$$\max_{\substack{\alpha_c,\ \alpha_r,\ \psi_c^{(t)},\ \psi_r^{(t)} \\ p\times 1\ \ p\times 1\ \ m\times p\ \ m\times p}} \sum_{j=1}^{p} \alpha_{r_j} + \sum_{t=1}^{\tau}\sum_{j=1}^{p}\sum_{i=1}^{m} w_i \, \psi_{r_{ij}}^{(t)}$$

weights for control

Subject to

$$0 \le \sum_{j=1}^{p} \psi_{r_{ij}}^{(t)} \le \left(\psi_j^{\mathbf{max}} + \psi_j^{\mathbf{min}}\right)/2, \quad \forall\ i=1,\dots,m, \quad \forall\ t=1,\dots,\tau$$

$$0 \le \psi_{r_{ij}}^{(t)} \le \left(\psi_j^{\mathbf{max}} + \psi_j^{\mathbf{min}}\right)/2, \quad \psi_j^{\mathbf{min}} + \psi_{r_{ij}}^{(t)} \le \psi_{c_{ij}}^{(t)} \le \psi_j^{\mathbf{max}} - \psi_{r_{ij}}^{(t)}$$

$$-1 + \alpha_{r_j} \le \alpha_{c_j} \le 1 - \alpha_{r_j}, \quad \forall\ j=1,\dots,p$$

$$0 < \varepsilon \le \alpha_{r_j} \le 1$$

$$\max_{\substack{\alpha_c,\ \alpha_r,\ \psi_c^{(t)},\ \psi_r^{(t)} \\ p\times 1\ \ p\times 1\ \ m\times p\ \ m\times p}} \sum_{j=1}^{p} \alpha_{r_j} + \sum_{t=1}^{\tau}\sum_{j=1}^{p}\sum_{i=1}^{m} w_i \, \psi_{r_{ij}}^{(t)}$$

weights for control

Subject to

$$0 \le \sum_{j=1}^{p} \psi_{r_{ij}}^{(t)} \le 1, \quad \forall\ i=1,\dots,m, \quad \forall\ t=1,\dots,\tau$$

$$0 \le \psi_{r_{ij}}^{(t)} \le 1$$

$$-1 + \alpha_{r_j} \le \alpha_{c_j} \le 1 - \alpha_{r_j}, \quad \forall\ j=1,\dots,p$$

$$0 < \varepsilon \le \alpha_{r_j} \le 1$$

// Scaling is within constraint set

$$(c^{(0)} + G^{(0)}\alpha_c) + |G^{(0)}| \cdot \alpha_r \leq X_{IS}^{max} \leftarrow \text{initial set}$$

$$(c^{(0)} + G^{(0)} \cdot \alpha_c) - |G^{(0)}| \cdot \alpha_r \geq X_{IS}^{min}$$

// for $t = 1, \ldots, T$

$$A^t (c^{(0)} + G^{(0)}\alpha_c) + \sum_{j=1}^{p} \sum_{s=1}^{t} A^{s-1} B u_{c_j}^{(t-s+1)} + \sum_{j=1}^{p} \left| A^t \alpha_{r_j} g_j^{(0)} + \sum_{s=1}^{t} A^{s-1} B u_{r_j}^{(t-s+1)} \right| \leq X_{CS}^{max} \leftarrow \text{constraint set}$$

$$A^{(t)} (c^{(0)} + G^{(0)} \cdot \alpha_c) + \sum_{j=1}^{p} \sum_{s=1}^{t} A^{s-1} B u_{c_j}^{(t-s+1)} - \sum_{j=1}^{p} \left| A^t \alpha_{r_j} g_j^{(0)} + \sum_{s=1}^{t} A^{s-1} B u_{r_j}^{(t-s+1)} \right| \geq X_{CS}^{min}$$

There is only a single scaling at time $t=0$, represented by 2 vectors:

$\underset{(p\times1)}{\alpha_c}$ and $\underset{(p\times1)}{\alpha_r}$, where p is the number of generators

Reach set updating:

$$R^{(t)} = \left\langle \underset{n\times n}{A^t} \left(\underset{n\times1}{c^{(0)}} + \underset{n\times p}{G^{(0)}} \cdot \underset{p\times1}{\alpha_c} \right) + \sum_{j=1}^{p} \sum_{s=1}^{t} A^{s-1} \cdot B \cdot \underset{m\times1}{u_{c_j}^{(t-s+1)}} \right.$$

$$\overbrace{A^{t} \cdot \alpha_{r_1} \cdot g_1^{(0)}}^{\text{Center}} + \overbrace{\sum_{s=1}^{t} A^{s-1} \cdot B \cdot \mu_{r_1}^{(t-s+1)}}^{\text{Sum over all generator input element centers}}, \ldots,$$

$$A^{t} \alpha_p g_p^{(0)} + \sum_{s=1}^{t} A^{s-1} \cdot B \cdot \mu_{r_p}^{(t-s+1)} \rangle$$

$\mu_c^{(t)}$ and $\mu_r^{(t)}$ are $m \times p$. As the scaling is done beforehand (at $t=0$), we don't need to worry about nonlinearity.