Exam project: Enumerate all neighborly 4-polytopes on 7 vertices

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1 Introduction

Our goal is to solve the following problem:

Problem 1. Enumerate all simplicial neighbourly 4-polytopes on 8 vertices up to combinatorial equivalent.

The rough idea of how we solve this problem is the following. We start from all possible affine Gale diagrams of of a 4-polytope on 8 vertices. These are point configurations of 8 points in the plane, each with an assigned positive or negative sign. We check which of these affine Gale diagrams correspond to neighbourly, simplicial polytopes, and consider only these configurations. Then, check whether they correspond to the same polytope (i.e. to a combinatorially equivalent polytope) and once we have all possible Gale diagrams, retrieve the polytope they correspond to.

2 Results

Since this is a studied problem (see [1]), we know that the result we must obtain is that there are 3 different simplicial neighbourly 4-polytopes on 8 vertices.

3 Discussion

4 Methods

4.1 Program structure

4.2 Program details

5 Proof

References

[1] B. Grünbaum, V.P. Sreedharan, An enumeration of simplicial 4-polytopes with 8 vertices. *Journal of Combinatiorial Theory 2*, 437-465, (1967).