for $t = 1:T$

$$\max_{\underline{\gamma}_i, \bar{\gamma}_i} \quad \sum_{i=1}^{p} \bar{\gamma}_i^{(t)} - \sum_{i=1}^{p} \underline{\gamma}_i^{(t)}$$

for $i = 1, \ldots, p$

p is number of generators, n is dimension of state space and t is current time step $t = 0$ corresponds to initial condition

subject to

$$-1 \le \underline{\gamma}_i^{(t)} \le 1$$
$$-1 \le \bar{\gamma}_i^{(t)} \le 1$$
$$\underline{\gamma}_i^{(t)} + \varepsilon \le \bar{\gamma}_i^{(t)}$$

ε is small number > 0 to ensure intervals are nonempty

// scaling is within constraint set

$$\left(c^{(t-1)} + G^{(t-1)} \cdot \alpha_c^{(t)} \right) + \left| G^{(t-1)} \right| \cdot \alpha_x^{(t)} \le X_{cs}^{max}$$

$$\left(c^{(t-1)} + G^{(t-1)} \cdot \alpha_c^{(t)} \right) - \left| G^{(t-1)} \right| \alpha_x^{(t)} \ge X_{cs}^{min} \leftarrow \text{constraint set infimum}$$

↑ previous center

↗ previous generator matrix; generators are concatenated horizontally: $n \times p$ where p is number of generators

↗ center shift scalars: $p \times 1$

↘ generator scalars: $p \times 1$

$$G^{(t-1)} \alpha_c^{(t)} = \sum_{i=1}^{n} g_i^{(t-1)} \cdot \alpha_i^{(t)} \leftarrow (100\% \text{ sure this is equivalent})$$

// reach set of scaling is within constraint set

$$A_d \cdot \left(c^{(t-1)} + G^{(t-1)} \cdot \alpha_c^{(t)} \right) + \left| A_d \cdot G^{(t-1)} \right| \alpha_x^{(t)} \le X_{cs}^{max}$$

$$\to A_d \cdot \left(c^{(t-1)} + G^{(t-1)} \alpha_c^{(t)} \right) - \left| A_d \cdot G^{(t-1)} \right| \alpha_x^{(t)} \ge X_{cs}^{min}$$

discretized continuous

// When $t=1$ additionally check that scaled initial set with generators is inside unscaled initial set without generators.

$$\left(c^{(t-1)} + G^{(t-1)}\alpha_c^{(t)}\right) + |G^{(t-1)}| \cdot \alpha_x^{(t)} \leq x_{IC}^{max} \leftarrow \text{initial set}$$

$$\left(c^{t-1} + G^{(t-1)}\alpha_c^{(t)}\right) - |G^{(t-1)}| \cdot \alpha_x^{(t)} \geq x_{IC}^{min}$$

Now, $\alpha_c = \dfrac{\bar{\gamma}+\underline{\gamma}}{2}$ in vector form, or individually,

$\alpha_x = \dfrac{\bar{\gamma}-\underline{\gamma}}{2}$

$\alpha_{c_i} = \dfrac{\bar{\gamma}_i + \underline{\gamma}_i}{2}$

$\alpha_{x_i} = \dfrac{\bar{\gamma}_i - \underline{\gamma}_i}{2}$

After each optimization, I construct α_c and α_x to be used for both forward reachability in the next iteration, as well as for mapping backward in the end to get the "safe set".

Forward:

$$Z^{(t)} = \left\langle Ad \cdot \left(c^{(t-1)} + G^{(t-1)} \cdot \alpha_c^{(t)}\right); \alpha_1^{(t)} \cdot Ad \cdot g_1^{(t-1)}; \dots \right.$$

$$\left. \dots, \alpha_p^{(t)} \cdot Ad \, g_p^{(t-1)} \right\rangle$$

Final Backward computation for safe set: ↙ exponent superscripts

$$\left\langle c^{(0)} + \sum_{t-1}^{t} \left(A_d^{-1}\right)^{t-1} \cdot G^{(t-1)} \cdot \alpha_c^{(t)}; \prod_{t=1}^{T} \alpha_{x_1}^{(t)} \cdot g_1^{(0)}, \dots \right.$$

$$\left. \dots; \prod_{t=1}^{T} \alpha_{x_p}^{(t)} \cdot g_p^{(0)} \right\rangle$$