AM170A SIR Model

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Feb 6, 2022

1 Numerical Approach

We are interested in the question, 'given some initial conditions and knowing the model parameters, can we predict how many people will eventually get infected'?

The differential equations which govern this SIR model are,

$$\frac{dS}{dt} = -c_1 SI \tag{1}$$
$$\frac{dI}{dt} = c_1 SI - c_2 I \tag{2}$$
$$\frac{dR}{dt} = c_2 I \tag{3}$$

where S is the number of succeptible individuals, I is the number of infected individuals, and R is the number of removed individuals.
The model assumes no-one moves in or out of the area such that

$$N = S + I + R$$

is constant over time.
We can make these model variables dimensionless by dividing by N, $x = \frac{S}{N}$, $y = \frac{I}{N}$, $z = \frac{R}{N}$.
Now we have,

$$\frac{dx}{dt} = -kxy \tag{4}$$
$$\frac{dy}{dt} = kxy - ly \tag{5}$$
$$\frac{dz}{dt} = ly \tag{6}$$
$$x + y + z = 1 \tag{7}$$

We know that $z(T)$ represents the total number of removed individuals at the end of the simulation, i.e., the total number of people who were infected.
Below are some ways to visualize $Z(T)$ as a function of k (infection rate), l (recovery rate), and $x(0)$ (initial amount of susceptible individuals). We assume that initially the total number of individuals in the simulation are split between x and y, meaning there is initially a group of susceptible individuals and the rest of the population falls under the category of initially infected. In other words, $z(0) = 0$, nobody is removed at the initial start of the simulation.

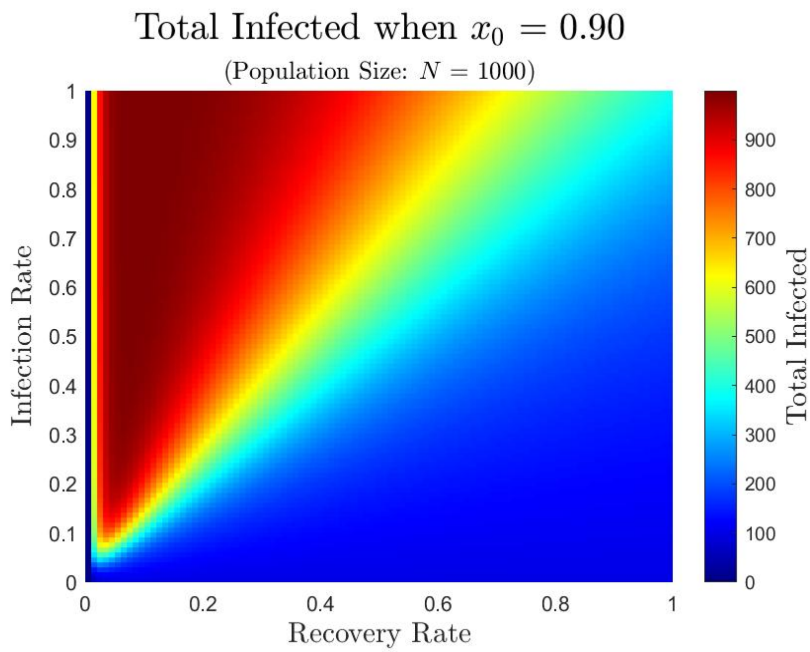

Figure 1: SIR model ($x_0 = 0.9$)

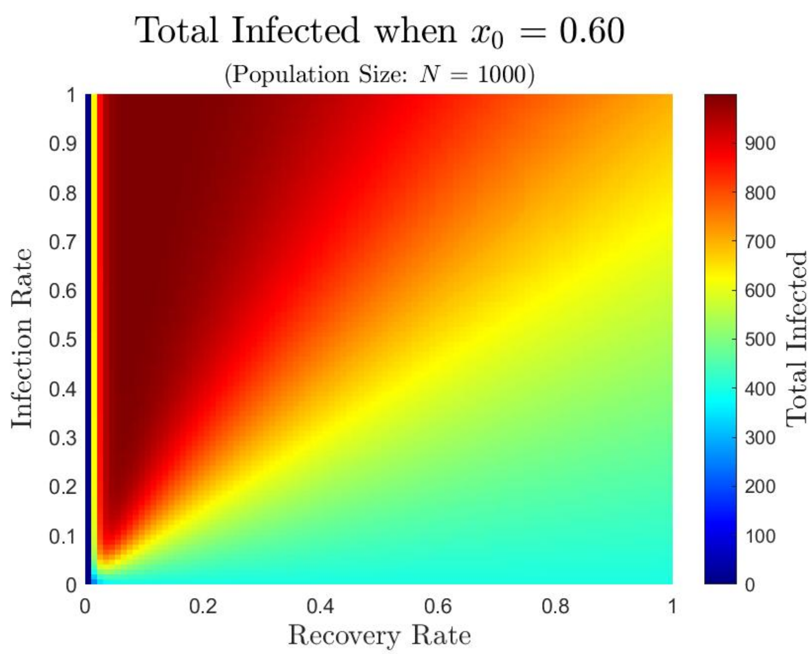

Figure 2: SIR model ($x_0 = 0.6$)

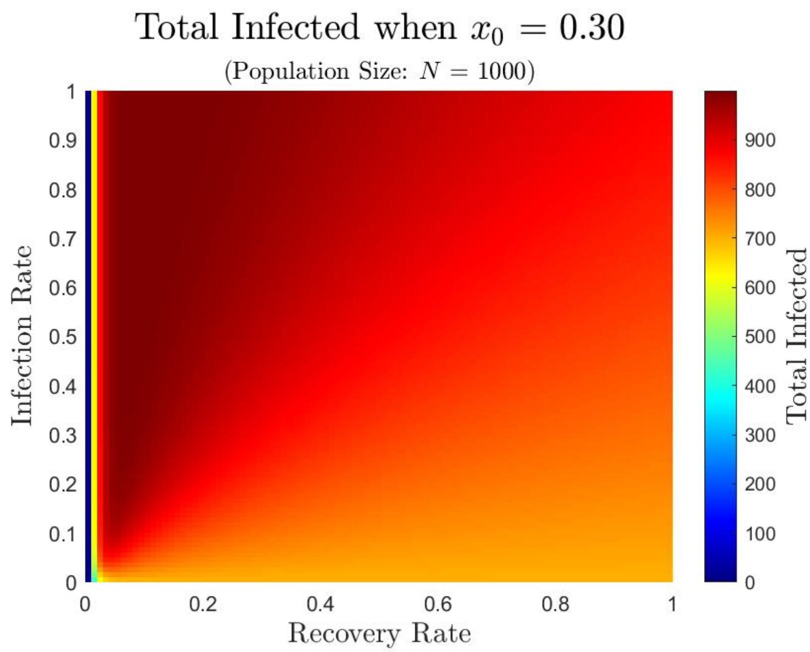

Figure 3: SIR model ($x_0 = 0.3$)

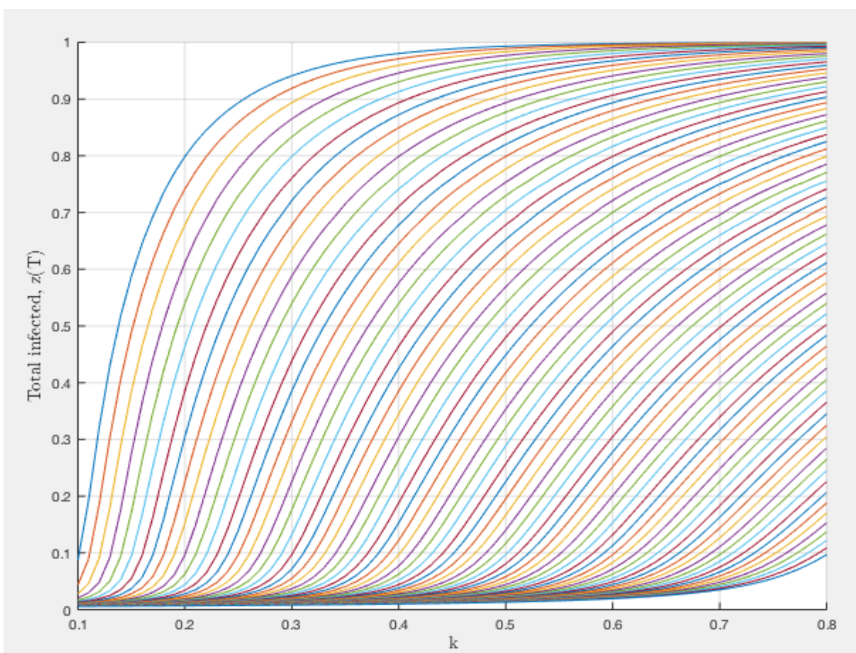

Figure 4: SIR model, Z(T) vs. k for varying values of l. Outermost curve corresponds to L = 0.1, innermost curve corresponds to L = 0.8.

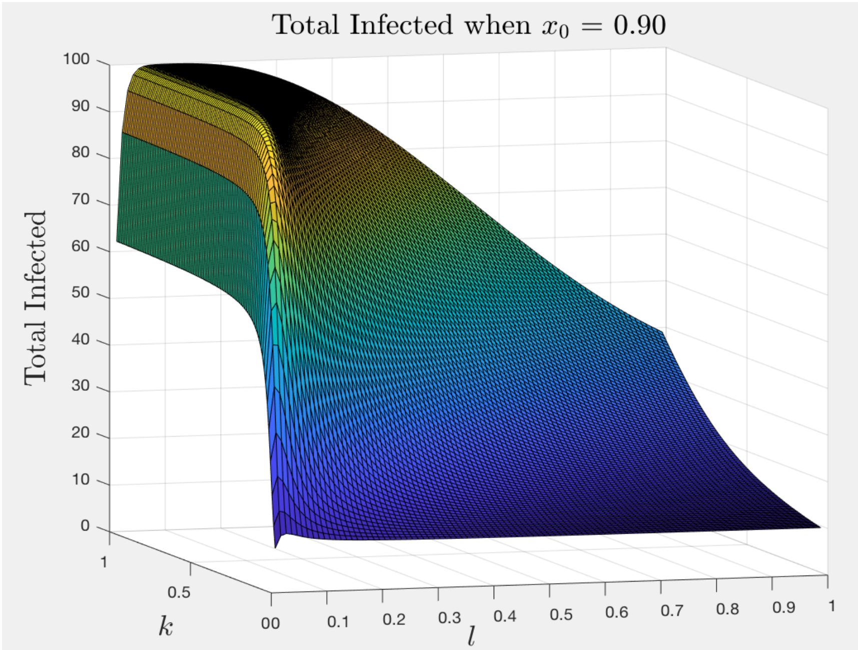

Figure 5: SIR model, Z(T) vs. k and l.