Norwegian University of Science and Technology
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TTT4120 Digital Signal Processing
Problem Set 10

The main topics for this problem set are Wiener filtering, filter structures and quantization effects. Relevant chapters from the textbook are 12.7.1, and 9. The maximum score for each problem is given in parentheses.

Problem 1 (3 points)

Given a noise corrupted signal $x(n) = s(n) + w(n)$, we wish to design a Wiener filter of length $M = 3$ to estimate the desired signal $s(n)$. $s(n)$ is an AR(1)-process given by $s(n) = 0.9s(n-1) + v(n)$, where $v(n)$ is a white noise sequence with variance $\sigma_v^2 = 0.09$.

$w(n)$ is a white noise process uncorrelated with $s(n)$, with variance $\sigma_w^2 = 1$.

Find the coefficients of the Wiener Filter. You can use MATLAB to solve the normal equations.

Problem 2 (3 points)

Given a stable, causal filter $H(z)$ on the form

$$H(z) = H_1(z)H_2(z) \tag{1}$$

where $H_1(z)$ is given as:

$$H_1(z) = \frac{z^{-1} - \frac{1}{2}}{1 - \frac{1}{2}z^{-1}} \tag{2}$$

and $H_2(z)$ is given as

$$H_2(z) = \frac{1}{1 + \frac{1}{2}z^{-1}} \tag{3}$$

(a) Show that $H(z)$ can be transformed to the following parallel form

$$H(z) = H_3(z) + H_4(z) = \frac{\frac{3}{4}}{1 - \frac{1}{2}z^{-1}} + \frac{-\frac{5}{4}}{1 + \frac{1}{2}z^{-1}} \tag{4}$$

(b) Sketch the following structures for $H(z)$:

- Direct form 2 (DF2)
- Parallel
- Caskade (based on equation 1)

Problem 3 (4 points)

Given a causal filter $H(z)$ with transfer function:

$$H(z) = \frac{1}{3} \cdot \frac{1 - 3z^{-1}}{1 - \frac{1}{3}z^{-1}} \tag{5}$$

(a) Sketch a direct form 2 (DF2) structure for the discrete filter with the gain factor 1/3 placed at the filter input.

(b) Show that the unit pulse response of the filter $H(z)$ is given by:

$$h(n) = \begin{cases} \frac{1}{3}\left(\frac{1}{3}\right)^n - \left(\frac{1}{3}\right)^{n-1} = -\frac{8}{9}\left(\frac{1}{3}\right)^{n-1} & n > 0 \\ \frac{1}{3} & n = 0 \\ 0 & n < 0 \end{cases} \tag{6}$$

The direct form 2 (DF2) structure in 3a is to be implemented using fixed point arithmetics with $B+1$ bits and dynamic range $[-1, 1)$. Rounding (quantisation) is performed after each multiplication.

(c) Derive an expression for the corresponding noise power σ_e^2 in terms of B (the number of bits excluding sign).

(d) Find the resulting noise power at the filter output in terms of σ_e^2.

(e) The input signal $x(n)$ has a uniformely distributed amplitude with full dynamic range; i.e. $x_{max} = \max_n |x(n)| = 1$.

- Show that, in order to avoid overflow, a scaling factor of 3/5 (i.e. a total gain factor of 1/5) has to be used at the filter input.

- How does the scaling influence on the signal-noise ratio (S/N) at the filter output?

- Find the signal-noise ratio at the filter output when $B + 1 = 8$ bit.