AM 147: Computational Methods and Applications: Winter 2022

Homework #3

Instructor: Abhishek Halder

Due: January 26, 2022

NOTE: Please submit your Homework as a single zip file named `YourlastnameYourfirstnameHW3.zip` via CANVAS. For example, `HalderAbhishekHW3.zip`. Please strictly follow the capital and small letters in the filename of the zip file you submit. You may not receive full credit if you do not follow the file-naming conventions. Your zip file should contain all .m files (MATLAB scripts) for the questions below.

Your zip file must be uploaded to CANVAS by 11:59 PM Pacific Time on the due date. The uploads in CANVAS are time-stamped, so please don't wait till last moment. Late homework will not be accepted.

Problem 1

Bisection method **($15 + 35 = 50$ points)**

(a) Write a MATLAB .m function named `bisection.m` that computes a real root for any nonlinear equation of the form $f(x) = 0$ within numerical tolerance ε, where f is continuous in the interval $[a, b]$. Your MATLAB function `bisection.m` should take the inputs: a, b, f, ε and return an approximation for the root x_{approx}. It is a good practice (but not mandatory) to pass an additional input for maximum number of iterations. The function f should be defined in another MATLAB function `func.m`.

(b) For any nonnegative integer n, the function $p_n(x)$ is defined recursively as

$$p_{n+1}(x) = 2xp_n(x) - p_{n-1}(x), \quad \text{where} \quad p_0(x) = 1, \quad p_1(x) = x.$$

It is known that $p_n(x)$ has n real roots, and all of them are located in the interval $[-1, 1]$.

Write a MATLAB code `YourlastnameYourfirstnameHW3p1.m` that plots a graph of the function $p_4(x)$. By visually inspecting this plot, call the function `bisection.m` from part (a) in your code `YourlastnameYourfirstnameHW3p1.m` to numerically compute all 4 roots of $p_4(x)$ within tolerance $\varepsilon = 10^{-4}$. Recall that `bisection.m` in turn calls appropriately defined `func.m`.

Executing your code `YourlastnameYourfirstnameHW3p1.m` should generate a plot of $p_4(x)$, AND print its 4 real roots approximated via bisection method in MATLAB command window.

Hint for part (b): Before you code, use hand calculation to first derive $p_4(x)$ as a function of x. You don't need to show/submit your hand calculations.