$$\max_{\substack{\alpha_c,\ \alpha_r,\ \psi_c^{(t)},\ \psi_r^{(t)} \\ p\times 1 \quad p\times 1 \quad m\times p \quad m\times p}} \sum_{j=1}^{p} \alpha_{r_j} + \sum_{t=1}^{\tau}\sum_{j=1}^{p}\sum_{i=1}^{m} \underbrace{w_i}_{\text{weights for control}} \psi_{r_{ij}}^{(t)}$$

Subject to

$$0 \le \sum_{j=1}^{p} \psi_{r_{ij}}^{(t)} \le 1, \quad \forall\ i=1,\dots,m \quad \forall\ t=1,\dots,\tau$$

$$0 \le \psi_{r_{ij}}^{(t)} \le 1$$

$$-1+\alpha_{r_j} \le \alpha_{c_j} \le 1-\alpha_{r_j}, \quad \forall\ j=1,\dots,p$$

$$0 < \varepsilon \le \alpha_{r_j} \le 1$$

// scaling is within constraint set

$$\left(c^{(0)} + G^{(0)}\alpha_c\right) + \left|G^{(0)}\right| \cdot \alpha_r \le X_{IS}^{max}$$

$$\left(c^{(0)} + G^{(0)}\alpha_c\right) - \left|G^{(0)}\right| \cdot \alpha_r \ge X_{IS}^{min}$$

initial set

// for $t=1,\dots,\tau$

$$A^{t}\left(c^{(0)} + G^{(0)}\alpha_c\right) + \sum_{j=1}^{p}\sum_{s=1}^{t} A^{s-1} B \psi_{c_j}^{(t-s+1)} + \sum_{j=1}^{p}\left| A^{t}\alpha_{r_j} g_j^{(0)} + \sum_{s=1}^{t} A^{s-1} B \psi_{r_j}^{(t-s+1)}\right| \le X_{CS}^{max}$$

$$A^{(t)}\left(c^{(0)} + G^{(0)}\alpha_c\right) + \sum_{j=1}^{p}\sum_{s=1}^{t} A^{s-1} B \psi_{c_j}^{(t-s+1)} - \sum_{j=1}^{p}\left| A^{t}\alpha_{r_j} g_j^{(0)} + \sum_{s=1}^{t} A^{s-1} B \psi_{r_j}^{(t-s+1)}\right| \ge X_{CS}^{min}$$

constraint set

There is only a single scaling at time $t=0$, represented by 2 vectors:

$\underset{(p\times 1)}{\alpha_c}$ and $\underset{(p\times 1)}{\alpha_r}$, where p is the number of generators

Reach set updating:

$$R^{(t)} = \Big\langle \underbrace{\underset{n\times n}{A}^{t}\big(\underset{n\times 1}{c}^{(0)} + \underset{n\times p}{G}^{(0)} \cdot \underset{p\times 1}{\alpha_c}\big) + \sum_{j=1}^{p}\sum_{s=1}^{t} A^{s-1} \cdot B \cdot \underset{m\times 1}{u}_{c_j}^{(t-s+1)}}_{\text{Center}},$$

Sum over all generator input element centers

$$A^{t} \cdot \alpha_{r_1} \cdot g_1^{(0)} + \sum_{s=1}^{t} A^{s-1} \cdot B \cdot u_{r_1}^{(t-s+1)}, \ldots,$$

$$A^{t} \alpha_p g_p^{(0)} + \sum_{s=1}^{t} A^{s-1} \cdot B \cdot u_{r_p}^{(t-s+1)} \Big\rangle$$

$u_c^{(t)}$ and $u_r^{(t)}$ are $m\times p$. As the scaling is done beforehand (at $t=0$), we don't need to worry about nonlinearity.