AM 147: Computational Methods and Applications: Winter 2022

Homework #4

Instructor: Abhishek Halder

Due: February 02, 2022

NOTE: Please submit your Homework as a single zip file named `YourlastnameYourfirstnameHW4.zip` via CANVAS. For example, `HalderAbhishekHW4.zip`. Please strictly follow the capital and small letters in the filename of the zip file you submit. You may not receive full credit if you do not follow the file-naming conventions. Your zip file should contain all .m files (MATLAB scripts) for the questions below.

Your zip file must be uploaded to CANVAS by 11:59 PM Pacific Time on the due date. The uploads in CANVAS are time-stamped, so please don't wait till last moment. Late homework will not be accepted.

Problem 1

Newton's method **($15 + 15 = 30$ points)**

(a) Using `format long`, write a MATLAB script `YourlastnameYourfirstnameHW4p1.m` that performs 6 Newton iterations to compute $3^{1/5}$ starting with $x_0 = 2$, that is, the script should compute the Newton iterates $x_1, x_2, x_3, x_4, x_5, x_6$.

(b) During Lecture 7 (slide 3, approx. 8-13 min in that lecture video) we explained why the nonlinear equation $f(x) := 2x^5 - 5x^4 + 20x^3 - 10x^2 + 10x - 1 = 0$ has unique real root in $[0, 1]$. Upon hearing such cool things you are learning in this class, your mathematician friend claims that this root equals

$$x_{\text{true}} = \frac{3^{1/5} - 1}{3^{1/5} + 1}.$$

To verify this claim, in the same code as in part (a), compute

$$x_{\text{approx}} = \frac{x_6 - 1}{x_6 + 1}$$

where x_6 is the last iterate from part (a). In the same code, plot the function $f(x)$ over $[0, 1]$ together with x_{approx} (to provide numerical evidence that your friend possibly got the formula right). For plotting x_{approx}, you can do

```
plot(x_approx,0,'ro','LineWidth', 2, 'MarkerSize', 10)
```

Problem 2

Fixed point recursion **(20 points)**

Using `format long`, write a MATLAB code with filename `YourlastnameYourfirstnameHW4p2.m` that computes the unique real root of the nonlinear equation

$$x - \cos(\sin(x)) = 0$$

using a fixed point recursion that is guaranteed to be globally convergent. In other words, you need to design a fixed point recursion that will provably converge to the unique real root of this equation for arbitrary initial guess.

Use your code to perform 30 recursions $x_1, x_2, \ldots, x_{30}$ starting with the initial guess $x_0 = 5$. You don't need to submit any hand calculations to justify/prove why your recursion is indeed globally convergent.