Department of Engineering Cybernetics

TTK4215 System Identification and Adaptive Control
Solution 10

Problem 4.13 from I&S
Let $\Lambda(s) = (s+1)^2$ and filter the equation

$$y = \rho^* \left(u - m\ddot{y} - \beta\dot{y}\right), \tag{1}$$

by $1/\Lambda(s)$. Then, we have

$$\frac{1}{\Lambda(s)} y = \rho^* \left(\frac{1}{\Lambda(s)} u - m \frac{s^2}{\Lambda(s)} y - \beta \frac{s}{\Lambda(s)} y \right). \tag{2}$$

Defining

$$z = \frac{1}{\Lambda(s)} y, \tag{3}$$
$$\theta^* = \begin{bmatrix} m & \beta \end{bmatrix}^T, \tag{4}$$
$$\phi = \begin{bmatrix} -\frac{s^2}{\Lambda(s)} y & -\frac{s}{\Lambda(s)} y \end{bmatrix}^T, \tag{5}$$
$$z_1 = \frac{1}{\Lambda(s)} u \tag{6}$$

we obtain the desired bilinear parametric form

$$z = \rho^* \left(\theta^{*^T} \phi + z_1 \right). \tag{7}$$

Since $\rho^* = 1/k$ and $k > 0$, we know the sign of ρ^*. We may apply the gradient method with instantaneous cost, for instance, which gives

$$\xi = \theta^T \phi + z_1 \tag{8}$$
$$\epsilon = \frac{z - \rho\xi}{m^2}, \tag{9}$$
$$\dot{\theta} = \Gamma \epsilon \phi, \tag{10}$$
$$\dot{\rho} = \gamma \xi \epsilon, \tag{11}$$

where we have used the fact that $sign(\rho^*) = 1$ in (10). The normalizing signal $m^2 = 1 + n_s^2$ must be designed to bound ϕ and z_1 from above, so one possible option would be

$$n_s^2 = \phi^T \phi + z_1^2. \tag{12}$$

Simulations.