for $t = 1:T$

$$\max_{\underline{\gamma}_i, \overline{\gamma}_i \text{ for } i=1,\dots,p} \sum_{i=1}^{p} \overline{\gamma}_i^{(t)} - \sum_{i=1}^{p} \underline{\gamma}_i^{(t)}$$

, p is number of generators, n is dimension of state space and t is current time step
t=0 corresponds to initial condition

subject to
$$-1 \le \underline{\gamma}_i^{(t)} \le 1$$
$$-1 \le \overline{\gamma}_i^{(t)} \le 1$$
$$\underline{\gamma}_i^{(t)} + \varepsilon \le \overline{\gamma}_i^{(t)}$$
, ε is small number > 0 to ensure intervals are nonempty

// scaling is within constraint set

$$\left(C^{(t-1)} + G^{(t-1)} \cdot \alpha_c^{(t)}\right) + \left|G^{(t-1)}\right| \cdot \alpha_x^{(t)} \le X_{cs}^{max}$$

$$\left(C^{(t-1)} + G^{(t-1)} \cdot \alpha_c^{(t)}\right) - \left|G^{(t-1)}\right| \alpha_x^{(t)} \ge X_{cs}^{min}$$ ← constraint set infimum

previous center

previous generator matrix; generators are concatenated horizontally : n×p where p is number of generators

center shift scalars: p×1

generator scalars: p×1

$$G^{(t-1)} \alpha_c^{(t)} = \sum_{i=1}^{n} g_i^{(t-1)} \cdot \alpha_i^{(t)}$$ ← (100% sure this is equivalent)

// reach set of scaling is within constraint set

$$A_d \cdot \left(C^{(t-1)} + G^{(t-1)} \cdot \alpha_c^{(t)}\right) + \left|A_d \cdot G^{(t-1)}\right| \alpha_x^{(t)} \le X_{cs}^{max}$$

$$A_d \cdot \left(C^{(t-1)} + G^{(t-1)} \alpha_c^{(t)}\right) - \left|A_d \cdot G^{(t-1)}\right| \alpha_x^{(t)} \ge X_{cs}^{min}$$

discretized continuous

more next page →

// When $t=1$ additionally check that scaled initial set with generators is inside unscaled initial set without generators.

$$\left(c^{(t-1)} + G^{(t-1)} \alpha_c^{(t)}\right) + \left|G^{(t-1)}\right| \cdot \alpha_x^{(t)} \leq x_{IC}^{max} \leftarrow \text{initial set}$$

$$\left(c^{t-1} + G^{(t-1)} \alpha_c^{(t)}\right) - \left|G^{(t-1)}\right| \cdot \alpha_x^{(t)} \geq x_{IC}^{min}$$

Now, $\alpha_c = \dfrac{\overline{\gamma} + \underline{\gamma}}{2}$ in vector form, or individually, $\alpha_{c_i} = \dfrac{\overline{\gamma}_i + \underline{\gamma}_i}{2}$

$\alpha_x = \dfrac{\overline{\gamma} - \underline{\gamma}}{2}$ $\qquad \alpha_{x_i} = \dfrac{\overline{\gamma}_i - \underline{\gamma}_i}{2}$

After each optimization, I construct α_c and α_x to be used for both forward reachability in the next iteration, as well as for mapping backward in the end to get the "safe set".

Forward:

$$Z^{(t)} = \left\langle Ad \cdot \left(c^{(t-1)} + G^{(t-1)} \cdot \alpha_c^{(t)}\right); \alpha_1^{(t)} \cdot Ad \cdot g_1^{(t-1)}; \ldots, \alpha_p^{(t)} \cdot Ad\, g_p^{(t-1)} \right\rangle$$

Final Backward computation for safe set:

exponent

superscript

$$\left\langle c^{(0)} + \sum_{t-1}^{T} \left(A_d^{-1}\right)^{t-1} \cdot G^{(t-1)} \cdot \alpha_c^{(t)}; \prod_{t=1}^{T} \alpha_{x_1}^{(t)} \cdot g_1^{(0)}, \ldots, \prod_{t=1}^{T} \alpha_{x_p}^{(t)} \cdot g_p^{(0)} \right\rangle$$