Simon Lee
AM 129

HW 5 Report

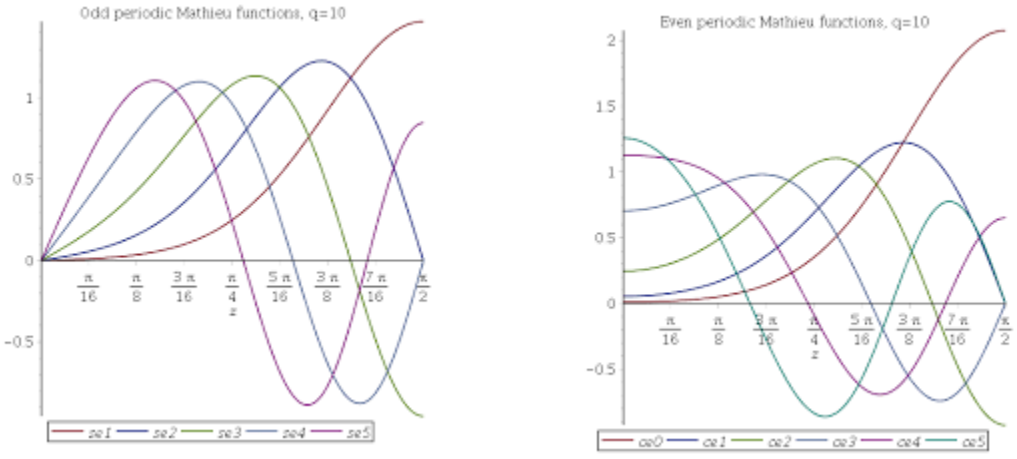
Introduction

In this assignment we were asked to plot a Methieu's functions. Mathieu's functions are differential equations introduced by Emile Leonard Mathieu who discovered them while studying elliptical vibrating drumheads. They have many applications in science and engineering especially in general relativity, quantum mechanics and optics. They are in a wide array of mathematical problems, some that we are very familiar with such as the boundary value problem. This equation can be modeled as the following equation:

$$y'' + (a - 2q\,cos(2x))y = 0$$

Plotting the solutions of the Mathieu Functions

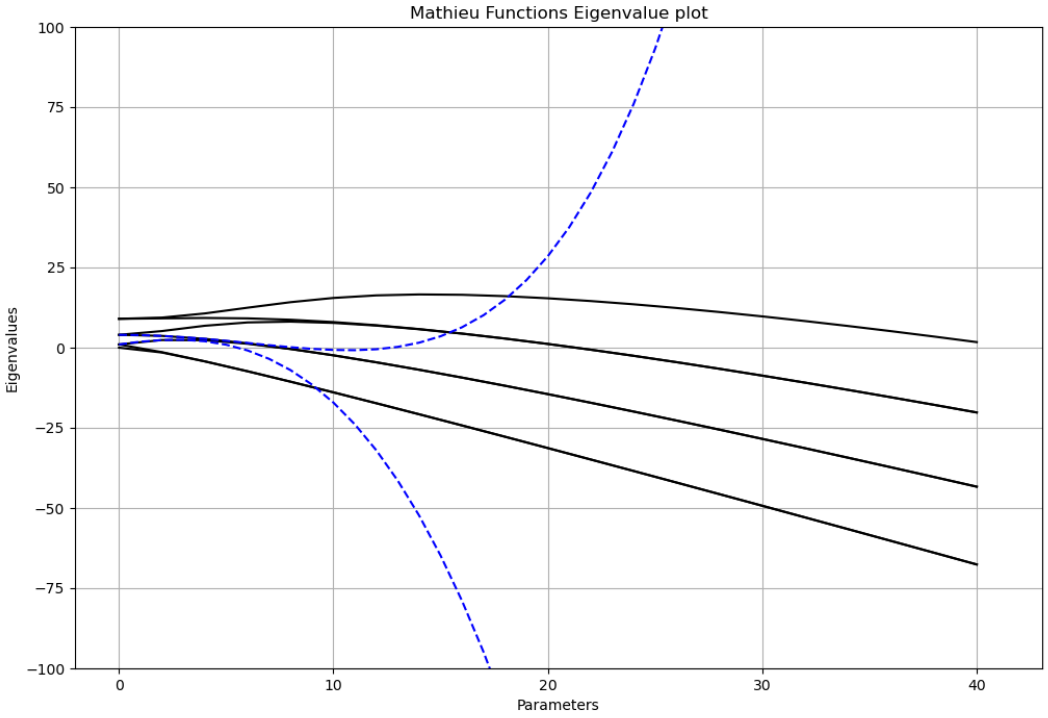
A common trend in applications tends to be in periodic motion which is present in the boundary value problem and partial differential equations. Periodic motion tends to have an oscillating type behavior and to get a visual of the Mathie Function, we will share an image (this graphic was not generated by me):



Now that we understand the behavior of these functions, we can provide some analysis on differential equations as well. Taking some previous knowledge of differential equations, we can recall that second order differential equations can be solved through obtaining eigenvalues. Therefore another way we can plot these functions is from their eigenvalues given a certain boundary.

In our homework we obtained some Fortran code that allowed us to plot these mathematical functions. Because we were given this code, we wanted to build a program to help us visualize what these functions looked like. Though I was quite unfamiliar with the Mathieu functions beforehand, I was able to learn a lot through this assignment. And since Python has the matplotlib library, we were able to visualize these functions through building a program that calls terminal commands that run our Fortran code. So in our

homework we were asked to graph the Mathieu functions within a boundary of 0-42 and obtained the following structure.



To provide some analysis of this graph, as shown above the x-axis represents the parameters or boundaries of this particular graph. And the y-axis represents the eigenvalues you obtain from systems. However when looking at the graph it is apparent that we have two different kinds of lines. The ones we graphed in black are the Mathieu function lines and we can see that the eigenvalues begin to grow quite big after a while. However the blue dashed lines are two other lines that start off with similar curves but grow exponentially at a really rapid rate. What this was supposed to demonstrate was that it is hard to get these projections that follow this model. These two equations are the following:

$$a(q) = 1 + q - \frac{1}{8}q^2 - \frac{1}{64}q^3$$

$$b(q) = 4 - \frac{1}{12}q^2 + \frac{5}{13824}q^4$$

Coding analysis

As currently constructed two of the functions rely solely on fixed q values (parameters). Though this is fine for the purpose of this class, if we were to alter the different values of q we would be able to see different relationships between the code.

We learned in class that within the matplotlib library that there were interactive shells that could allow us to update our plots live. If we were to have live code demos we could easily see the Mathieu function given some crazy parameters to comprehend the function itself even more. I would also try to look at

different ways to visualize this system more than its eigenvalue representation. The Periodic motion aspect of these functions actually appears to be more interesting but I think regardless it is still really cool and how unique these projections are.

Conclusion

Overall this assignment was fun to see Python used as both a visualization and wrapper program. I was relatively new to using the operating system (os) library but its capabilities of using terminal commands can be huge for future projects in scientific computing. This has now been our second project in data visualization and we have seen that we can map a wide array of mathematical functions. This power of visualization and computing in general has helped us understand real world systems overall. I am glad this assignment is a nice little prelude to what we should expect on the final project.