Bombay Modeling

Simon Lee, Ayrton Uranga

February 2022

1 Taylor Expansion to extract k & l

We have this equation with respect to $\frac{dR}{dt}$ because the variable R is linearly related to the number of deaths. Because our objective is to provide a curve that best fits the available data, we first begin by Taylor expanding to extract the values of R & R_0 from the below equation. S_0 and N are roughly the same and therefore we can cancel the terms out.

$$\frac{dR}{dt} = l(S_0 - (R - R_0) - S_0 e^{-}\frac{k(R - R_0)}{NL}) \tag{1}$$

We assume that the $R - R_0$ is small. After performing the Taylor expansion we obtain the following equation.

$$f(x) = \sum_{k=1}^{2} l(S_0 - (R - R_0) - S_0 e^{-}\frac{k(R - R_0)}{NL} = R(\frac{kS}{N} - l) - \frac{R^2(k^2 S)}{2(lN^2)} \tag{2}$$

Now we approximate the curve using MatLab's built-in curve fitting tool. We use a custom equation and plot the general quadratic equation $aR - bR^2$ where a and b are some unknown constants.

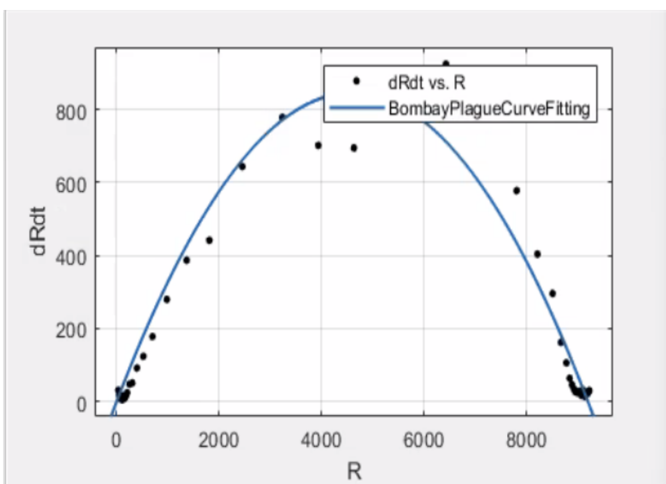

Figure 1: Curve Fitting of the Bombay Plague Data

After obtaining our curve we now have two known values for the coefficients a and b, where a = 0.3681, and b = 4.005e-05. And now from this value we can isolate the variables and solve for k and l respectively. Below are the following steps:

$$a = k - l -> 0.3681 = k - l -> k = 0.3681 + l \tag{3}$$

We decided that we wanted to show our work for how we obtained the values of k and l but warning it the arithmetic is trivial but ugly.

$$b = \frac{k^2 S}{2lN^2} \tag{4}$$

$$4.005e - 05 = \frac{k^2}{2lN} \tag{5}$$

$$\frac{801}{200} * \frac{1}{10^5} = \frac{(\frac{2681}{10000} + 2)^2}{1800000l} \tag{6}$$

$$\frac{801}{20000000} = \frac{(\frac{3681)}{10000} + l)^2}{1800000l} \tag{7}$$

$$\frac{801}{20000000} = \frac{3681^2 + 73620000l + 10000^2 l^2}{1800000 * 10000^2 l} \tag{8}$$

$$1441800000 * 10000^2 l = 2000000(3681^2 + 7362000l + 10000^2 l^2)/200000 \tag{9}$$

$$7209 * 10000^2 l = 100(3681^2 + 7362000l + 10000^2 l^2) \tag{10}$$

$$7209 - 10000^2 l - 100 * 3681^2 - 73620000l * 100 - 100 * 10000^2 l^2 = 0 \tag{11}$$

$$-100^5 l^2 + (7209 * 10000^2 - 100 * 73620000)l + 100 * 3681^2 = 0 \tag{12}$$

$$l = -(-(7209 * 10000^2 - 100 * 73620000) \pm (-7209 * 10000^2 - 100 * 73620000)^2) \tag{13}$$

$$\frac{-4(100^5)(100 - 3681^2)^(1/2)}{2(100)^5} \tag{14}$$

$$k = 0.369999 \tag{15}$$

$$l_1 = 0.001899 \tag{16}$$

$$l_2 = 71.2519 \tag{17}$$

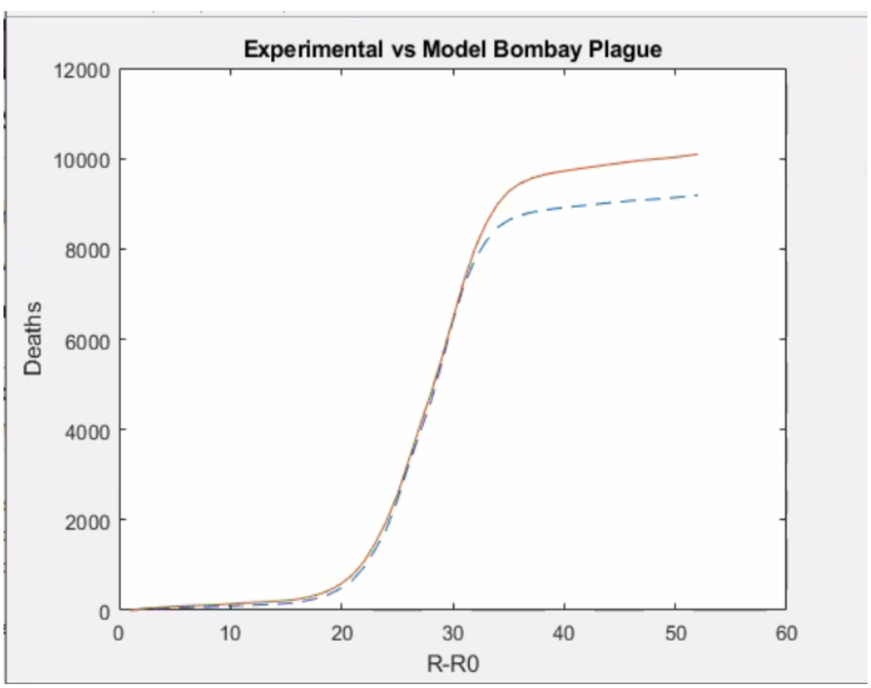

Figure 2: Our k and l plugged into a graph comparing $\frac{dR}{dt}$ by $R - R_0$