

$$\max_{\substack{\alpha_c, d_r, u_c^{(t)}, u_r^{(t)} \\ p \times 1 \quad p \times 1 \quad m \times p \quad m \times p}} \sum_{j=1}^P d_{r_j} + \sum_{t=1}^T \sum_{j=1}^P \sum_{i=1}^m w_i \cdot u_{r_{ij}}^{(t)}$$

Weights for control

Subject to

$$0 \leq \sum_{j=1}^P u_{r_{ij}}^{(t)} \leq 1, \quad \forall i=1, \dots, m, \quad \forall t=1, \dots, T$$

$$0 \leq u_{r_{ij}}^{(t)} \leq 1$$

$$-1 + d_{r_j} \leq \alpha_{c_j} \leq 1 - d_{r_j}, \quad \forall j=1, \dots, P$$

$$0 < \varepsilon \leq d_{r_j} \leq 1$$

// Scaling is within constraint set

$$(c^{(0)} + G^{(0)} \alpha_c) + |G^{(0)}| \cdot d_r \leq X_{IS}^{\max} \leftarrow \text{initial set}$$

$$(c^{(0)} + G^{(0)} \alpha_c) - |G^{(0)}| \cdot d_r \geq X_{IS}^{\min}$$

// for $t=1, \dots, T$

$$A^t (c^{(0)} + G^{(0)} \alpha_c) + \sum_{j=1}^P \sum_{s=1}^{t-1} A^{s-1} B u_{c_j}^{(t-s+1)} + \sum_{j=1}^P |A^t d_{r_j} g_j^{(0)} + \sum_{s=1}^{t-1} A^{s-1} B u_{r_j}^{(t-s+1)}| \leq X_{CS}^{\max}$$

constraint set

$$A^t (c^{(0)} + G^{(0)} \alpha_c) + \sum_{j=1}^P \sum_{s=1}^{t-1} A^{s-1} B u_{c_j}^{(t-s+1)} - \sum_{j=1}^P |A^t d_{r_j} g_j^{(0)} + \sum_{s=1}^{t-1} A^{s-1} B u_{r_j}^{(t-s+1)}| \geq X_{CS}^{\min}$$

There is only a single scaling at time $t=0$,
represented by 2 vectors:

\mathcal{L}_c and \mathcal{L}_r , where p is the number
($p \times 1$) ($p \times 1$) of generators

Reach set updating:

$$R^{(t)} = \left(A^{(t)} \left(C^{(0)} + G^{(0)} \cdot \mathcal{L}_c \right) + \sum_{j=1}^p \sum_{s=1}^{t-1} A \cdot B \cdot y_{c,j}^{(t-s+1)} \right)$$

$n \times n$ $n \times 1$ $n \times p$ $p \times 1$ $p \times 1$ $s-1$ $(t-s+1)$
 $m \times 1$

Center Sum over all generator input element centers

$$A^{(t)} \cdot \mathcal{L}_{r_1} \cdot g_1^{(0)} + \sum_{s=1}^t A^{s-1} \cdot B \cdot y_{r_1}^{(t-s+1)}, \dots,$$

$$A^{(t)} \cdot \mathcal{L}_{r_p} \cdot g_p^{(0)} + \sum_{s=1}^t A^{s-1} \cdot B \cdot y_{r_p}^{(t-s+1)}$$

$y_c^{(t)}$ and $y_r^{(t)}$ are $m \times p$. As the scaling is done
beforehand (at $t=0$), we don't need to worry
about nonlinearity.