Reprojection error based on Scaramuzza's one point algorithm

Akhil Nagariya

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Let $X_i = (x_i, y_i, z_i)$ and $X_i' = (x_i', y_i', z_i')$ are the corresponding points in the two image frames. Considering the usual orientation of coordinate axes (Y axis pointing down, X axis pointing right and Z axis pointing away from the camera) the rotation about Y axis according to Scaramuzza's one point method can be given as:

$$\theta = tan^{-1}\left(\frac{x_i' y_i - x_i y_i'}{z_i' y_i + z_i y_i'}\right) \tag{1}$$

Suppose that we are given 2D to 2D imperfect image correspondences $\{X_i\} \leftrightarrow \{X_i'\}$. We wish to find an angle $\hat{\theta}$ and pairs of perfectly matched points $\hat{X}_i$ and $\hat{X}_i'$, i.e. points that satisfy

$$\theta = tan^{-1}\left(\frac{\hat{x_i}' \hat{y_i} - \hat{x_i} \hat{y_i}'}{\hat{z_i}' \hat{y_i} + \hat{z_i} \hat{y_i}'}\right) \tag{2}$$

and minimize the reprojection error function given by

$$\sum_{i}^{n} d(X_i, \hat{X}_i) + d(X_i', \hat{X_i}') \tag{3}$$

So the correspondences can be interpreted as imperfect images of a world point and the reprojection error quantifies their deviation from the true image projections $\hat{X}_i$,$\hat{X_i}'$