Robotics 34753
Educational Robotic Arm

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1 Problem 1

Find, by the use of Figure 1, the direct kinematic transformations, T_4^0 for the robot stylus, and T_5^0 for the robot camera, as function of all joint angles.

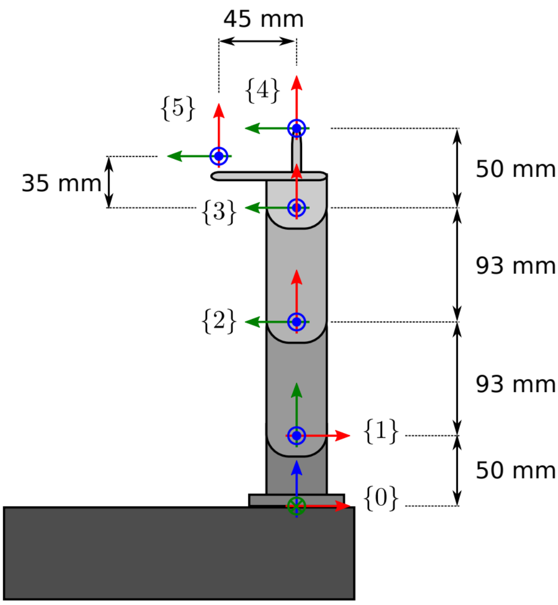


Figure 1: *Diagram of the 4-joint robot and axes layout. The x, y, and x axes are the red, green, and blue arrows respectively.*

For this problem, the red, green, and blue arrows were interpreted as x, y, and z-axes respectively. Using this interpretation, the Denavit-Hartenberg convention was used to find T_4^0 as the following conditions have been met:

- x_i is perpendicular to the z_{i-1} axis
- x_i intersects with the z_{i-1} axis

The values of each parameter can be seen in Table 1. Using these parameters and the formula from equation 1, matrices T_1^0, T_2^1, T_3^2, T_4^3 have been calculated by substituting the values of Table 1 inside equation 1.1.

$$\begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.1}$$

$$T_1^0 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & 50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Table 1: Denavit-Hartenberg paremeters for joints 1, 2, 3, and 4.

	θ_i	d_i	a_i	α_i
1	θ_1	50	0	$\frac{\pi}{2}$
2	θ_2	0	93	0
3	θ_3	0	93	0
4	θ_4	0	50	0

$$T_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 93\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & 93\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 93\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & 93\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 50\cos\theta_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & 50\sin\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The above 4 matrices were multiplied together to calculate T_4^0. The result can be seen in equation 1.2.

$$T_4^0 = \begin{bmatrix} \sigma_3\cos\theta_1 & -\sigma_1\cos\theta_1 & \sin\theta_1 & \sigma_2\cos\theta_1 \\ \sigma_3\sin\theta_1 & -\sigma_1\sin\theta_1 & -\cos\theta_1 & \sigma_2\sin\theta_1 \\ \sigma_1 & \sigma_3 & 0 & 50\sigma_1 + 93\sin(\theta_2+\theta_3) + 93\sin\theta_2 + 50 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.2}$$

Where:

$$\sigma_1 = \sin(\theta_2+\theta_3+\theta_4)$$
$$\sigma_2 = 50\sigma_3 + 93(cos(\theta_2) + cos(\theta_2+\theta_3))$$
$$\sigma_3 = \cos(\theta_2+\theta_3+\theta_4)$$

For T_5^4 a normal transformation matrix was used. There is no rotation from position 4 to 5, only a translation as seen below.

$$T_5^4 = \begin{bmatrix} 1 & 0 & 0 & -15 \\ 0 & 1 & 0 & 45 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To calculate T_5^0 (Equation 1.3), the last matrix, T_5^4 is multiplied to T_4^0.

$$T_5^0 = \begin{bmatrix} \sigma_3 \cos\theta_1 & -\sigma_2 \cos\theta_1 & \sin\theta_1 & \sigma_1 \cos\theta_1 \\ \sigma_3 \sin\theta_1 & -\sigma_2 \sin\theta_1 & -\cos\theta_1 & \sigma_1 \sin\theta_1 \\ \sigma_2 & \sigma_3 & 0 & \gamma \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.3}$$

Where:

$$\begin{gathered} \gamma = 0.035\sigma_2 + 0.045\sigma_3 + 0.093[\sin(\theta_2) + \sin(\theta_2 + \theta_3)] + 0.05 \\ \sigma_1 = -0.045\sigma_2 + 0.035\sigma_3 + 0.093[\cos(\theta_2) + \cos(\theta_2 + \theta_3)] \\ \sigma_2 = \sin(\theta_2 + \theta_3 + \theta_4) \\ \sigma_3 = \cos(\theta_2 + \theta_3 + \theta_4) \end{gathered}$$

2 Problem 2

Determine the inverse kinematic transformation

$$q = [q_1, q_2, q_3, q_4]^T = f(x_4^0, o_4^0)$$

where x_4^0 are the first 3 components of the first column of T_4^0, and o_4^0 are the first 3 components of the last column of T_4^0, respectively. Satisfy all position components in o_4^0 and only the last component of x_4^0.

First, to solve for θ_1, the first and second values in o_4^0 are used.

$$o_4^0(1) = x = \sigma_2 \cos\theta_1 \tag{2.1}$$

$$o_4^0(2) = y = \sigma_2 \sin\theta_1 \tag{2.2}$$

Therefore,

$$\theta_1 = \arctan\frac{y}{x} \tag{2.3}$$

Next, the law of cosine can be used to calculate θ_3 and θ_2. This is done as follows:

$$\theta_{\text{TOTAL}} = \arctan\frac{z_4}{\sqrt{x_4^2 + y_4^2}} \tag{2.4}$$

Therefore, we can find the equation for θ_3 as

$$z_3 = z_4 + a4\sin\theta_{TOTAL} \tag{2.5}$$

$$x_3 = x_4 + a4\cos\theta_{TOTAL}\cos\theta_1 \tag{2.6}$$

$$y_3 = x_3\tan\theta_1 \tag{2.7}$$

$$r = \sqrt{x_3^2 + y_3^2} \tag{2.8}$$

$$s = z_3 - d_1 \tag{2.9}$$

$$\cos\theta_3 = \frac{r^2 + s^2 - a_2^2 - a_3^2}{2a_2a_3} \tag{2.10}$$

$$\theta_3 = \arctan\frac{\pm\sqrt{1 - \cos{\theta_3}^2}}{\cos\theta_3} \tag{2.11}$$

Where $\pm$ indicates the elbow up or down configurations respectively. Using θ_3, θ_2 can be found using the following formula

$$\theta_2 = \arctan\frac{s}{r} - \arctan\frac{a_3\sin\theta_3}{a_2 + a_3\cos\theta_3} \tag{2.12}$$

Lastly, θ_4 can be calculated as follows

$$\theta_4 = \theta_{TOTAL} - \theta_2 - \theta_3 \tag{2.13}$$

3 Problem 3

Find a sequence of 37 robot configurations

$$q(j) = [q_1^{(j)}, q_2^{(j)}, q_3^{(j)}, q_4^{(j)}]^T = f(x_4^0 = [?, ?, 0]^T, o_4^0 = p^0(\varphi + j)), j = 0, 1, ..., 36)$$

that are necessary for the stylus tip to track 36 equidistant points on a circle with R=32 mm and $p_c^0 = [150, 0, 120]^T mm$. The tracked points start at $\varphi_0 = 0$ and end at $\varphi_36 = 2\pi$, while the stylus remains horizontal at all configuration.

The circle is defined by the equation:

$$p^0(\varphi) = p_c^0 + R \begin{bmatrix} 0 \\ \cos\varphi \\ \sin\varphi \end{bmatrix} \quad \textit{for } 0 \leq \varphi \leq 2\pi \tag{3.1}$$

In order for the four q values to be calculated for each value of φ the following steps need to be taken.

1. Substitute p_c^0, R and the current φ value into the circle equation.
2. θ_1 can be found using the x and y values from p^0, the position of the end-effector.

$$\theta_1 = \arctan\frac{y_c}{x_c} \tag{3.2}$$

3. The values for x_c and y_c can be found using the following:

$$x_c = o_4^0(1) \tag{3.3}$$

$$y_c = o_4^0(2) \tag{3.4}$$

4. By following the same steps seen in Problem 2 (2.4 to 2.13, it is possible to calculate the joint angles for each angle φ given as input

Using this process for each value of φ, a sequence of 37 robot configurations is found. The matrix in Figure 2 shows the results of the joint angles for the 37 equidistant points on the circle.

$$
\begin{bmatrix}
12.0426 & 49.5165 & -49.9695 & 0.4529 \\
11.8648 & 49.4983 & -46.5147 & -2.9837 \\
11.3357 & 49.3658 & -42.9646 & -6.4012 \\
10.4675 & 49.1159 & -39.4039 & -9.7120 \\
9.2814 & 48.7584 & -35.9362 & -12.8222 \\
7.8082 & 48.3200 & -32.6913 & -15.6287 \\
6.0885 & 47.8489 & -29.8321 & -18.0169 \\
4.1732 & 47.4160 & -27.5551 & -19.8609 \\
2.1215 & 47.1058 & -26.0709 & -21.0348 \\
0.0000 & 46.9923 & -25.5532 & -21.4391 \\
-2.1215 & 47.1058 & -26.0709 & -21.0348 \\
-4.1732 & 47.4160 & -27.5551 & -19.8609 \\
-6.0885 & 47.8489 & -29.8321 & -18.0169 \\
-7.8082 & 48.3200 & -32.6913 & -15.6287 \\
-9.2814 & 48.7584 & -35.9362 & -12.8222 \\
-10.4675 & 49.1159 & -39.4039 & -9.7120 \\
-11.3357 & 49.3658 & -42.9646 & -6.4012 \\
-11.8648 & 49.4983 & -46.5147 & -2.9837 \\
-12.0426 & 49.5165 & -49.9695 & 0.4529 \\
-11.8648 & 49.4326 & -53.2573 & 3.8247 \\
-11.3357 & 49.2656 & -56.3157 & 7.0502 \\
-10.4675 & 49.0391 & -59.0893 & 10.0502 \\
-9.2814 & 48.7801 & -61.5286 & 12.7485 \\
-7.8082 & 48.5167 & -63.5896 & 15.0728 \\
-6.0885 & 48.2768 & -65.2346 & 16.9578 \\
-4.1732 & 48.0850 & -66.4327 & 18.3477 \\
-2.1215 & 47.9613 & -67.1611 & 19.1998 \\
0.0000 & 47.9186 & -67.4055 & 19.4869 \\
2.1215 & 47.9613 & -67.1611 & 19.1998 \\
4.1732 & 48.0850 & -66.4327 & 18.3477 \\
6.0885 & 48.2768 & -65.2346 & 16.9578 \\
7.8082 & 48.5167 & -63.5896 & 15.0728 \\
9.2814 & 48.7801 & -61.5286 & 12.7485 \\
10.4675 & 49.0391 & -59.0893 & 10.0502 \\
11.3357 & 49.2656 & -56.3157 & 7.0502 \\
11.8648 & 49.4326 & -53.2573 & 3.8247 \\
12.0426 & 49.5165 & -49.9695 & 0.4529
\end{bmatrix} \tag{3.5}
$$

Figure 2: The 37 calculated equidistant points of a circle using the elbow-down solution. The values are in degrees.

4 Problem 4

Determine the Jacobian of the manipulator for the robot end-effector and the Jacobian for the robot camera (as a function of the joint configuration q). Report the numerical results for the two Jacobians at $\varphi = 0$, $\varphi = \pi/2$, $\varphi = \pi$, and $\varphi = 3\pi/2$ along the path studies in Problem 3

The formula for revolute joints was used to calculate the Jacobian matrix.

$$J = [J_1 ... J_i ... J_n], \text{ where } J_i = \begin{bmatrix} z_{i-1} \times (o_n - O_{i-1}) \\ z_{i-1} \end{bmatrix} \tag{4.1}$$

Using this formula the general equations for the matrices J_0^4 and J_0^5 were found. The Jacobian matrix for the end-effector is:

$$J_0^4 = \begin{bmatrix} -\sigma_3 \sin\theta_1 & -\sigma_6 \cos\theta_1 & -\sigma_5 \cos\theta_1 & -\sigma_4 \cos\theta_1 \\ \sigma_3 \cos\theta_1 & -\sigma_6 \sin\theta_1 & -\sigma_5 \sin\theta_1 & -\sigma_4 \sin\theta_1 \\ 0 & \sigma_3 & \sigma_2 & \sigma_1 \\ 0 & \sin\theta_1 & \sin\theta_1 & \sin\theta_1 \\ 0 & -\cos\theta_1 & -\cos\theta_1 & -\cos\theta_1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{4.2}$$

Where:

$$\begin{aligned} \sigma_1 &= 50\cos(\theta_2 + \theta_3 + \theta_4) \\ \sigma_2 &= \sigma_1 + 93\cos(\theta_2 + \theta_3) \\ \sigma_3 &= \sigma_2 + 93\cos(\theta_2) \\ \sigma_4 &= 50\sin(\theta_2 + \theta_3 + \theta_4) \\ \sigma_5 &= \sigma_4 + 93\sin(\theta_2 + \theta_3) \\ \sigma_6 &= \sigma_5 + 93\sin(\theta_2) \end{aligned}$$

And, the Jacobian matrix for the camera.

$$J_0^5 = \begin{bmatrix} -\sigma_3 \sin\theta_1 & -\sigma_6 \cos\theta_1 & -\sigma_5 \cos\theta_1 & -\sigma_4 \cos\theta_1 \\ \sigma_3 \cos\theta_1 & -\sigma_6 \sin\theta_1 & -\sigma_5 \sin\theta_1 & -\sigma_4 \sin\theta_1 \\ 0 & \sigma_3 & \sigma_2 & \sigma_1 \\ 0 & \sin\theta_1 & \sin\theta_1 & \sin\theta_1 \\ 0 & -\cos\theta_1 & -\cos\theta_1 & -\cos\theta_1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{4.3}$$

Where:

$$\begin{aligned} \sigma_1 &= 35\cos(\theta_2 + \theta_3 + \theta_4) - 45\sin(\theta_2 + \theta_3 + \theta_4) \\ \sigma_2 &= \sigma_1 + 93\cos(\theta_2 + \theta_3) \\ \sigma_3 &= \sigma_2 + 93\cos(\theta_2) \\ \sigma_4 &= 35\sin(\theta_2 + \theta_3 + \theta_4) + 45\cos(\theta_2 + \theta_3 + \theta_4) \end{aligned}$$

$$\begin{aligned}\sigma_5 &= \sigma_4 + 93\sin(\theta_2 + \theta_3) \\ \sigma_6 &= \sigma_5 + 93\sin(\theta_2)\end{aligned}$$

The solutions from Problem 3 (Figure 2) are used to provide the corresponding q_i values for each value of φ. The values of q_i for each φ value are found in Table 2.

Table 2: q_i values for each φ value

	θ_1	θ_2	θ_3	θ_4
$\varphi = 0$	12.0426	49.5165	-49.9695	0.4529
$\varphi = \pi/2$	0	46.9923	-25.5532	-21.4391
$\varphi = \pi$	-12.0426	49.5165	-49.9695	0.4529
$\varphi = 3\pi/2$	0	47.9186	-67.4055	19.4869

Substituting in each of these q_i values for each Jacobian matrix and each φ value gives us the following solutions in Figure 3 to 10.

$$J^4_{\varphi=0} = \begin{bmatrix} -42.4319 & -68.4595 & 0.7190 & 0 \\ 198.8996 & -14.6047 & 0.1534 & 0 \\ 0 & 203.3754 & 142.9971 & 50 \\ 0 & 0.2086 & 0.2086 & 0.2086 \\ 0 & -0.9780 & -0.9780 & -0.9780 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{4.4}$$

Figure 3: *End-effector Jacobian when* $\varphi = 0$

$$J^5_{\varphi=0} = \begin{bmatrix} -39.3025 & -112.4690 & -43.2905 & -44.0096 & -44.0097 \\ 184.2298 & -23.9934 & -9.2353 & -9.3887 & -9.3888 \\ 0 & 188.3755 & 127.9972 & 35 & -15 \\ 0 & 0.2086 & 0.2086 & 0.2086 & 0.2086 \\ 0 & -0.9780 & -0.9780 & -0.9780 & -0.9780 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.5}$$

Figure 4: *Camera Jacobian when* $\varphi = 0$

$$J^4_{\varphi=\frac{\pi}{2}} = \begin{bmatrix} 0 & -102 & -33.9926 & 0 \\ 200 & 0 & 0 & 0 \\ 0 & 200 & 136.5650 & 50 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{4.6}$$

Figure 5: *End-effector Jacobian when* $\varphi = \frac{\pi}{2}$

$$J^5_{\varphi=\frac{\pi}{2}} = \begin{bmatrix} 0 & -147 & -78.9926 & -45 & -45 \\ 185 & 0 & 0 & 0 & 0 \\ 0 & 185 & 121.5650 & 35 & -15 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.7}$$

Figure 6: *Camera Jacobian when* $\varphi = \frac{\pi}{2}$

$$J^4_{\varphi=\pi} = \begin{bmatrix} 42.4319 & -68.4595 & 0.7190 & 0 \\ 198.8996 & 14.6047 & -0.1534 & 0 \\ 0 & 203.3754 & 142.9971 & 50 \\ 0 & -0.2086 & -0.2086 & -0.2086 \\ 0 & -0.9780 & -0.9780 & -0.9780 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{4.8}$$

Figure 7: *End-effector Jacobian when* $\varphi = \pi$

$$J^5_{\varphi=\pi} = \begin{bmatrix} 39.3025 & -112.4690 & -43.2905 & -44.0096 & -44.0097 \\ 184.2298 & 23.9934 & 9.2353 & 9.3887 & 9.3888 \\ 0 & 188.3755 & 127.9972 & 35 & -14.9999 \\ 0 & -0.2086 & -0.2086 & -0.2086 & -0.2086 \\ 0 & -0.9780 & -0.9780 & -0.9780 & -0.9780 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.9}$$

Figure 8: *Camera Jacobian when* $\varphi = \pi$

$$J^4_{\varphi=3\frac{\pi}{2}} = \begin{bmatrix} 0 & -38 & 31.0240 & 0 \\ 200 & 0 & 0 & 0 \\ 0 & 200 & 137.6727 & 50 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{4.10}$$

Figure 9: *End-effector Jacobian when* $\varphi = 3\frac{\pi}{2}$

$$J^5_{\varphi=3\frac{\pi}{2}} = \begin{bmatrix} 0 & -83 & -13.9760 & -45 & -45 \\ 185.0000 & 0 & 0 & 0 & 0 \\ 0 & 185 & 122.6728 & 35 & -15 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.11}$$

Figure 10: *Camera Jacobian when* $\varphi = 3\frac{\pi}{2}$

5 Problem 5

Compute the joint velocities $\dot{q}$ at $\varphi = \pi/2$, along the path from Problem 3, so that the stylus tip velocity is $v_4^0 = [0, -3, 0] mm/s$ and $\dot{x_4} = [?, ?, 0]$.

Since the Jacobian matrix is not square, the formula $v = J\dot{q}$ is rearranged to form equation 5.1, which is used to find the joint velocities. Mathematically speaking, inverting a matrix is not possible. However, since we are engineers, the pseudo inverse (Equation 5.2) has been used to approximate the inverse of the Jacobian matrix.

$$\dot{q} = J^{+}v + (NN^{+})\dot{q_{ns}} \tag{5.1}$$

Where J^{+} is the pseudo-inverse Jacobian defined as

$$J^{+} = \begin{cases} A^T(AA^T)^{-1},\ m < n \\ A^{-1},\ m = n \\ (A^TA)^{-1}A^T,\ m > n \end{cases} \tag{5.2}$$

In this case, $m > n$, since the robot has an under-actuated arm (the number of joints is less than 6), and, since $(NN^{+})\dot{q}_{ns}$ does not affect the end pose, it is possible to reduce the Equation 5.1 to

$$\dot{q} = J^{+}v \tag{5.3}$$

Using the Jacobian matrix from Figure 5, it is possible to calculate $\dot{q}$ as

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0.005 & 0 & 0 & 0 & 2.5\text{e-}5 \\ -0.0232 & 0 & -0.00911 & 0 & -0.456 & 0 \\ 0.0402 & 0 & 0.0273 & 0 & 1.37 & 0 \\ -0.017 & 0 & -0.0182 & 0 & -1.91 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.015 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5.4}$$

6 Problem 6

Use the inverse computed joint configurations $q^{(0)}, q^{(9)}, q^{(18)}, q^{(27)}, q^{(36)}$ from Problem 3, to find suitable interpolation polynomials for the following segments:

Segment A: $q(0) \rightarrow q(9)$**,** $0 \leq t_A \leq 2s$

$$\begin{aligned}
q_1(t_A) &= A_{15} \cdot t_A^5 + A_{14} \cdot t_A^4 + A_{13} \cdot t_A^3 + A_{12} \cdot t_A^2 + A_{11} \cdot t_A + A_{10} \\
q_2(t_A) &= A_{25} \cdot t_A^5 + A_{24} \cdot t_A^4 + A_{23} \cdot t_A^3 + A_{22} \cdot t_A^2 + A_{21} \cdot t_A + A_{20} \\
q_3(t_A) &= A_{35} \cdot t_A^5 + A_{34} \cdot t_A^4 + A_{33} \cdot t_A^3 + A_{32} \cdot t_A^2 + A_{31} \cdot t_A + A_{30} \\
q_4(t_A) &= A_{45} \cdot t_A^5 + A_{44} \cdot t_A^4 + A_{43} \cdot t_A^3 + A_{42} \cdot t_A^2 + A_{41} \cdot t_A + A_{40}
\end{aligned} \tag{6.1}$$

Segment B: $q(9) \rightarrow q(18)$**,** $0 \leq t_B \leq 2s$

$$\begin{aligned}
q_1(t_B) &= B_{15} \cdot t_B^5 + B_{14} \cdot t_B^4 + B_{13} \cdot t_B^3 + B_{12} \cdot t_B^2 + B_{11} \cdot t_B + B_{10} \\
q_2(t_B) &= B_{25} \cdot t_B^5 + B_{24} \cdot t_B^4 + B_{23} \cdot t_B^3 + B_{22} \cdot t_B^2 + B_{21} \cdot t_B + B_{20} \\
q_3(t_B) &= B_{35} \cdot t_B^5 + B_{34} \cdot t_B^4 + B_{33} \cdot t_B^3 + B_{32} \cdot t_B^2 + B_{31} \cdot t_B + B_{30} \\
q_4(t_B) &= B_{45} \cdot t_B^5 + B_{44} \cdot t_B^4 + B_{43} \cdot t_B^3 + B_{42} \cdot t_B^2 + B_{41} \cdot t_B + B_{40}
\end{aligned} \tag{6.2}$$

Segment C: $q(18) \rightarrow q(27)$**,** $0 \leq t_C \leq 2s$

$$\begin{aligned}
q_1(t_C) &= C_{15} \cdot t_C^5 + C_{14} \cdot t_C^4 + C_{13} \cdot t_C^3 + C_{12} \cdot t_C^2 + C_{11} \cdot t_C + C_{10} \\
q_2(t_C) &= C_{25} \cdot t_C^5 + C_{24} \cdot t_C^4 + C_{23} \cdot t_C^3 + C_{22} \cdot t_C^2 + C_{21} \cdot t_C + C_{20} \\
q_3(t_C) &= C_{35} \cdot t_C^5 + C_{34} \cdot t_C^4 + C_{33} \cdot t_C^3 + C_{32} \cdot t_C^2 + C_{31} \cdot t_C + C_{30} \\
q_4(t_C) &= C_{45} \cdot t_C^5 + C_{44} \cdot t_C^4 + C_{43} \cdot t_C^3 + C_{42} \cdot t_C^2 + C_{41} \cdot t_C + C_{40}
\end{aligned} \tag{6.3}$$

Segment D: $q(27) \rightarrow q(36)$**,** $0 \leq t_D \leq 2s$

$$\begin{aligned}
q_1(t_D) &= D_{15} \cdot t_D^5 + D_{14} \cdot t_D^4 + D_{13} \cdot t_D^3 + D_{12} \cdot t_D^2 + D_{11} \cdot t_D + D_{10} \\
q_2(t_D) &= D_{25} \cdot t_D^5 + D_{24} \cdot t_D^4 + D_{23} \cdot t_D^3 + D_{22} \cdot t_D^2 + D_{21} \cdot t_D + D_{20} \\
q_3(t_D) &= D_{35} \cdot t_D^5 + D_{34} \cdot t_D^4 + D_{33} \cdot t_D^3 + D_{32} \cdot t_D^2 + D_{31} \cdot t_D + D_{30} \\
q_4(t_D) &= D_{45} \cdot t_D^5 + D_{44} \cdot t_D^4 + D_{43} \cdot t_D^3 + D_{42} \cdot t_D^2 + D_{41} \cdot t_D + D_{40}
\end{aligned} \tag{6.4}$$

Determine the coefficients A_{ij}, B_{ij}, C_{ij}, D_{ij} so that

$$\begin{aligned}
&v(t_{A=0}) = [0,0,0]^T \text{ mm/s} \quad \ddot{q}(t_{A=0}) = [0,0,0,0]^T \text{ rad/s}^2 \\
&v(t_{A=2}) = v(t_{B=0}) = [0,27,0]^T \text{ mm/s} \quad \ddot{q}(t_{A=2}) = \ddot{q}(t_{B=0}) = [0,0,0,0]^T \text{ rad/s}^2 \\
&v(t_{B=2}) = v(t_{C=0}) = [0,0,27]^T \text{ mm/s} \quad \ddot{q}(t_{B=2}) = \ddot{q}(t_{C=0}) = [0,0,0,0]^T \text{ rad/s}^2 \\
&v(t_{C=2}) = v(t_{D=0}) = [0,27,0]^T \text{ mm/s} \quad \ddot{q}(t_{C=2}) = \ddot{q}(t_{D=0}) = [0,0,0,0]^T \text{ rad/s}^2 \\
&v(t_{D=2}) = [0,0,0]^T \text{ mm/s} \quad \ddot{q}(t_{D=2}) = [0,0,0,0]^T \text{ rad/s}^2
\end{aligned} \tag{6.5}$$

Before managing the trajectory planning, the velocities must be calculated just as it has been done for the previous exercise using Equation 5.1. To get our quintic polynomial coefficients, we have to solve the following matrix operation:

$$tc = q \tag{6.6}$$

Where:

$$q = \begin{bmatrix} q_{in} \\ v_{in} \\ a_{in} \\ q_a \\ v_a \\ a_a \end{bmatrix} \tag{6.7}$$

$$t = \begin{bmatrix} 1 & t_{in} & t_{in}^2 & t_{in}^3 & t_{in}^4 & t_{in}^5 \\ 0 & 1 & 2t_{in} & 3t_{in}^2 & 4t_{in}^3 & 5t_{in}^4 \\ 0 & 0 & 2 & 6t_{in} & 12t_{in}^2 & 20t_{in}^3 \\ 1 & t_a & t_a^2 & t_a^3 & t_a^4 & t_a^5 \\ 0 & 1 & 2t_a & 3t_a^2 & 4t_a^3 & 5t_a^4 \\ 0 & 0 & 2 & 6t_a & 12t_a^2 & 20t_a^3 \end{bmatrix} \tag{6.8}$$

By substituting the contour conditions given in the exercise, the following solutions are presented for each segment:

$$A = \begin{bmatrix} 0.2102 & 0 & 0 & -0.1377 & 0.08767 & -0.01597 \\ 0.8642 & 0 & 0 & -0.05507 & 0.0413 & -0.00826 \\ -0.8721 & 0 & 0 & 0.5327 & -0.3995 & 0.0799 \\ 0.007905 & 0 & 0 & -0.4776 & 0.3582 & -0.07164 \end{bmatrix} \tag{6.9}$$

$$B = \begin{bmatrix} 0 & -0.125 & 0 & -0.07523 & 0.07205 & -0.01597 \\ 0.8202 & 0 & 0 & 0.05785 & -0.04373 & 0.008781 \\ -0.446 & 0 & 0 & -0.2684 & 0.1683 & -0.03036 \\ -0.3742 & 0 & 0 & 0.2106 & -0.1246 & 0.02157 \end{bmatrix} \tag{6.10}$$

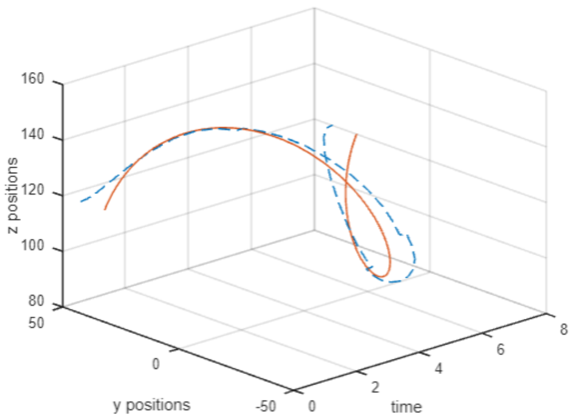
$$C = \begin{bmatrix} -0.2102 & 0 & 0 & 0.1377 & -0.08767 & 0.01597 \\ 0.8642 & -0.002775 & 0 & -0.0307 & 0.02337 & -0.004709 \\ -0.8721 & -0.2642 & 0 & 0.01598 & 0.02105 & -0.007513 \\ 0.007905 & 0.267 & 0 & 0.01472 & -0.04442 & 0.01222 \end{bmatrix} \tag{6.11}$$

$$D = \begin{bmatrix} 0 & 0.125 & 0 & 0.07523 & -0.07205 & 0.01597 \\ 0.8363 & 0 & 0 & 0.03486 & -0.02615 & 0.005229 \\ -1.176 & 0 & 0 & 0.3804 & -0.2853 & 0.05706 \\ 0.3401 & 0 & 0 & -0.4153 & 0.3114 & -0.06229 \end{bmatrix} \tag{6.12}$$

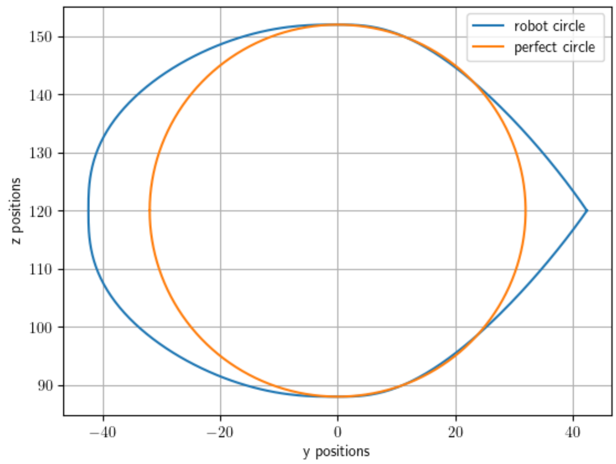
7 Problem 7

Plot the actual path of the end-effector for the entire period from = 0 to 8 s for the interpolated trajectory from problem 6 and compare it to the exact desired circular path. Try to improve the approximation either by using more knot-points or by using different interpolation functions that those found in Problem 6.

We assume that x values remain fairly constant in the trajectory path, hence we replace it with time (0-8s). The orange trajectory is the ideal trajectory, and the blue one is the generated trajectory.



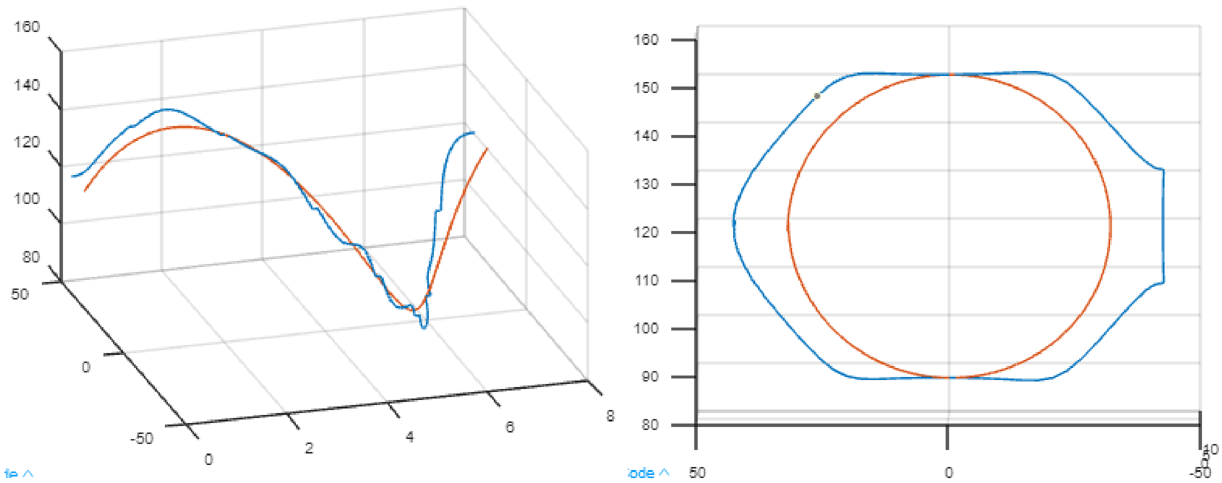
(a) Plot of y and z as a function of time



(b) Comparison of trajectory path y and z in 2D

Figure 11: Plot of end effector path for time period *t=0* to *t=8*

The generated trajectory used 40 pairs of coordinates (time, y, z) while in the perfect trajectory, 315 pairs were used (one for each rad, from 0 to 2π).



(a) Plot of y and z as a function of time (b) Comparison of trajectory paths y and z in 2D

Figure 12: 8 knots

From the plot it can be seen that the trajectory became less faithful once the number of knot points was increased to 8, the trajectory was generated the trajectory using 80 pairs of coordinates (time, y, z).

8 Problem 8

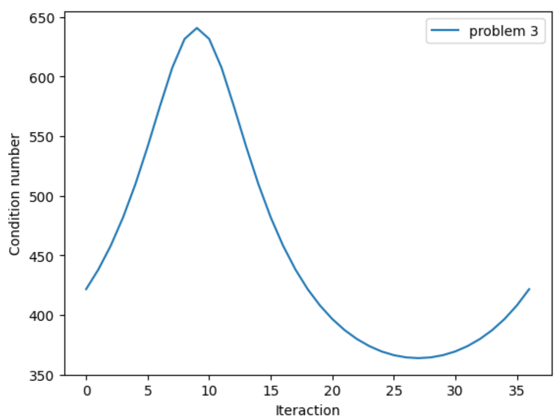
Plot the condition number of the Jacobian matrix of the manipulator along the path from Problem 3 as well as along the actual path from Problem 6 or 7, and evaluate if the path includes any singularities.

For each interaction, the Jacobian has been calculated with the same methods as Problem 4 (Equation 4.2) and the condition number has been calculated with NumPy's function 'linalg.cond()':

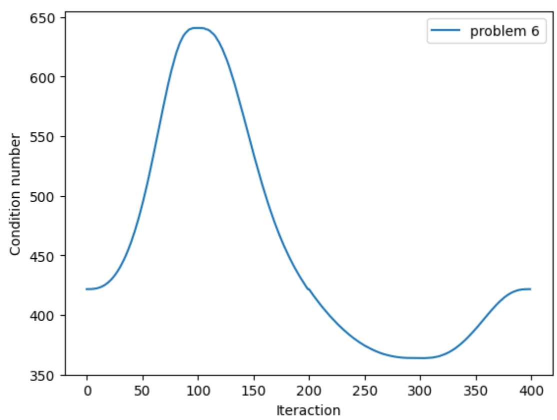
```
cond3.append(np.linalg.cond(J4);

cond6.append(np.linalg.cond(J4);
```

Where 'cond3' is a 37 elements long vector, while 'cond6' is a 400 elements long vector. The plots of the two functions look as follows:



(a) Conditioning number using data from problem 3



(b) Conditioning number using data from problem 6

Figure 13: Comparison of the two conditioning numbers over iteractions

A large condition number indicates that the matrix is close to being singular, and values of near 1 indicate a well-conditioned matrix. In mathematical terms, a singularity happens when the determinant of the Jacobian is close to 0. This implies that:

- Certain directions of motion may be unattainable;
- Infinite joint velocity may correspond to bounded end-effector velocities;
- Bounded joint torques may correspond to infinite end-effector torques/forces;
- Positions that may become unreachable by small perturbations of the robot link geometry.

Moreover, in practical terms, if the values assumed by the condition number are continuous in time, the matrix associated with the condition number is regarded as non-singular. In our case, as shown in Figure 13, it is possible to appreciate the continuity of the condition number curves for our solution. Therefore, the Jacobian matrix of the manipulator along the path of Problems 3 and 7 does not include singularities .

9 Problem 9

Neglecting the own mass of the robot arm, and assuming a weight of $1N$ along the negative z_0 direction, calculate and plot all joint torques $\tau_1, \tau_2, \tau_3, \tau_4$ as a function of the position $\varphi \in [0, 2\pi]$.

According to Velocity Kinematics & Jacobian Matrix theory, the joint torques vector of the robot can be found from the equation:

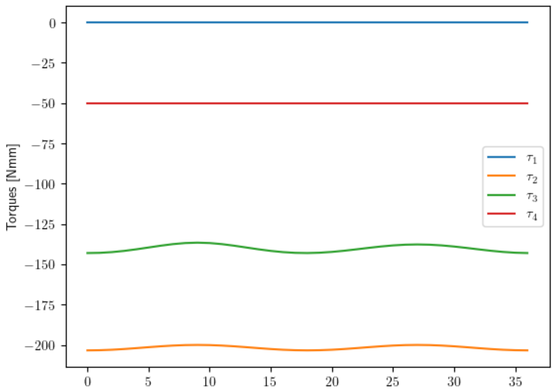
$$\tau = J^T(q)F \tag{9.1}$$

where:

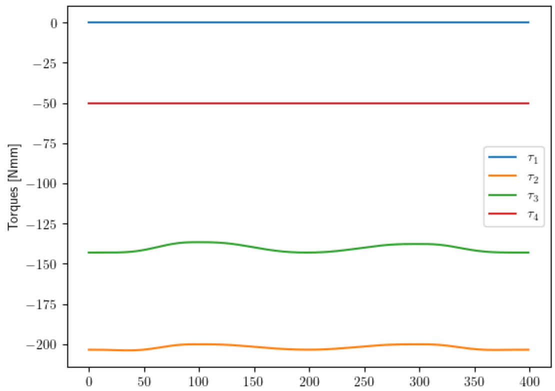
$$F = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{9.2}$$

and $J^T(q)$ represents the transposed matrix of the Jacobian J_4 calculated in Problem 4 (Equation 4.2):

By combining Equations 9.1, 9.2 and 4.2 the vector of torques is calculated as a function of θ_1, θ_2, θ_3 and θ_4. The resulting joint torques τ_1, τ_2, τ_3, and τ_4 are plotted for both input from Problem 3 and Problem 6 for the positions of $\varphi \in [0, 2\pi]$.



(a) Torque using data from problem 3



(b) Torque using data from problem 6

Figure 14: Comparison of the torques numbers over iteractions

10 Problem 10

Provide a rough but realistic estimate for I_0 based on the dimensions and masses of the links. Then derive the dynamic system for the robot arm in its standard form $D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau(38)$ and plot the required joint torques τ for the trajectory of Problem 6 or 7.

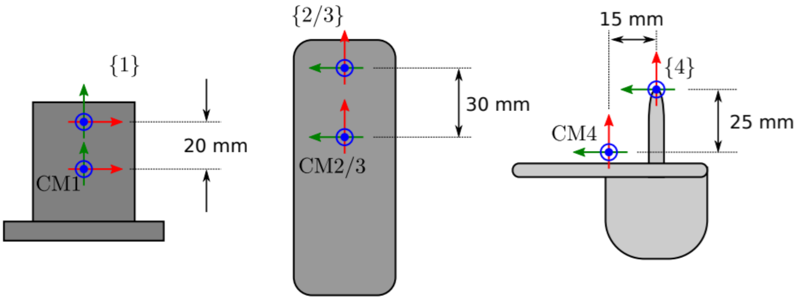


Figure 15: Centers of mass and principal axes of inertia for links 1, 2, 3 and 4

In order to provide a rough but realistic estimate for I_0 based on the dimensions and masses of the links, it is preferable to commence calculating from link 1. Due to the geometry of this link, the calculation of the inertia could be done by dividing this joint into a cylinder (base) and a rectangular parallelepiped (motor) as shown in the figure below.

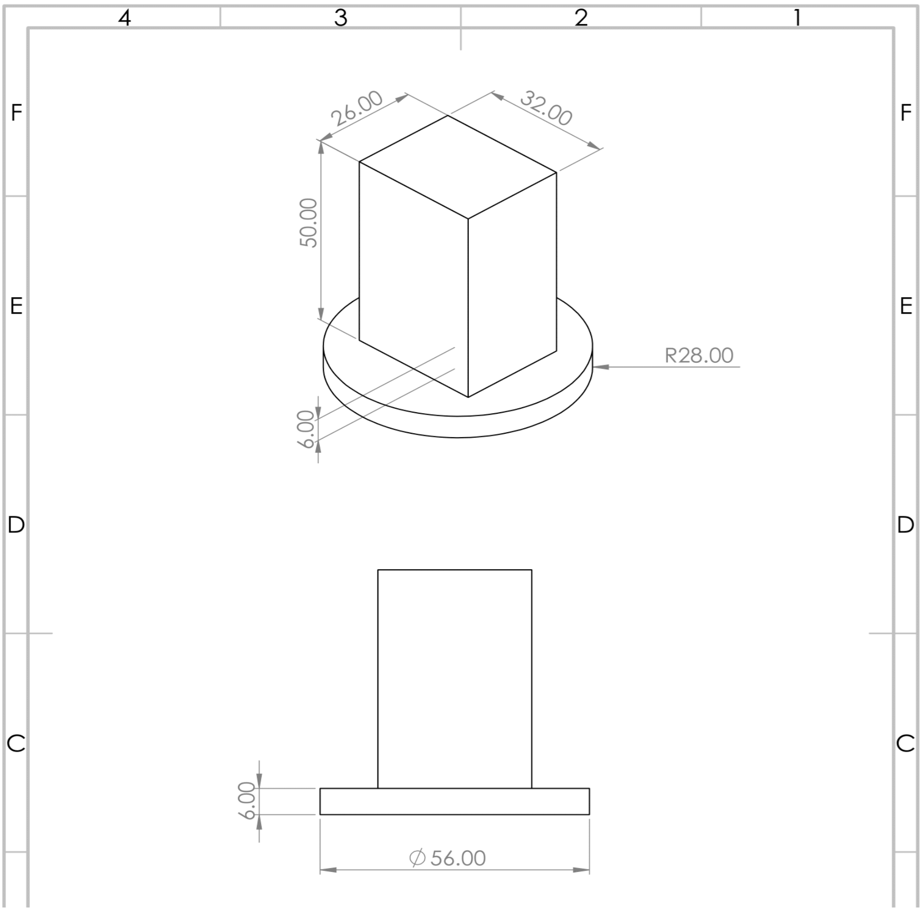


Figure 16: Link 1 geometry

According to DYNAMIXEL AX-12A servo motor drawings and as it is depicted the dimensions of the motor are $H \times W \times D = 50 \times 32 \times 26\ mm^3$ with $\mathrm{m_{motor}} = 50\,\mathrm{g}$ and for the base: radius $= 28mm$, *height* $= 6mm$ and $\mathrm{m_{base}} = 10\,\mathrm{g}$.From the given matrices for inertia in the frames of CM_i it can be derived that I_0 is equal to I_{xx} of the first link.

$$I_{xx} = \frac{1}{12} m_{\text{motor}} (H^2 + D^2) \tag{10.1}$$

Since the rotation axis of the parallelepiped does not pass through the center of mass of link 1, the Huygens–Steiner theorem will be applied with the distance being $d = 20mm$ as shown in the figure above.

$$I_{xx_m} = \frac{1}{12} m_{\text{motor}} (H^2 + D^2) + m_{\text{motor}} d^2 \tag{10.2}$$

and for the base:

$$I_{xx_m} = \frac{1}{12} m_{\text{base}} (3r^2 + h^2) \tag{10.3}$$

Combining the above equations we get the total inertia of I_{xx} for link 1:

$$I_{xx_{t_1}} = \frac{1}{12} m_{\text{motor}} (H^2 + D^2) + m_{\text{motor}} d^2 + \frac{1}{12} m_{\text{base}} (3r^2 + h^2) \tag{10.4}$$

From Equation 10.4 it is calculated that $I_{xx_{t_1}} = 40\ kgmm^2$ which is equal to $I_0 = 40\ kgmm^2$. Based on part 4's dynamics description of the problem the inertia matrices for the frames of CM_i can be calculated as:

$$\bar{D}_1 = \begin{bmatrix} 35.223 & 0 & 0 \\ 0 & 14.09 & 0 \\ 0 & 0 & 31.7 \end{bmatrix}$$

$$\bar{D}_2 = \bar{D}_3 = \begin{bmatrix} 15.85 & 0 & 0 \\ 0 & 49.312 & 0 \\ 0 & 0 & 42.268 \end{bmatrix} \tag{10.5}$$

$$\bar{D}_4 = \begin{bmatrix} 17.612 & 0 & 0 \\ 0 & 17.612 & 0 \\ 0 & 0 & 17.612 \end{bmatrix}$$

In order to proceed with the formulation of the dynamics of the robot it is needed to compute the Jacobian matrices of the center of mass for each link. For this step, updated transformation matrices are required that establish the connection between the base frame and the centers of mass. According to the first figure of this chapter, the Denavit-Hartenberg parameters for the first three links are:

Making the calculations the following matrices are obtained:

$$T^0_{\text{CM}_1} = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

θ_i	d_i	α_i	a_i
θ_1	30	0	$\frac{\pi}{2}$
θ_2	0	63	0
θ_3	0	63	0

Table 3: D-H parameters for the centers of mass for the first three links

$$T^1_{\mathrm{CM}_2} = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 63\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 63\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^2_{\mathrm{CM}_3} = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 63\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 63\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Calculation of the matrices $T^0_{CM_i}$, where $\{0\}$ refers to the base frame and CM_i refers tho the center of mass of link i. Furthermore, the matrices of T^0_1, T^1_2, T^2_3, T^3_4 will be used from Problem 1.

$$T^0_{\mathrm{CM}_2} = T^0_1 \cdot T^1_{\mathrm{CM}_2} \Rightarrow$$

$$T^0_{\mathrm{CM}_2} = \begin{bmatrix} \cos(\theta_1)\cos(\theta_2) & -\cos(\theta_1)\sin(\theta_2) & \sin(\theta_1) & 63\cos(\theta_1)\cos(\theta_2) \\ \cos(\theta_2)\sin(\theta_1) & -\sin(\theta_1)\sin(\theta_2) & -\cos(\theta_1) & 63\cos(\theta_2)\sin(\theta_1) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 63\sin(\theta_2)+50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_{\mathrm{CM}_3} = T^0_1 \cdot T^1_2 \cdot T^2_{\mathrm{CM}_3} \Rightarrow$$

$$T^0_{\mathrm{CM}_3} = \begin{bmatrix} \cos(\theta_1)\cdot\cos(\theta_2+\theta_3) & -\sin(\theta_2+\theta_3)\cdot\cos(\theta_1) & \sin(\theta_1) & (93\cos(\theta_2)+63\cos(\theta_2+\theta_3))\cdot\cos(\theta_1) \\ \sin(\theta_1)\cdot\cos(\theta_2+\theta_3) & -\sin(\theta_1)\cdot\sin(\theta_2+\theta_3) & -\cos(\theta_1) & (93\cos(\theta_2)+63\cos(\theta_2+\theta_3))\cdot\sin(\theta_1) \\ \sin(\theta_2+\theta_3) & \cos(\theta_2+\theta_3) & 0 & 93\cdot\sin(\theta_2)+63\cdot\sin(\theta_2+\theta_3)+50 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where: $\sigma_1 = 21\cos(\theta_2+\theta_3)+31\cos(\theta_2)$.

Additionally, for the computation of $T^0_{\mathrm{CM}_4}$ the following translation matrix will be used :

$$Tr^4_{\mathrm{CM}_4} = \begin{bmatrix} 1 & 0 & 0 & -25 \\ 0 & 1 & 0 & 15 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^0_{\mathrm{CM}_4} = T^0_1 \cdot T^1_2 \cdot T^2_3 \cdot T^3_4 \cdot Tr^4_{\mathrm{CM}_4} \Rightarrow$$

$$T^0_{\mathrm{CM}_4} = \begin{bmatrix} \sigma_2\cos(\theta_1) & -\sigma_1\cos(\theta_1) & \sin(\theta_1) & \sigma_4\cos(\theta_1) \\ \sigma_2\sin(\theta_1) & -\sigma_1\sin(\theta_1) & -\cos(\theta_1) & \sigma_4\sin(\theta_1) \\ \sigma 1 & \sigma_2 & 0 & \sigma_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\sigma_1 = \sin(\theta_2 + \theta_3)$$
$$\sigma_2 = \cos(\theta_2 + \theta_3)$$
$$\sigma_3 = 93\sin(\theta_2) + 68\sigma_1 + 15\sigma_2 + 50$$
$$\sigma_4 = 93\cos(\theta_2) + 68\sigma_2 - 15\sigma_1$$

The computation of Jacobian matrices for the centers of mass of the links can now be performed using the same method as explained in Problem 4 (Equation 4.2).

$$J_{\mathrm{CM}_1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_{\mathrm{CM}_2} = \begin{bmatrix} -63 \cdot \sin(\theta_1) \cdot \cos(\theta_2) & -(63 \cdot \sin(\theta_2) + 20) \cdot \cos(\theta_1) \\ 63 \cdot \cos(\theta_1) \cdot \cos(\theta_2) & -(63 \cdot \sin(\theta_2) + 20) \cdot \sin(\theta_1) \\ 0 & 63 \cdot \cos(\theta_2) \\ 0 & \sin(\theta_1) \\ 0 & -\cos(\theta_1) \\ 1 & 0 \end{bmatrix}$$

$$J_{\mathrm{CM}_3} = \begin{bmatrix} -\sigma_5 \sin(\theta_1) & -\sigma_4 \cos(\theta_1) & -\sigma_3 \cos(\theta_1) \\ \sigma_5 \cos(\theta_1) & -\sigma_4 \sin(\theta_1) & -\sigma_3 \sin(\theta_1) \\ 0 & \sigma 2 & 30\sigma_1 \\ 0 & \sin(\theta_1) & \sin(\theta_1) \\ 0 & -\cos(\theta_1) & -\cos(\theta_1) \\ 1 & 0 & 0 \end{bmatrix}$$

Where:

$$\sigma_1 = 30\cos(\theta_2) + 63\cos(\theta_2 + \theta_3)$$
$$\sigma_2 = 93\cos(\theta_2) + 63\cos(\theta_2 + \theta_3)$$
$$\sigma_3 = 30\sin(\theta_2) + 63\sin(\theta_2 + \theta_3)$$
$$\sigma_4 = 93\sin(\theta_2) + 63\sin(\theta_2 + \theta_3) + 20$$
$$\sigma_5 = 93\cos(\theta_2) + 63\cos(\theta_2 + \theta_3)$$

$$J_{\mathrm{CM}_4} = \begin{bmatrix} -\sigma_3 \sin(\theta_1) & -\sigma_6 \cos(\theta_1) & -\sigma_5 \cos(\theta_1) & -\sigma_4 \cos(\theta_1) \\ \sigma_3 \cos(\theta_1) & -\sigma_6 \sin(\theta_1) & -\sigma_5(\theta_1) & -\sigma_4 \sin(\theta_1) \\ 0 & \sigma_3 & \sigma_2 & \sigma_1 \\ 0 & \sin(\theta_1) & \sin(\theta_1) & \sin(\theta_1) \\ 0 & -\cos(\theta_1) & -\cos(\theta_1) & -\cos(\theta_1) \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Where:

$$\sigma_1 = -15\sin(\theta_2 + \theta_3) + 5\cos(\theta_2 + \theta_3)$$

$$\sigma_2 = \sigma_1 + 68\cos(\theta_2 + \theta_3)$$
$$\sigma_3 = \sigma_2 + 93\cos(\theta_2)$$
$$\sigma_4 = 15\cos(\theta_2 + \theta_3) + 5\sin(\theta_2 + \theta_3)$$
$$\sigma_5 = \sigma_4 + 68\sin(\theta_2 + \theta_3)$$
$$\sigma_6 = \sigma_5 + 93\sin(\theta_2) + 20$$

Moving on to the calculation of the potential energy of the robot the following equation will be used:

$$P = \sum_{i=1}^{4} m_i g^T \mathbf{r}_{\mathrm{CM}_i}(\mathbf{q}) \tag{10.6}$$

where:

- m_i: mass of the i-th link,
- $\mathbf{g} = [0, 0, 9.81]^T$: gravitational acceleration vector,
- $\mathbf{r}_{\mathrm{CM}_i}$: vector from the base frame to the Center of Mass of the i-th link

The result is visible in the Jupyter Notebook present in the 12.

The gravitational terms, expressed by $g(q)$, are determined from the following equation:

$$g(q) = \begin{bmatrix} g_1(q) \\ g_2(q) \\ g_3(q) \\ g_4(q) \end{bmatrix} \tag{10.7}$$

where each g_k is defined as:

$$g_k = \frac{\partial P}{\partial q_k} \tag{10.8}$$

Hence, from Equations 10.6, 10.8, and 10.7 g is computed as follows:

$$g(q) = \begin{bmatrix} 0 \\ 5886.0 \cdot \sin(\theta_2 + \theta_3) - 1.589 \times 10^5 \cdot \cos(\theta_2) - 7.613 \times 10^4 \cdot \cos(\theta_2 + \theta_3) \\ 5886.0 \cdot \sin(\theta_2 + \theta_3) - 7.613 \times 10^4 \cdot \cos(\theta_2 + \theta_3) \\ 0 \end{bmatrix}$$

(10.9)

Continuing with the dynamic analysis of the robot the following terms of inertia matrix $D(q)$ and Christoffel symbols $[C_{q,\dot{q}}]_{kj}$ are computed:

$$D(q) = \sum_{i=1}^{4} m_i J_{v,\mathrm{CM}_i}^T v_{\mathrm{CM}_i} + J_{\omega_i}^T \left(R_i I_i R_i^T \omega_i + J_{\omega_i} \right), \tag{10.10}$$

where:

- m_i: mass of the i-th link,
- J_{v,CM_i}: first three rows (linear velocities) of the Jacobian for the center of mass of the i-th link,
- J_{ω_i}: last three rows (rotational velocities) of the Jacobian for the i-th link,
- R_i: rotation matrix associated with the i-th link,
- I_i: inertia tensor associated with the i-th link.

$$[C_{q,\dot{q}}]_{kj} = \sum_{i=1}^{4} c_{ijk}(q)\dot{q} = \sum_{i=1}^{4} \frac{1}{2}\left(\frac{\partial d_{kj}}{\partial \dot{q}_i} + \frac{\partial d_{ki}}{\partial \dot{q}_j} - \frac{\partial d_{ij}}{\partial \dot{q}_k}\right)\dot{q}_i \tag{10.11}$$

Considering the substantial size of the symbolic matrices $D(q)$ and $[C_{q,\dot{q}}]$, it is deemed impractical to present them analytically in this report. However, a computed output from Python is present in the Jupyter Notebook present in 12.

The dynamic equation of motion (matrix form of Euler-Lagrange Equations) is given by:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau(q) \tag{10.12}$$

Through 10.12 and the numerical values of the trajectory planning from problem 6, the required joint torques of each joint are computed and depicted in the figure below.

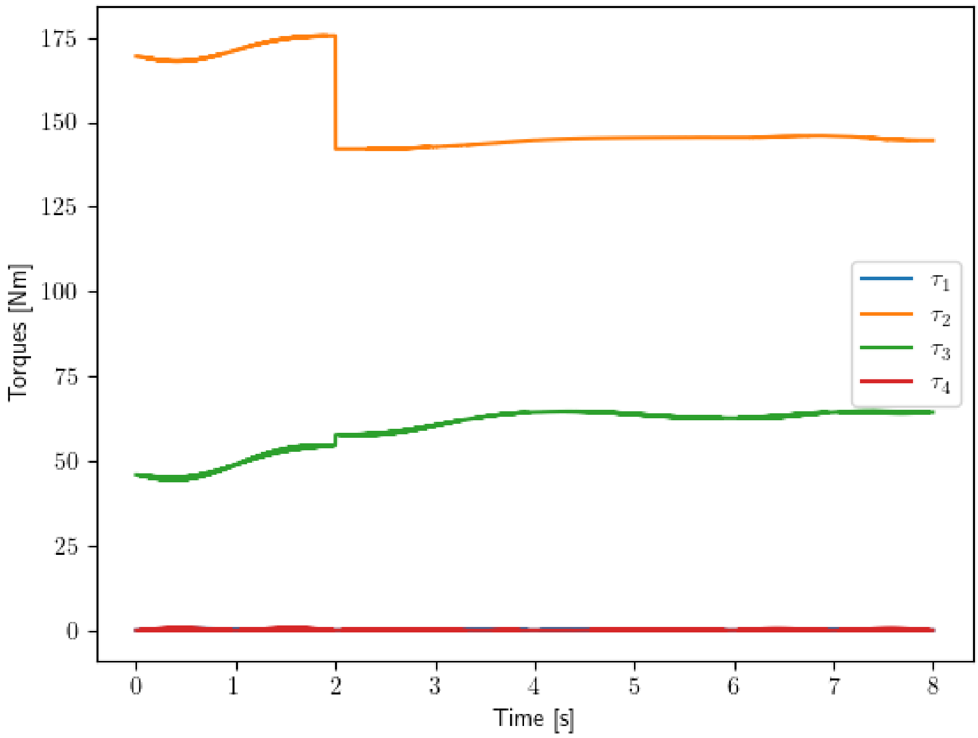


Figure 17: Joint torques τ corresponding to the trajectory path of Problem 6

11 Problem 11b

Create a software that enables the robot arm to recognize all red smarties among a cluster of colorful smarties on a surface in front of it, and to probe all of them with its stylus.

The group decided to use Matlab as the language for solving this task. It is composed of two files:

- **MyRobot.m**: contains the robot initialization and all the functions needed to move the robot. It is heavily based on this GitHub repository;
- **Vision.m**: uses computer vision techniques to recognize red dots.

First of all, the robot is initialized along its webcam. Once the robot is in position, a snapshot is taken. The following imaging processing techniques are used on it: removal of the red color extraction; noise filtering; binary conversion; removal of non-significant objects; and labeling of the remaining components.

Once the red objects are recognized and saved into a variable, the pixel distance is converted from the center of the camera to meters for each detected object. In this case, the pixel-to-meter ratio that has been used is calculated by taking a direct measure followed by checking the pixel dimensions from the snapshot. Ideally, it should have been done using a stereo vision system with two cameras or photos, and using the camera's intrinsic and extrinsic parameters as input. The distance triangulation should have been done mathematically as follows:

$$D = \frac{b * \lambda}{d} \tag{11.1}$$

With D as distance, b as baseline, which is the distance between the camera frames, λ as the focal length, and d as disparity, which is the distance between the features detected when feature extraction is done on the two images.

Once the camera frame has been converted to the end effector frame, further transformation is needed to get the distance w.r.t. frame 0. It is done by calculating the pose of the feature w.r.t. frame 0 using the transformation matrix 1.3 and the following formula for the pose:

$$p^0 = R_5^0 p^5 + o_5^0 \tag{11.2}$$

Once the pose in frame 0 is found, it is possible to make the robot move to the detected object by using it as input for the inverse kinematics formulas explained in problems 2 and 3. In case there is more than one object, a trajectory planning algorithm should be implemented by interpolating a polynomial such as the ones found in problem 6 with the positions of every red smartie found by the camera.

Unfortunately, it has not been possible for our team to make the robot move effectively to the position we desired with the inverse kinematics as shown in the demonstration video shown in the 12. Due to time constraints, it is not able to apply changes in time for the deadline. However, we believe that the issue relies on how we implemented distance triangulation since it was an approximation of the optimal method.

The code and the demonstration video can be found in the 12.

12 Appendix

GitHub repository containing the code used for the robot, the demonstration video, and the Python code used to solve the problems: GitHub.

Problems 1 to 10 were solved mostly in Python, while Problem 11b was solved in Matlab 2023b.