AM 147: Computational Methods and Applications: Winter 2022

Homework #7

Instructor: Abhishek Halder

Due: February 23, 2022

NOTE: Please submit your Homework as a single zip file named `YourlastnameYourfirstnameHW7.zip` via CANVAS. For example, `HalderAbhishekHW7.zip`. Please strictly follow the capital and small letters in the filename of the zip file you submit. You may not receive full credit if you do not follow the file-naming conventions. Your zip file should contain all .m files (MATLAB scripts) and .pdf files for the questions below.

Your zip file must be uploaded to CANVAS by 11:59 PM Pacific Time on the due date. The uploads in CANVAS are time-stamped, so please don't wait till last moment. Late homework will not be accepted.

Problem 1

Finite difference approximation of derivative and numerical errors (25 points)

Let $f(x) = \exp(100x)$ for real scalar x.

Write a MATLAB file named `YourlastnameYourfirstnameHW7p1.m` to approximate $f'(x_0)$ at $x_0 = 0$ using the forward difference (Lec. 19, p. 6) and the central difference (Lec. 19, p. 8) approximations with step size $h := 2^{-k/4}$ where k increments from 20 to 200 in the steps of one, i.e., $k = 20, 21, \ldots, 199, 200$. Notice that as k becomes larger, h becomes smaller.

In this case, you can compute by hand the true value of $f'(x_0)$ at $x_0 = 0$. Use that true value to make your MATLAB file plot two relative error curves in the same figure window. One curve should be for the forward difference, and another should be for the central difference approximation. Specifically, plot relative errors (in vertical axis) versus h (in horizontal axis) in log-log scale. With appropriately defined variables, the plotting commands in your file should look as follows:

```
figure(1)
loglog(h,rel_err_forward,'--ro','LineWidth',2,'MarkerSize',10)
hold on
loglog(h,rel_err_central,'-bs','LineWidth',2,'MarkerSize',10)
legend('Forward difference','Central difference')
set(gca,'FontSize',30)
xlabel('h','FontSize',30);
ylabel('Relative error','FontSize',30);
grid on
axis tight
```

The figure will reveal interesting trade-off in the choice of step size h with respect to the truncation and round-off errors. (You don't need to submit any explanation of the plots.)

Problem 2

MathWorks tutorial for ordinary differential equations (25 **points)**

We will soon cover numerical algorithms to solve the ordinary differential equation (ODE) initial value problems. Since some of you may not have seen ODEs before, this exercise will prepare you to understand what they are and what does it mean to solve ODEs.

In your browser, go to
`https://www.mathworks.com/learn/tutorials/solving-ordinary-differential-equations-with-matlab.html`
On the top right corner of that page, login to your UCSC MathWorks account that you used to install MATLAB, and finish the ODE tutorial. This is similar to the OnRamp tutorial you did earlier in HW1. When finished, generate the certificate pdf file using "View/Print certificate". Submit this certificate as `YourlastnameYourfirstname_HW7p2.pdf`