Mathematical Model for Water & Ice Cube Experiment

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1 Mathematical Model

Our mathematical model involves the physics of melting and how a given ice cube melts in a body of fresh water at a fixed temperature. Through this model we wish to extract the melt time from the melting velocity and the height of the ice cube. Below we show equations needed for the final model

Melting Velocity (v_m)

$$v_m = \frac{\rho_{water} c_p v_{conv} \Delta T}{\Lambda \rho_{ice}} \tag{1}$$

numerator is known as the convective heat flux denoted as Q

$\rho_{water} = 1000 kg/m^3$ - density of water (fixed constant)
$c_p = 4200 J/(kgK)$ - specific heat at constant pressure (fixed constant)
v_{conv} = *input parameter* - convection velocity in m/s
ΔT = *input parameter* - temperature difference between ice and water
$\Lambda = 3.34x10^5 J/kg$ - heat capacity (fixed constant)
$\rho_{ice} = 917 kg/m^3$ - density of ice (fixed constant)

Rayleigh's Theorem

$$\delta \approx 10(\frac{v_{visc} k_T}{\alpha g \Delta T})^{\frac{1}{3}}, v_{conv} \approx \frac{K_T}{\delta} \tag{2}$$

$v_{visc} = 1.3x10^{-6} m^2/s$ - viscosity (fixed constant)
$k_T = 1.3x10^{-7} m^2/s$ - thermal diffusivity (fixed constant)
$\alpha = 10^{-4} K^{-1}$ - coefficient of thermal expansion
$g = 9.8 m/s^2$ - gravity (constant)
ΔT = *input parameter* (same from Equation 1)

My Model for time

$$t_m = \frac{H}{2 v_m} \tag{3}$$

H = *input parameter* - height of the ice cube
v_m = melting velocity from Equation 1

From the following model based on all these constants and input parameters we are able to model the melt time for an ice cube in fresh water

2 Data

Test	ρ_{water}	c_p	ν_{conv}	ΔT	Λ	ρ_{ice}	ν_{visc}	k_T	α	g	H	t_m
Trial 1	1000	4200	0.00064943541	21.5	$3.34x10^5$	917	$1.3x10^{-6}$	$1.3x10^{-7}$	10^{-4}	9.8	0.0288	587
Trial 2	1000	4200	0.00064127838	20.7	$3.34x10^5$	917	$1.3x10^{-6}$	$1.3x10^{-7}$	10^{-4}	9.8	0.0256	481
Trial 3	1000	4200	0.00064640064	21.2	$3.34x10^5$	917	$1.3x10^{-6}$	$1.3x10^{-7}$	10^{-4}	9.8	0.0357	688

** sorry for not including units...*

In this experiment I varied the H variable which resembles the height of the ice cube coming from my mathematical model (Equation 3). Through these experiments I ran, I was able to see how this height variable affected the melting time of the overall ice cube. In my experiments, I constructed 3 different sized ice cubes listed in the table labeled H and put it inside the ΔT temperature water (not constant) and timed the ice cubes melting using a stop watch. Doing so helped generate my data points which shows how long it took to melt the ice cubes given the size. In my results my $0.0256m$, and $0.0288m$ sized ice cubes melted a little slower than my expected result, and my $0.0357m$ sized ice cube melted faster than expected. These results were skewed because I believe the ΔT variable was not fixed and had some small effect on the melting time. The mathematical model was also plotted below and is being shown as a linear line that would show how long it would take to melt given the size.

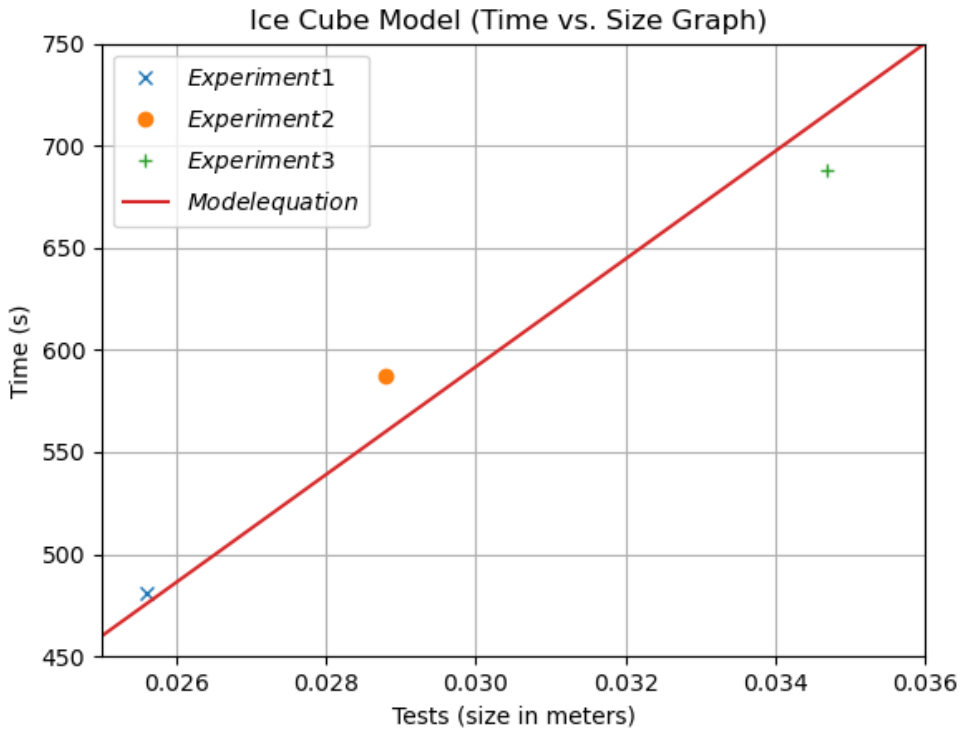

Figure 1: Using the different sized ice cubes (input parameters) I was able to graph a few lines based on my experimental data