$$\max_{\substack{\alpha_c,\ \alpha_r,\ \psi_c^{(t)},\ \psi_r^{(t)} \\ p\times 1\ \ p\times 1\ \ m\times p\ \ m\times p}} \sum_{j=1}^{p} \alpha_{r_j} + \sum_{t=1}^{\tau}\sum_{j=1}^{p}\sum_{i=1}^{m} \underbrace{w_i}_{\text{weights for control}} \psi_{r_{ij}}^{(t)}$$

Subject to

$$0 \le \sum_{j=1}^{p} \psi_{r_{ij}}^{(t)} \le \left\{\psi_j^{\max} + \psi_j^{\min}\right\}/2 \ , \ \forall\ i=1,\dots,m \quad \forall\ t=1,\dots,\tau$$

$$0 \le \psi_{r_{ij}}^{(t)} \le 1 \ , \ \psi_j^{\min} + \psi_{r_{ij}}^{(t)} \le \psi_{c_{ij}}^{(t)} \le \psi_j^{\max} - \psi_{r_{ij}}^{(t)}$$

$$-1 + \alpha_{r_j} \le \alpha_{c_j} \le 1 - \alpha_{r_j} \ , \ \forall\ j=1,\dots,p$$

$$0 < \varepsilon \le \alpha_{r_j} \le 1$$

$$\max_{\substack{\alpha_c,\ \alpha_r,\ \psi_c^{(t)},\ \psi_r^{(t)} \\ p\times 1\ \ p\times 1\ \ m\times p\ \ m\times p}} \sum_{j=1}^{p} \alpha_{r_j} + \sum_{t=1}^{\tau}\sum_{j=1}^{p}\sum_{i=1}^{m} \underbrace{w_i}_{\text{weights for control}} \psi_{r_{ij}}^{(t)}$$

Subject to

$$0 \le \sum_{j=1}^{p} \psi_{r_{ij}}^{(t)} \le 1 \ , \ \forall\ i=1,\dots,m \quad \forall\ t=1,\dots,\tau$$

$$0 \le \psi_{r_{ij}}^{(t)} \le 1$$

$$-1 + \alpha_{r_j} \le \alpha_{c_j} \le 1 - \alpha_{r_j} \ , \ \forall\ j=1,\dots,p$$

$$0 < \varepsilon \le \alpha_{r_j} \le 1$$

// Scaling is within constraint set

$$\left(c^{(0)} + G^{(0)} \alpha_c\right) + \left| G^{(0)} \right| \cdot \alpha_r \leq X_{IS}^{max} \leftarrow \text{initial set}$$

$$\left(c^{(0)} + G^{(0)} \cdot \alpha_c\right) - \left| G^{(0)} \right| \cdot \alpha_r \geq X_{IS}^{min}$$

// for $t = 1, \ldots, \tau$

$$A^{t}\left(c^{(0)} + G^{(0)} \alpha_c\right) + \sum_{j=1}^{p} \sum_{s=1}^{t} A^{s-1} B\, u_{c_j}^{(t-s+1)} + \sum_{j=1}^{p} \left| A^{t} \alpha_{r_j} g_j^{(0)} + \sum_{s=1}^{t} A^{s-1} B\, u_{r_j}^{(t-s+1)} \right| \leq X_{CS}^{max} \leftarrow \text{constraint set}$$

$$A^{(t)}\left(c^{(0)} + G^{(0)} \cdot \alpha_c\right) + \sum_{j=1}^{p} \sum_{s=1}^{t} A^{s-1} B\, u_{c_j}^{(t-s+1)} - \sum_{j=1}^{p} \left| A^{t} \alpha_{r_j} g_j^{(0)} + \sum_{s=1}^{t} A^{s-1} B\, u_{r_j}^{(t-s+1)} \right| \geq X_{CS}^{min}$$

There is only a single scaling at time $t=0$, represented by 2 vectors:

$\underset{(p\times 1)}{\alpha_c}$ and $\underset{(p\times 1)}{\alpha_r}$, where p is the number of generators

Reach set updating:

$$R^{(t)} = \left\langle \underset{n\times n}{A^{t}} \left(\underset{n\times 1}{c^{(0)}} + \underset{n\times p}{G^{(0)}} \cdot \underset{p\times 1}{\alpha_c} \right) + \sum_{j=1}^{p} \sum_{s=1}^{t} A^{s-1} \cdot B \cdot \underset{m\times 1}{u_{c_j}^{(t-s+1)}}, \right.$$

$$\underbrace{A^t \cdot \alpha_{r_1} \cdot g_1^{(0)}}_{\text{Center}} + \underbrace{\sum_{s=1}^{t} A^{s-1} \cdot B \cdot u_{r_1}^{(t-s+1)}}_{\text{Sum over all generator input element centers}}, \ldots,$$

$$A^t \alpha_p g_p^{(0)} + \sum_{s=1}^{t} A^{s-1} \cdot B \cdot u_{r_p}^{(t-s+1)} \rangle$$

$u_c^{(t)}$ and $u_r^{(t)}$ are $m \times p$. As the scaling is done beforehand (at $t=0$), we don't need to worry about nonlinearity.