Optimization Inquiry Outline

Simon Lee

Audience: The audience of this paper is applied mathematicians who are interested in optimizations or finding minimums of a given function or system.

1 Introduction

- Introduce the field of optimization. Optimization is a field in applied mathematics and numerical analyis where we are looking for an optimal solution of some system or equation in finite time.
- Explain how the field has had many applications in the real world, and how the power of better numerical algorithms combined with stronger computing power have made it possible to approach complex problems.
- Briefly speak about our problem and what we wish to tackle. Given that we have a puck or ball that slides down a frictionless slide connecting from point A to B (with considerations to potential and kinetic energy), what is the function that yields the best time.

2 Model & Method

2.1 The Euler Lagrange Equation

- Introduce the problem once more with more detail. We first begin by adding a coordinate system for these points A and B, as well as the time equation that we first derived.

$$T = \int_0^{x_b} \frac{\sqrt{1+y'^2}}{\sqrt{2g(y_A - y(x))}} dx. \tag{1}$$

- Explain how this time equation (Functional) is the basis of this problem and how we will use it as a measure to find the most optimal solution.
- Explain that if we discretize the curve from point A to point B with an infinite amount of lines, that we could write this equation out in terms of our new coordinate system.

$$T(\{x_i, y_i\}) = \sum_{i=0}^{N} \int_{x_i}^{x_{i+1}} \frac{\sqrt{1+(\frac{y_{i+1}-y_i}{x_{i+1}-x_i})^2}}{\sqrt{2g(y_a - (y_i + \frac{y_{i+1}-y_i}{x_{i+1}-x_i}(x-x_i))}} dx. \tag{2}$$

- Now that we have this new equation we can write out the summation of this equation which eventually yields an 2pt boundary value problem ODE. **Alot of derivation in between from last point** We end up with the following equation:

$$\boxed{(y_a - y)(1 + y'^2) = \text{constant}} \tag{3}$$

with the boundary conditions, $f(x=0) = y_a$ and $f(x=b) = 0$

- This 2pt ODE can now be redefined into what we know today as the Euler Lagrange Equation

$$\frac{\partial \mathscr{L}}{\partial y} = \frac{d}{dx}(\frac{\partial \mathscr{L}}{\partial y'}). \tag{4}$$

- At last we can perform a few more derivations and we end up with a second order ODE of the form

$$y'' = \frac{1 + y'^2}{2(y_a - y)}. \tag{5}$$

**** The goal of all this derivation is for the reader to know that your assumption makes sense****

2.2 Methods

- Introduce the numerical experiments in Matlab that you wish to run of your newly found
- Can also check the accuracy of the code by putting it up against an analytical solution to verify that it does indeed work
- By showing that this numerical method obtains the correct and most optimal function, we can acknowledge the power of computation
- Discuss how we put this up against a method of brute forcing a family of functions and how guessing and checking is not as efficient to getting the actual optimal function (however it does get you really close for the family of functions $y = (x-1)^n$

3 Conclusion

- Take a step back from the problem and talk about what was just covered in the paper (summarization)
- Talk about how this Euler Lagrange equation can be used in more than just a basic problem like a frictionless slide and can be applied to more complex functions with multiple variables for example
- Leave the readers excited by explaining that the field of optimization is only growing with the new developments of numerical methods and the how they can be used real time situations