Norwegian University of Science and Technology
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TTT4120 Digital Signal Processing
Problem Set 6

This problem set is concerned with practical frequency analysis of discrete-time signals using Discrete Fourier Transform (DFT). You will examine the relationship between DFT and DTFT, compute convolution in the frequency domain using DFT, and estimate the spectrum of an infinite signal using a finite segment of the signal.

In order to successfully accomplish this exercise you need to study chapters 6.1, 7.1, 7.1.2, 7.2.1, 7.3.1, 7.4, 8.1.3 from the textbook.

Problem 1: Relationship between DTFT and DFT [3 points]

Consider the following sequence of length N_x,

$$x(n) = \begin{cases} 0.9^n & n = 0, \ldots, N_x - 1, \\ 0 & \text{otherwise.} \end{cases}$$

where $N_x = 28$. We would like to analyze the sequence in the frequency domain.

(a) Compute the spectrum $X(f)$ of $x(n)$ using the DTFT and plot its magnitude for $f \in [0, 1)$.

(b) Use the function `fft` to compute $X(k) = DFT\{x(n)\}$ with the DFT length equal to $N_x/4$, $N_x/2$, N_x and $2N_x$.

(c) What is the relationship between the DFT index k and the normalized frequency f? Find f that corresponds to the $k = 1$ for the four DFTs computed in (b).

(d) Plot the magnitude of each DFT (use `stem`) together with the magnitude of the DTFT (use `plot`) as a function of f. What is the relationship between DFT and DTFT for the different DFT lengths? Explain the results.

(e) Why is it sufficient to compute the values of the DTFT and DFT corresponding only to the frequency range $f \in [0, 0.5]$ for any real signal?

Problem 2: Linear Convolution [2.5 points]

The sequence $x(n)$ from Problem 1 is filtered through a FIR filter with unit sample response given by

$$h(n) = \begin{cases} 1 & n = 0, \ldots, N_h - 1, \\ 0 & \text{otherwise}, \end{cases}$$

where $N_h = 9$.

(a) Use Matlab to compute and plot the output signal $y(n)$ in the time domain. Useful Matlab functions: `ones, conv, stem`.

What is the length of $y(n)$, N_y? Does it agree with theory? Explain.

(b) Use Matlab to compute and plot the output signal $y(n)$ via the frequency domain using DFT/IDFT. Useful Matlab functions: `fft, ifft`.

How should the DFT/IDFT lengths be chosen in order to obtain exact values of $y(n)$ using the above algorithm? Explain why.

Run your Matlab program with DFT/IDFT lengths set to $N_y/4$, $N_y/2$, N_y and $2N_y$. Plot each result together with the output signal obtained in (a) (use different colors) and compare. Explain your observations.

Problem 3: Spectral Estimation [2.5 points]

In this problem we will use DFT to estimate the spectrum of a discrete-time signal based on finite signal segments. The following signal is considered

$$x(n) = \sin(2\pi f_1 n) + \sin(2\pi f_2 n),$$

where $f_1 = 7/40$ and $f_2 = 9/40$.

(a) Sketch the magnitude spectrum of the sampled signal, $|X(f)|$ for $f \in [0, 0.5]$.

(b) Use Matlab to generate a segment of length 100 of the signal $x(n)$.

Use DFT of length 1024 to estimate the spectrum $X(f)$ based on this signal segment.

Plot the estimated magnitude spectrum for $f \in [0, 0.5]$.

Repeat the above with segment lengths equal to 1000, 30 and 10.

Compare with the sketch in (a) and explain the similarities and differences.

(c) Repeat (b) with segment of length 100 of the signal $x(n)$, using DFT lengths equal to 256 and 128.

What effect does the DFT length have on spectral estimation.

Problem 4: Fast Fourier Transform (FFT) [2 points]

(a) What is the Fast Fourier Transform?

(b) Explain briefly the idea of radix-2 FFT (decimation-in-time).

(c) Given the signal $x(n)$ of length N.

Find the number of operations (multiplications) needed for the direct computation of the N-point DFT of $x(n)$.
(Assume that the values of W_N^k are stored in a table in advance.)

(d) Given the signals $f_1(n) = x(2n)$ and $f_2(n) = x(2n+1)$ of length $M = N/2$.

Show that

$$X(k) = F_1(k) + W_N^k F_2(k) \quad k = 0, 1, \cdots, M-1 \tag{1}$$
$$X(k+M) = F_1(k) - W_N^k F_2(k) \quad k = 0, 1, \cdots, M-1 \tag{2}$$

where $X(k)$, $F_1(k)$ and $F_2(k)$ are DFTs of $x(n)$, $f_1(n)$, and $f_2(n)$, respectively.

(e) Find the number of operations needed to compute $X(k)$ according to (1) and (2). Assume that $F_1(k)$ and $F_2(k)$ are computed using direct DFT computation.

Compare to the result in (c) and comment.