Homework #5: Seafloor subsidence due to cooling **Due: 5:00 PM 10/17/16**

Please read the following questions carefully and make sure to answer the problems completely. In your MATLAB script(s), please include the problem numbers with your answers. Then use the *Publish* function in MATLAB to publish your script to a *pdf* document. For more on the *Publish* functionality within MATLAB see http://www.mathworks.com/help/matlab/matlab_prog/publishing-matlab-code.html. Upload your *pdf* file to Blackboard under Assignment #5. Your filename should be *GEOS397_HW5_Lastname.pdf*. Hint: You can achieve this automatically by calling your MATLAB script *GEOS397_HW5_Lastname.m*.

Background

Volcanoes, intrusions, earthquakes, mountain building and metamorphism are all controlled by the transfer and generation of heat. The Earth's thermal budget controls the activity of the lithosphere and asthenosphere as we as the developement of the innermost structure of the Earth. Three-fifths of the surface of the Earth is oceanic lithosphere, all of which has been formed in the last 160 Ma or so. New lithosphere is created at mid-ocean ridges and then subducted along oceanic trenches. Understanding the structure of the oceanic lithosphere and the mid-ocean ridges is particularly important because it provides a key to understanding the mantle.

The bottom of Challenger Deep in the Mariana Trench (Western Pacific Ocean) is 10.92 km below sea level, and Mauna Kea on the island of Hawaii rises to 4.2 km above sea level from an ocean basin more than 5 km deep. Such features dwarf Mount Everest (8.84 km above sea level). The seafloor can be classified into four main divisions: mid-ocean ridges, ocean basins, continental margins and oceanic trenches (Fowler, 2005).

In this homework you will model the age of the oceanic lithosphere using the current depth of the lithosphere. All programming will take place in the MATLAB editor; you only need to make one script. I suggest you build *sections* of the script and make sure each section runs as expected before you proceed to the next section.

Preliminaries

Thermodynamics is a branch of natural science concerned with heat and temperature and their relation to energy and work.

- Temperature (T) – a quantity that measures the degree of coldness or hotness of a body
- Heat (Q) – a form of energy possessed by a body
- Heat stored in a body is defined by the mean kinetic energy of its molecules
- If we add heat, the molecules speed up and the temperature of the body increases

Effects of temperature: Thermal expansion

- Changes in temperature can be accompanied by changes in pressure and/or volume
- Changing the temperature of a solid or liquid changes its volume
- This is the basis of the thermometer (Celsius, Kelvin, Fahrenheit)

Energy: Joules and Watts

- Heat is a form of energy
- Joule discovered the link between mechanical energy and heat energy
- Power is the rate of change of energy
- The unit of power is the [Watt = J/s]
- Watt used thermal energy (heat) to create mechanical energy

Heat

Heat arrives at the Earth's surface from its interior and from the Sun. We measure heat movement in units of Watts (W = J/s). Earth receives about 2e17 W from the Sun. This heat is often radiated back out into space. This number is significantly larger than the internal heat loss coming from Earth's interior (4.4e13 W); the approximate rate that earthquakes release energy is 1e11 W. From a geologic perspective, the Sun's heat is important because it drives the surface water cycle, the rainfall, and hence erosion. However, the heat source for igneous intrusion, metamorphism and tectonics is within the Earth, and it is this internal source which accounts for most geological phenomena.

Heat moves by *conduction*, *convection*, *radiation* and *advection*. We will focus in this homework on *conduction*. *Conduction* is the transfer of heat through a material by atomic or molecular interaction within the material. As everyone knows, heat flows from a hot body to a cold body, not the other way around. The *rate* at which heat is conducted through a solid is proportional to the temperature *gradient* (the difference in temperature per unit length). Heat is conducted faster when there is a large temperature gradient.

Part 1: Conductive heat flow *(25 pts.)*

Step 1: A model

Imagine an infinitely long and wide solid plate. The plate has thickness d. The temperature at the top of the plate is T_1 and the temperature at the bottom of the plate is T_2. Draw a diagram of this plate and label these parameters. (You can do this by hand and scan or try to do this with MATLAB.) *(5 pts.)*

Step 2: Heat flow

Assuming that $T_2 > T_1$, the rate of change of *heat flow per unit area* up through the plate is proportional to

$$\frac{T_2 - T_1}{d}.$$

In fact the rate of heat flow per unit area (Q) *down* though the plate is

$$Q = -k\frac{T_2 - T_1}{d}, \tag{1}$$

where k is called the *thermal conductivity*. Q is the rate of heat flow per unit area and has units Wm^{-2}; k has units $Wm^{-1}\,{}^{\circ}C^{-1}$.

Why does the heat flow *down* in this equation? *(5 pts.)*

Step 3: Thermal conductivities

Give the values of thermal conductivity for the following items. Make sure to cite where you found this value and make sure the units are in the SI units given in step 2. *(5 pts.)*

- Silver:
- Magnesium:
- Glass:
- Rock:
- Wood:

Step 4: The heat transport equation

Let's assume that the temperature of the upper surface (i.e. at z) is T and that the temperature at the lower surface (i.e. $z + \delta z$) is $T + \delta T$. Substitute these values into equation 1. (Think about how to replace d, T_1 and T_2.) *(2 pts.)*

In the limit that δz goes to zero, some part of the right-hand side of your new equation becomes a derivative.

Write this derivative. *(1 pt.)*

Write the heat flow (equation 1) using this derivative. *(1 pt.)*

Write this same equation using the *gradient* operator. *(1 pt.)*

*NOTE: Instead of the **sediment transport** equation we had in HW3, we now have the **heat transport** equation. Look back at equation 1 in HW3 if you do not remember. Instead of sediment moving up or down based on the topographic gradient, we now have heat moving up or down based on the temperature gradient.*

Step 5: The conservation equation

Thinking back to HW3, we also encountered the mass conservation equation (look at HW3 equation 2). In the case of heat we also have a conservation equation, the **heat energy** conservation equation:

$$c_p \rho \frac{\partial T}{\partial t} = A - \frac{\partial Q}{\partial z}. \tag{2}$$

This equation tells us how temperature T changes with time given a change in heat flow and internal heat generation. In this equation A represents the internal heat generation (e.g. from radio active decay), ρ is the material density and c_p is the specific heat of the material with units $W kg^{-1}\ {}^{\circ}C^{-1}$. *Specific heat is defined as the amount of heat necessary to raise the temperature of 1 kg of the material by 1°C.*

Use the heat transport equation and compute the derivative $\partial Q/\partial z$ and insert this into the conservation equation. *(2 pts.)*

Assume that internal heat generation is zero, write the updated the conservation equation from the previous step. *(2 pts.)*

We can further simplify this equation if we incorporate a new variable κ. We can then write

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}.$$

This is known as the *heat-conduction equation*. It is again a diffusion equation, just as in HW3. We will NOT solve this equation using finite differences this time. Instead, in the next part of the homework we will use an analytic solution to this PDE provided by someone else.

What is the value of κ in this case? *(1 pt.)*

Part 2: Oceanic lithosphere cooling *(15 pts.)*

In this section of the homework we will model how the oceanic lithosphere cools with time. We did not solve the partial differential equation governing temperature change in the lithosphere with time (see previous part), but here is **"a solution"**

$$T(z,t) = T_0 \times erf\left(\frac{z}{2\sqrt{\kappa t}}\right),$$

where T_0 is the temperature at the mid-ocean ridge (i.e. spreading center), z is depth, κ is the thermal diffusivity (**different from thermal conductivity** k) and t is time.

*NOTE: this is **"a solution"** because it is the solution derived when the boundary conditions are $T(x = 0) = T_0$ and $T(z = 0) = 0$. This solution would change if we changed the boundary conditions. If you are interested in this kind of stuff take GEOPH501 and GEOPH502. These are the courses where we do complete derivations of physical problems like this.*

Step 1: Setup the model domain and compute

Make depth vector that goes from 0 to 100 km. Make a time vector that goes from 0 to 100 Ma. You can choose the interval in each of these vectors. Use the value $\kappa = 1.0\text{e}{-6}\ m^2/s$. Take the temperature at the mid-ocean ridge to be 640°C. Use the MATLAB function **erf()** to compute the function $T(z,t)$ and plot. Make sure to label all axes and include a colorbar.

1. Use two *for* loops to compute the function $T(z,t)$. *(5 pts.)*
2. Use the **meshgrid()** command to setup matrices for z and t and then compute the function $T(z,t)$. *(5 pts.)*

*NOTE: **erf()** is a function like **exp()** or **plot()**; it has a built-in MATLAB call. Make sure to plot both of the temperature profiles that you compute in different ways.*

3 pts. extra credit: Time how long it takes to compute $T(z,t)$ using the two methods. If you don't notice a difference, try decreasing the sample interval in z and t. Discuss the differences in computation time.

Step 2: Analyze the model output

Answer the following questions in as much detail as possible. *(5 pts.)*

- Does your model make sense given the boundary conditions used to derive the solution?
- What controls the rate at which the temperature decays? (List all things you can think of.)
- How could we convert this model from *age* to *distance* from ridge axis?
- What would be a more appropriate boundary condition at $T(z = 0)$ given what we know about the oceans?
- Is 640°C an appropriate value for the temperature at a mid-ocean ridge? Why or why not?

Part 3: Plate velocity and the depth of oceans *(30 pts.)*

As an oceanic plate moves away from the ridge axis and cools, it contracts and thus increases in density (Fowler, 2005). This leads to deeper oceans above older oceanic lithosphere. Depending on the cooling model used for the oceanic lithosphere, the depth-age relationship can vary. For example, for ages less than 20 Ma, a simple relation between bathymetric depth d [km] and lithospheric age t [Ma] is observed:

$$d = 2.6 + 0.365t^{1/2}.$$

Thus depth increases with the square root of age. Alternatively, for ages greater than 20 Ma this simple relation dos not hold; depth increases more slowly with increasing age and approximates a negative exponential:

$$d = 5.65 - 2.47e^{-t/36}.$$

NOTE: We currently only have empirical models of depth-age relationships for oceanic crust. This is partly because we do not completely know the internal heat generation A. The other difficulty in making a mathematical model of depth is due to the fact that heat flux through the crust changes as sediment is deposited on the plate, which closes off the geothermal part of the system.

Step 1: Load and plot sea-floor depth data

Load the file *spreadingData.mat* in MATLAB. This will load the structure *Bath*. *(3 pts.)*

This structure contains four fields.

```
fields(Bath)
ans =
    'atlanticz'
    'atlanticx'
    'pacificx'
    'pacificz'
```

These data are for the Pacific and Atlantic Oceans. The fields that end in z are the ocean depths [m] measured along a profile normal to a mid-ocean ridge. The fields that end in x are the distances [km] from the ridge where the data were collected. Plot depth vs. distance from the ridge; plot each ocean in its own subplot. Make sure to label axes. *(5 pts.)*

Step 2: A half-space model

There are actually many models for ocean depth with age. In this part, we will use a half-space cooling model; this heat-flow model leads to the following relationship for depth

$$d = 2.65 + 0.345t^{1/2}. \tag{3}$$

(A half-space model just means that the physical properties, e.g. density, specific heat, etc., are constant at all depths and all times.) We know that age t and distance x are related by the plate velocity $v = x/t$. Try a number of velocities [km/Ma] and predict the ocean depth for the given offsets in *Bath.pacificx* and *Bath.atlanticx*. Plot your predicted depth models on your subplots with the observed data. *(10 pts.)*

What does 2.65 represent in the equation above? *(2 pts.)*

What are the plate velocities [km/Ma] that match your data the closest? List both oceans. *(5 pts.)*

Convert these plate velocities to [cm/yr] and compare with an estimate from the literature from these ocean basins. Do your estimates agree with published Pacific and Atlantic Ocean spreading rates? *(5 pts.)*

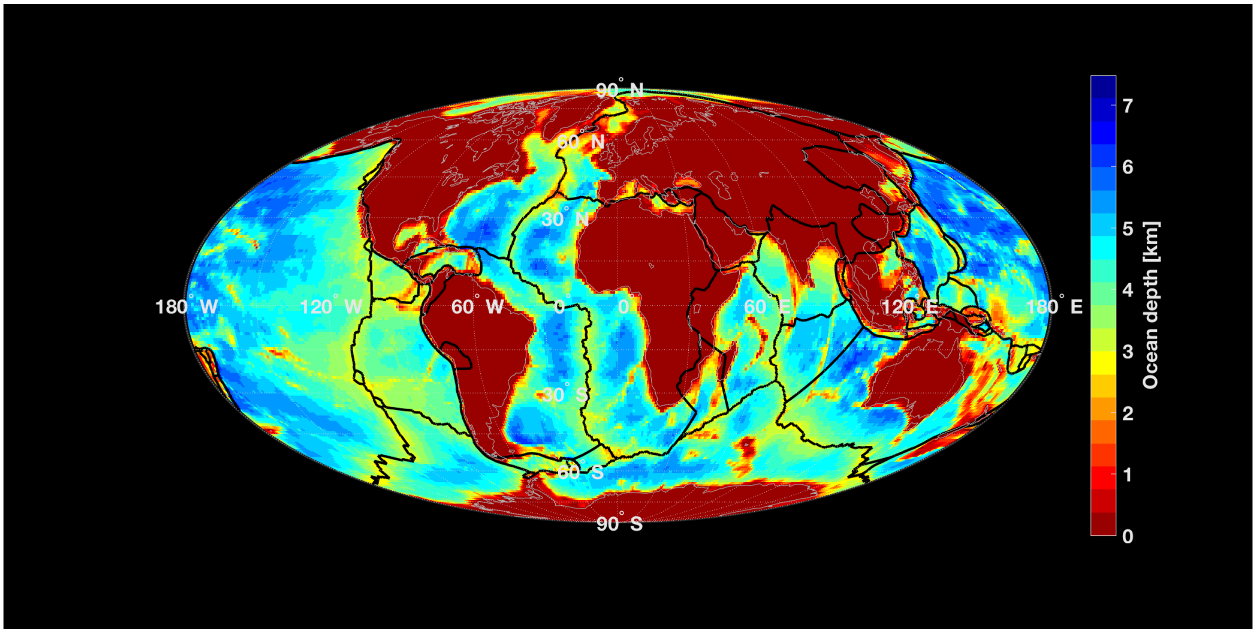

Figure 1: Global ocean depth map.

Part 4: Global oceanic plate ages *(30 pts.)*

Step 1: Load topo data and plot sea-floor depths

Load the files *topo* and *coastlines* into MATLAB, just like we did in HW4. Load the plate boundaries this time and plot these as well. Below is the MATLAB line needed to load the plate boundary data. These data are built into MATLAB; you do not need to download this file. *(5 pts.)*

```
[platelat, platelon] = importPlates('All_boundaries.txt');
```

Step 2: Kill the topography and get the units right

As in the previous homework, set the topography values above sea-level to zero. This is your ocean depth data in [m]. Convert the ocean depths to [km] so that we can use these depths in our equation to estimate age [Ma]. Plot these data and use the following code for your colorbar. *(5 pts. – make sure to check the 1 point in the comment below. Explain this colormap() function!)*

```
colormap( flipud( jet(20) ) ); % (1 pt.) your comment here
```

You should be able to produce something that looks like Figure 1.

Step 3: Compute sea-floor age

Rearrange equation 3 to solve for age t. *(5 pts.)*

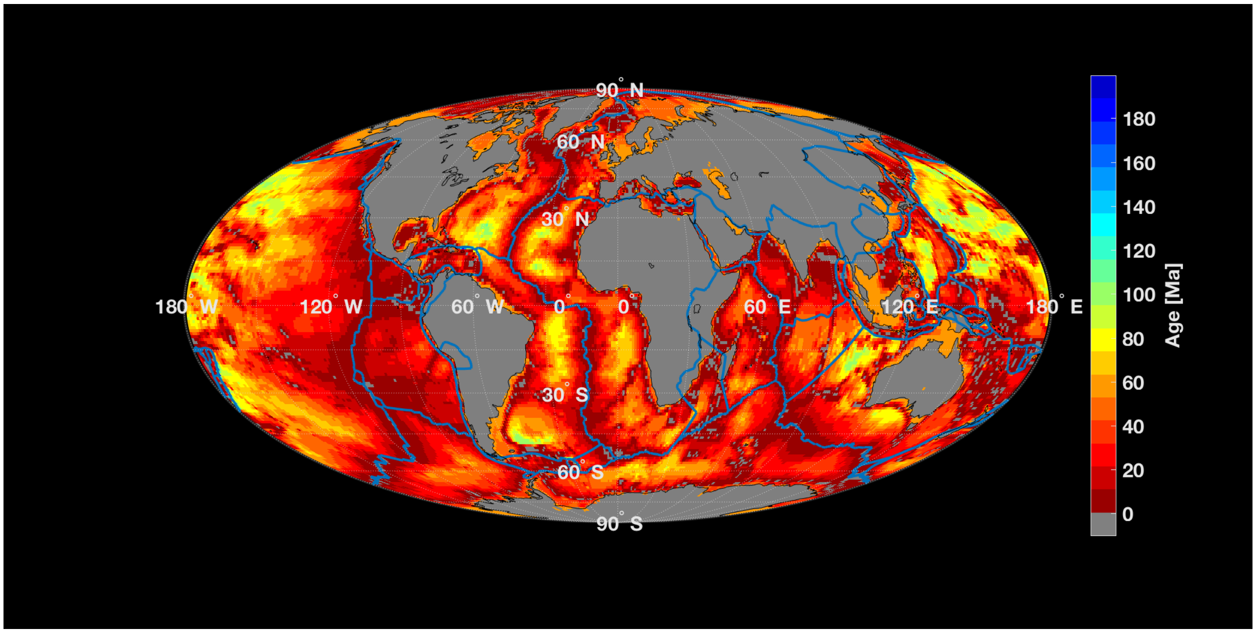

Figure 2: Global ocean age map.

Step 4: Plot the oceanic lithosphere age map

Now that you have an equation that computes age based on ocean depth, compute the age for all grid points in the ocean. *NOTE: Make sure to account for the continents. Even though you already set those equal to zero depth, check what happens in your equation when depth is zero! Hint: the equation stills computes an age, but you should force it not to or correct this after the fact. I force my continents to have an age of -10 Ma. The code below will then set the colormap so my continents are gray.* Plot your results and use the following colorbar information. *(10 pts.)*

```
cmap = flipud( jet(20) ); % (1 pt.) your comment here
cmap = [0.5 0.5 0.5 ; cmap]; % (1 pt.) your comment here
cmap(end,:) = []; % (1 pt.) your comment here
colormap(cmap); % (1 pt.) your comment here
```

Step 5: Discuss your results

Please answer the following questions in as much detail as possible. *(5 pts.)*

- Does your map of ocean ages make sense given the plate boundaries?
- What is the oldest age in your map?
- Where does this oldest age occur and does this make sense geologically?
- Where do the youngest ages occur? Does this conform to your knowledge of oceanic lithosphere generation?
- Are there any assumptions that have gone into this model that might not be accurate?

*NOTE: you may find the functions **max()**, **find()** and **ind2sub()** to be useful when looking for the oldest and youngest parts of the oceanic lithosphere.*

References

Fowler, C. R. 2005. *The Solid Earth: An Introduction to Global Geophysics*. 2nd edn. Cambridge.