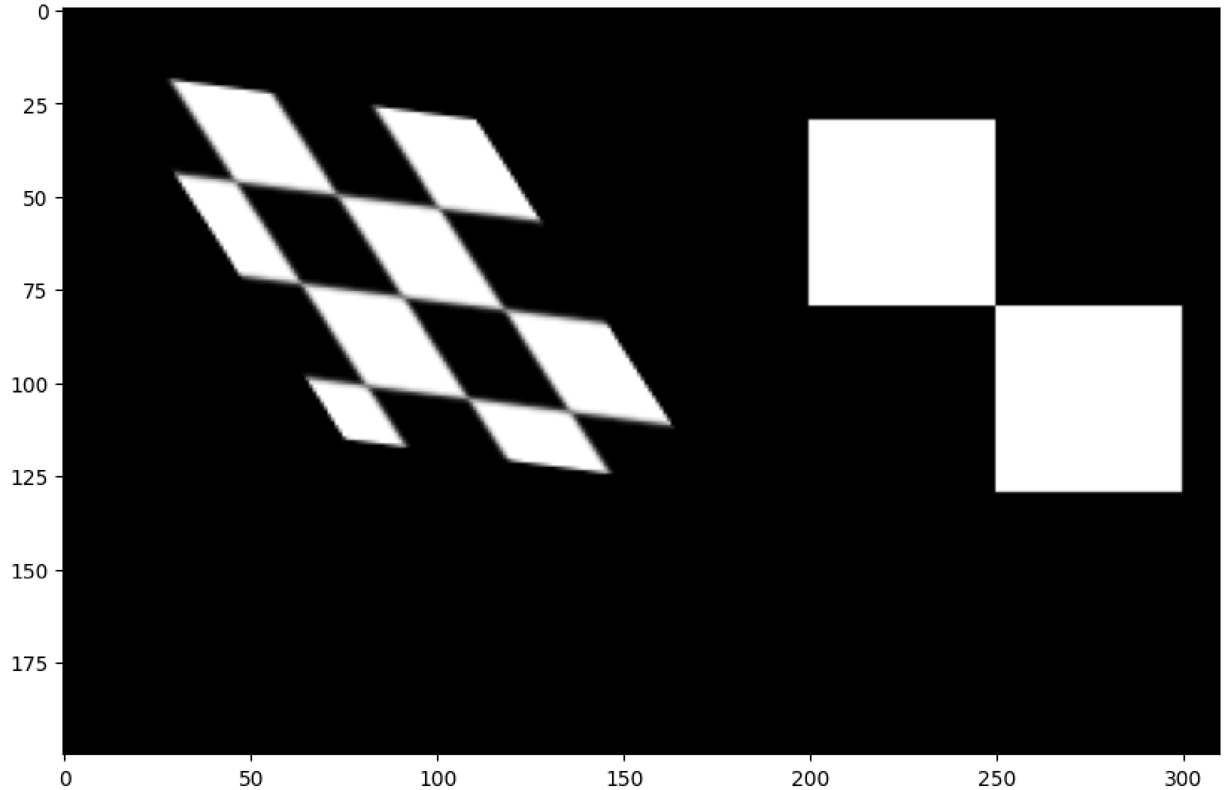
Question 3

Corner Detection

In this question, you will be implementing the Harris corner detector. Corners serve as useful features in images for a variety of reasons: they are salient, well localized, and invariant to a variety of transformations (illumination, rotation, and scale changes). The Harris corner detector also possesses some of these invariances (rotation and intensity shift) but is not invariant to image and intensity scaling. Nevertheless, the Harris corner detector is still a popular method for detecting corners in images.

A grayscale test image (normalized to be in [0, 1]) containing some squares and a warped checkerboard is constructed below.

In [ ]:
```
# Sheared checkerboard
tform = AffineTransform(scale=(1.3, 1.1), rotation=1, shear=0.7,
                        translation=(110, 30))
image = warp(data.checkerboard()[:90, :90], tform.inverse,
             output_shape=(200, 310))

# Two squares
image[30:80, 200:250] = 1
image[80:130, 250:300] = 1

display_gray(image)
```



Computing Image Gradients

The first step in the Harris corner detector is to compute the image gradients. While there are a variety of different methods to compute gradients, you will use the Sobel filter, which is defined below for the x and y directions.

In [ ]:
```
sobel_x = np.array([[1, 0, -1], [2, 0, -2], [1, 0, -1]])
sobel_y = np.array([[1, 2, 1], [0, 0, 0], [-1, -2, -1]])
```

After defining the Sobel filters, you need to apply them to obtain the gradients. Complete a function `compute_image_gradient(image)` that returns the horizontal (along the x direction) and vertical (along the y direction) image gradients using the provided Sobel filters. For this function (and all convolutions that follow), use the `conv2D(image, kernel)` function, which you should copy paste into the cell at the beginning of Question 2.

Answer 3.1

Copy paste your solution in the cell below on Overleaf for Question 3.1.

In [ ]:
```
# Write your code in this cell.

def compute_image_gradient(image: np.array):
    """
    Compute the horizontal and vertical image gradients using Sobel filters.

    Author Simon Lee
    """
```

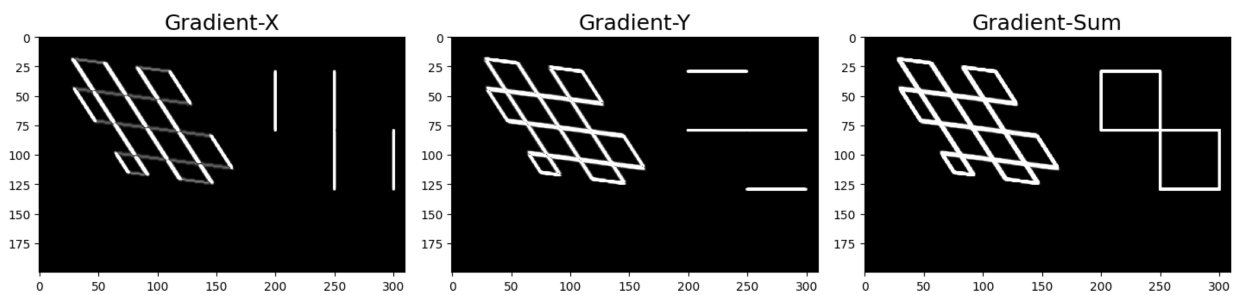
```
    gradient_x = conv2D(image, sobel_x)
    gradient_y = conv2D(image, sobel_y)

    return gradient_x, gradient_y
```

Display the absolute gradient along the horizontal and vertical directions and their sum. You should observe that the gradient in the horizontal (x-direction) is unable to capture the horizontal edges while the gradient in the vertical (y-direction) is unable to capture the vertical edges. Both gradients capture the diagonal edges, but they may appear dimmer than the horizontal and vertical edges.

Answer 3.2

Execute the cell below and copy the saved image on Overleaf for Question 3.2.

In [ ]:
```
fig, ax = plt.subplots(1,3,figsize=(1 + 3*4.5,4))
img_gradient_x, img_gradient_y = compute_image_gradient(image)
display_axis(ax[0], np.abs(img_gradient_x), 'Gradient-X')
display_axis(ax[1], np.abs(img_gradient_y), 'Gradient-Y')
display_axis(ax[2], np.abs(img_gradient_x) + np.abs(img_gradient_y), 'Gradient-
fig.tight_layout()
os.makedirs('Data/Solutions', exist_ok=True)
fig.savefig('Data/Solutions/question_3_2.pdf', format='pdf', bbox_inches='tigh
```



Computing the Covariance Matrix

After computing the gradients, the Harris corner detector then computes the covariance matrix of the gradients (see lecture 5 slide 26). Complete the function `grad_covariance(image, size)` that computes each pixel's covariance matrix using the `size x size` window centered at the pixel. This function should return three matrices I_{xx}, I_{xy}, I_{yy} containing the top-left, diagonal, and bottom-right terms, respectively, of every pixel's covariance matrix. When computing the covariance, you do not need to subtract the means of the image gradients. The average filter is provided below as a useful function.

In [ ]:
```
# This is the standard box filter which computes the mean of all the pixels ins
def average_filter(size: int):
    assert size%2 == 1
    return 1.0 * np.ones((size,size))/(size**2)
```

Answer 3.3

Copy paste your solution in the cell below on Overleaf for Question 3.3.

In [ ]:
```
# Write your code in this cell.

def grad_covariance(image: np.array, size: int):
    """
    Computes the covariance matrix

    Author Simon Lee
    """
    gradient_x, gradient_y = compute_image_gradient(image)

    # Compute the squares and product of the gradients
    Ixx = gradient_x ** 2
    Iyy = gradient_y ** 2
    Ixy = gradient_x * gradient_y
    avg_filter = average_filter(size) # make avergae filter to convolve

    # Convolve each matrix with the average filter
    Ixx = conv2D(Ixx, avg_filter)
    Iyy = conv2D(Iyy, avg_filter)
    Ixy = conv2D(Ixy, avg_filter)

    return Ixx, Ixy, Iyy
```

Harris Response Function

Finally, the Harris corner detector uses the covariance matrix to compute a response function, which is then thresholded to obtain the locations of the corners. Complete the function `harris_response(image, k, size)` which computes the Harris response function (see lecture 5 slide 43, Harris & Stephens (1988)) for an image using a `size x size` window around every pixel. The parameter `k` corresponds to the parameter in the Harris response function.

Answer 3.4

Copy paste your solution in the cell below on Overleaf for Question 3.4.

You can calculate the harris response using the following equation:

$$R = Det(M) - k * (Trace(M))^2$$

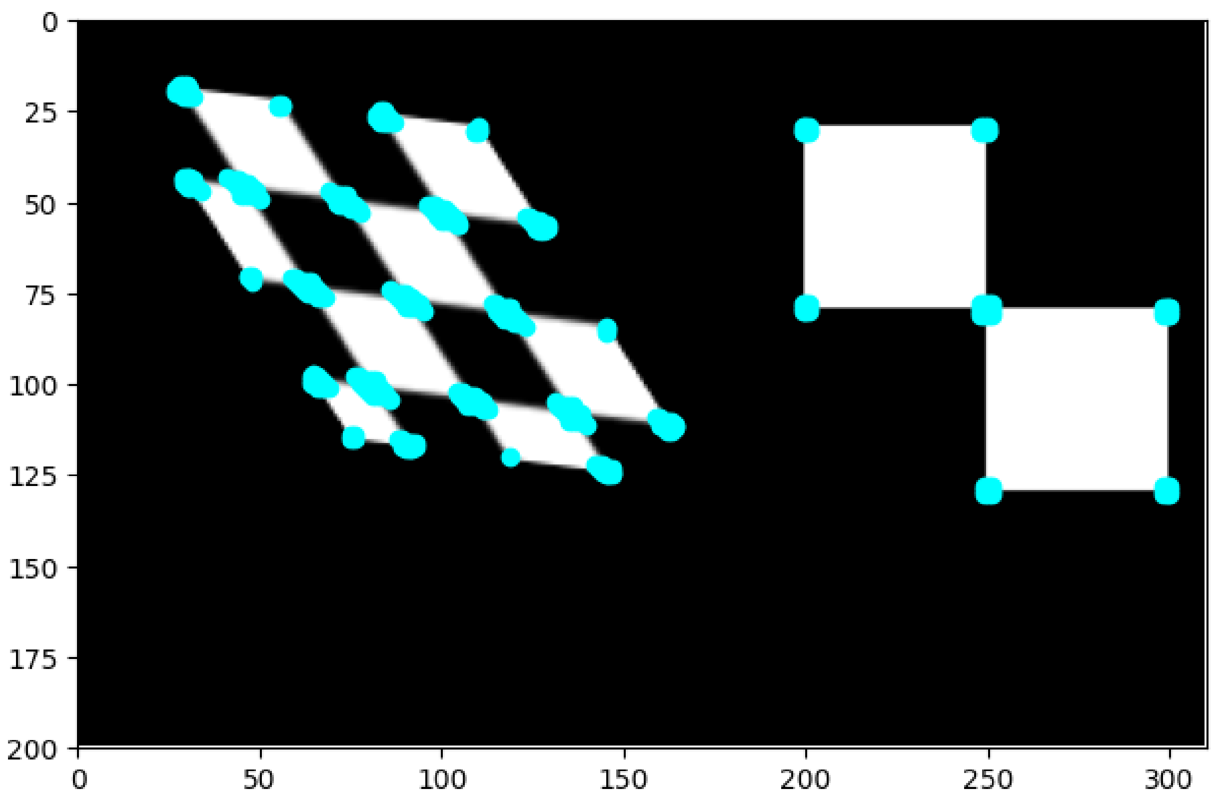
In [ ]:
```
# Write your code in this cell.

def harris_response(image: np.array, k: float, size: int):
    """
    Computes the Harris response function for each pixel in the image.

    Author: Simon Lee
    """
    try:
        # Compute the covariance matrix of gradients
        Ixx, Ixy, Iyy = grad_covariance(image, size)
    except ValueError as e:
        raise ValueError(f"Error computing gradient covariance: {e}")
```

```
    try:
        # Compute the Harris response for each pixel
        Det_M = Ixx * Iyy - Ixy ** 2
        Trace_M = Ixx + Iyy
        R = Det_M - k * (Trace_M ** 2)
    except Exception as e:
        raise ValueError(f"Error computing Harris response: {e}")

    return R
```

Answer 3.5

Execute the cell below and copy the saved image on Overleaf for Question 3.5. The cell below finds every pixel location where the Harris response function is above a certain threshold and then shows the locations of these pixels (the corner detections) in cyan.

In [ ]:
```
coords = np.argwhere(harris_response(image, 0.05, 3) > 0.02)
fig, ax = plt.subplots()
ax.imshow(image, cmap=plt.cm.gray)
ax.plot(coords[:, 1], coords[:, 0], color='cyan', marker='o',
        linestyle='None', markersize=6)
ax.axis((0, 310, 200, 0))
fig.tight_layout()
plt.show()
fig.savefig('Data/Solutions/question_3_5.pdf', format='pdf', bbox_inches='tigh
```



Non-Maximum Suppression

You should observe from the above image that all corners are detected. However, some corners are detected multiple times, which is due to simply thresholding the Harris response. To suppress multiple detections, you will implement, in steps, a function `non_max_suppression(harris_response, distance, threshold)` that applies non-maximum suppression to the Harris corner response and returns the remaining corner detections.

Non-maximum suppression works as follows:

1. Threshold the Harris response map to obtain the pixel locations where the response is greater than a certain threshold. These pixel locations form our candidate corner detections.
2. Sort these detections based on maximum response value.
3. Go through the sorted detections in order and for each detection, remove other detections that are within a certain Euclidean distance from the current detection. This step suppresses detections that are not local maxima.

You will implement the three steps of non-maximum suppression in order. First, complete the function `threshold_harris_response(harris_response, threshold)`, which returns the indices of the Harris response map corresponding to values that are greater than some threshold.

Answer 3.6

Copy paste your solution in the cell below on Overleaf for Question 3.6.

In [ ]:
```
# Write your code in this cell.

def threshold_harris_response(harris_response: np.array, threshold: float):
    """
    Thresholds the Harris response map to obtain pixel locations with
    response greater than the threshold.

    Author Simon Lee
    """
    # Find indices where the response is greater than the threshold
    indices = np.argwhere(harris_response > threshold)

    return indices
```

Then, complete the function `sort_detections(candidate_detections, harris_response)`, which returns the candidate detections sorted based on maximum Harris response value.

Answer 3.7

Copy paste your solution in the cell below on Overleaf for Question 3.7.

In [ ]:
```
# Write your code in this cell.

def sort_detections(candidate_detections: np.array, harris_response: np.array)
    """
    Sorts the candidate detections based on the Harris response value in descen

    Author: Simon Lee
    """
    # Extract the Harris response values for each candidate detection
    # and sort in descending order
    sorted_indices = np.argsort([harris_response[i, j] for i, j in candidate_de

    # Arrange the detections in sorted order
    sorted_detections = candidate_detections[sorted_indices]

    return sorted_detections
```

The final step is to go through the sorted detections and suppress detections that are not local maxima. Complete the function `local_max(sorted_detections, distance)`, which goes through the sorted detections and returns only the detections that are local maxima (using local neighborhoods defined by a Euclidean distance threshold `distance`).

A function that computes Euclidean distance between two points is provided below for convenience.

In [ ]:
```
def l2_distance(p1: np.array, p2: np.array):
    return np.linalg.norm(p1 - p2, ord=2)
```

Answer 3.8

Copy paste your solution in the cell below on Overleaf for Question 3.8.

In [ ]:
```
# Write your code in this cell.

def local_max(sorted_detections: np.array, distance: float):
    """
    Returns the detections that are local maxima.

    Author: Simon Lee
    """
    local_maxima = []
    for current_detection in sorted_detections:
        # Compare current detection to all detections that are already conside
        if all(l2_distance(current_detection, existing_detection) > distance fo
            local_maxima.append(current_detection)

    return np.array(local_maxima)
```

Now, combine the three previously implemented functions to complete the function `non_max_suppression(harris_response, distance, threshold)`, which applies non-maximum suppression to the Harris corner response and returns the remaining corner detections as a NumPy array of (row, col) locations.

Answer 3.9

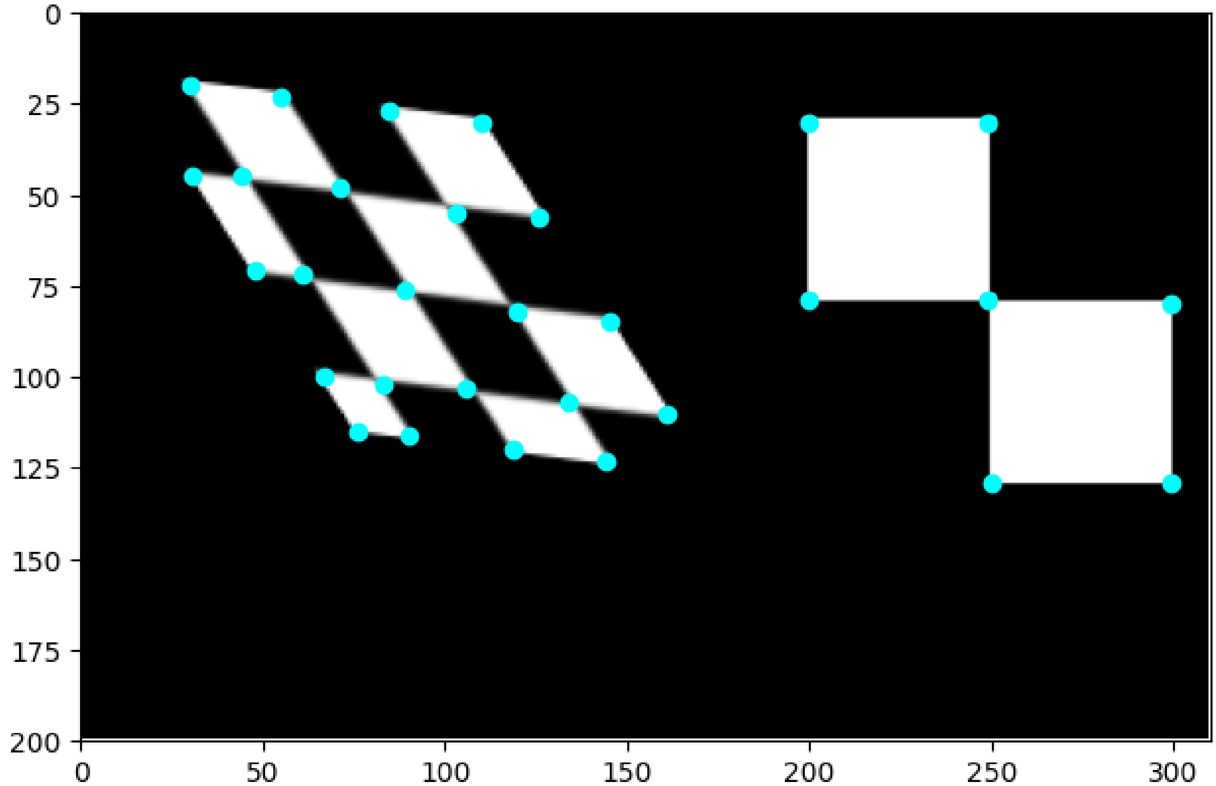
Copy paste your solution in the cell below on Overleaf for Question 3.9.

In [ ]:
```
# Write your code in this cell.

def non_max_suppression(harris_response: np.array, distance: float, threshold:
    """
    Applies non-maximum suppression to the Harris corner response to get corne
    using 3 previous functions we wrote

    Author: Simon Lee
    """
    # run all three functions
    candidate_detections = threshold_harris_response(harris_response, threshol
    sorted_detections = sort_detections(candidate_detections, harris_response)
    corners = local_max(sorted_detections, distance)

    return corners
```

Answer 3.10

Execute the cell below and copy the saved image on Overleaf for Question 3.10. The cell below runs non-maximum suppression on the Harris response map and then shows the locations of the corner detections in cyan. Duplicate corner detections should now be removed.

In [ ]:
```
coords = non_max_suppression(harris_response(image, 0.05, 3), 10.0, 0.02)
fig, ax = plt.subplots()
ax.imshow(image, cmap=plt.cm.gray)
ax.plot(coords[:, 1], coords[:, 0], color='cyan', marker='o',
        linestyle='None', markersize=6)
ax.axis((0, 310, 200, 0))
fig.tight_layout()
plt.show()
fig.savefig('Data/Solutions/question_3_10.pdf', format='pdf', bbox_inches='tig
```


0
25
50
75
100
125
150
175
200
0
50
100
150
200
250
300