

L16: class exercise – Try to implement/solve the following problems in MATLAB.

Numerical Integration

Numerically integrate the following using *quad* or *integral*:

$$\begin{aligned} & \int_0^3 \sqrt{y+1} dy \\ & \int_{-1}^1 \frac{5r}{(4+r^2)^2} dr \\ & \int_0^{\pi/6} \cos^{-3}(2\theta) \sin(2\theta) d\theta \\ & \int_0^{\pi/2} e^{\sin(x)} \cos(x) dx \\ & \int_0^{\sqrt{\ln \pi}} 2xe^{x^2} \cos(e^{x^2}) dx \\ & \int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2} \end{aligned}$$

Compare a simple function and discrete data

1. Plot the function $y(x) = xe^{-x}$ between $x = 0$ and $x = 5$ using 101 points.
2. Using integration by parts, show that the exact value of $I(a) = \int_0^a y(x) dx = \int_0^a xe^{-x} dx = 1 - e^{-a} - ae^{-a}$.
3. Write a **function** that computes the exact area of A using the solution in the previous step. Call this function *exactArea.m*.
4. Use your function to compute the value of $I(a = 5)$.
5. Use matlab's built in *trapz* function to compute the approximate integral of your discrete function in step 1.
6. Repeat the *trapz* integration when the number of points is 11 and 1001.
7. Compare the differences between the exact solution to the integral and the approximate solutions when you use 11, 101 and 1001 points to compute the integral of $y(x)$ from 0 to 5.
8. Repeat steps 5, 6 and 7 using *quad* or *integral* instead of *trapz*.