for $t = 1 : T$

$$\max_{\underline{\gamma}_i, \overline{\gamma}_i \text{ for } i=1,\dots,p} \sum_{i=1}^{p} \overline{\gamma}_i^{(t)} - \sum_{i=1}^{p} \underline{\gamma}_i^{(t)}$$

, p is number of generators, n is dimension of state space and t is current time step
$t=0$ corresponds to initial condition

subject to

$$-1 \le \underline{\gamma}_i^{(t)} \le 1$$

$$-1 \le \overline{\gamma}_i^{(t)} \le 1$$

$$\underline{\gamma}_i^{(t)} + \varepsilon \le \overline{\gamma}_i^{(t)}$$

, ε is small number > 0 to ensure intervals are nonempty

// . scaling is within constraint set

$$\left(c^{(t-1)} + G^{(t-1)} \alpha_c^{(t)}\right) + \left|G^{(t-1)}\right| \cdot \alpha_x^{(t)} \le X_{cs}^{max}$$

$$\left(c^{(t-1)} + G^{(t-1)} \cdot \alpha_c^{(t)}\right) - \left|G^{(t-1)}\right| \alpha_x^{(t)} \ge X_{cs}^{min}$$

← constraint set infimum

previous center

previous generator matrix; generators are concatenated horizontally : $n \times p$ where p is number of generators

center shift scalars : $p \times 1$

generator scalars: $p \times 1$

$$G^{(t-1)} \alpha_c^{(t)} = \sum_{i=1}^{n} g_i^{(t-1)} \cdot \alpha_i^{(t)}$$

← (100% sure this is equivalent)

// reach set of scaling is within constraint set

$$A_d \cdot \left(c^{(t-1)} + G^{(t-1)} \cdot \alpha_c^{(t)}\right) + \left|A_d \cdot G^{(t-1)}\right| \alpha_x^{(t)} \le X_{cs}^{max}$$

$$A_d \cdot \left(c^{(t-1)} + G^{(t-1)} \alpha_c^{(t)}\right) - \left|A_d \cdot G^{(t-1)}\right| \alpha_x^{(t)} \ge X_{cs}^{min}$$

discretized continuous

more next page

// When $t=1$ additionally check that scaled initial set with generators is inside unscaled initial set without generators.

$$\left(c^{(t-1)} + G^{(t-1)}\alpha_c^{(t)}\right) + \left|G^{(t-1)}\right|\cdot\alpha_x^{(t)} \le x_{IC}^{max} \leftarrow \text{initial set}$$

$$\left(c^{t-1} + G^{(t-1)}\alpha_c^{(t)}\right) - \left|G^{(t-1)}\right|\cdot\alpha_x^{(t)} \ge x_{IC}^{min}$$

Now, $\alpha_c = \dfrac{\overline{\gamma} + \underline{\gamma}}{2}$ in vector form, or individually, $\alpha_{c_i} = \dfrac{\overline{\gamma}_i + \underline{\gamma}_i}{2}$

$\alpha_x = \dfrac{\overline{\gamma} - \underline{\gamma}}{2}$ $\qquad \alpha_{x_i} = \dfrac{\overline{\gamma}_i - \underline{\gamma}_i}{2}$

After each optimization, I construct α_c and α_x to be used for both forward reachability in the next iteration, as well as for mapping backward in the end to get the "safe set".

Forward:

$$Z^{(t)} = \left\langle A_d\cdot\left(c^{(t-1)} + G^{(t-1)}\cdot\alpha_c^{(t)}\right); \alpha_1^{(t)}\cdot A_d\cdot g_1^{(t-1)}; \ldots \right.$$
$$\left. \ldots, \alpha_p^{(t)}\cdot A_d\, g_p^{(t-1)} \right\rangle$$

Final Backward computation for safe set:

exponent

superscript

$$\left\langle \sum_{t-1}^{T} \left(A_d^{-1}\right)^{t-1}\cdot G^{(t-1)}\cdot\alpha_c^{(t)}; \prod_{t=1}^{T}\alpha_{x_1}^{(t)}\cdot g_1^{(0)}, \ldots \right.$$
$$\left. \ldots, \prod_{t=1}^{T}\alpha_{x_p}^{(t)}\cdot g_p^{(0)} \right\rangle$$