TET4100 Kretsanalyse – Løsning

Institutt: Elkraftteknikk Dato: 2012.08.15

Øving 8

32) LaPlace-Analyse 8.A

32.1) $$I_{inn}(s)=\frac{I_{inn}}{s}$$

$$I_C(s)=\frac{sL}{sL+R+\frac{1}{sC}}\cdot I_{inn}(s)=I_{inn}\frac{s}{s^2+s\frac{R}{L}+\frac{1}{LC}}$$

$$V_C(s)=\frac{1}{sC}\cdot I_C(s)=\frac{I_{inn}}{sC}\frac{s}{s^2+s\frac{R}{L}+\frac{1}{LC}}=I_{inn}R\cdot\frac{\frac{1}{RC}}{s^2+s\frac{R}{L}+\frac{1}{LC}}$$

32.2) $\textit{Spenningssprang}$ over $\textit{kondensator ikke mulig}\rightarrow v_C(0)=0$

$\textit{også mulig}: v_C(0)=\lim_{s\rightarrow\infty}\left(s\cdot V_C(s)\right)$

$\textit{Spole er kortslutning}\rightarrow v_C(\infty)=0$

$\textit{også mulig}: v_C(\infty)=\lim_{s\rightarrow 0}\left(s\cdot V_C(s)\right)$

32.3) $$V_C(s)=I_{inn}R\cdot\frac{\frac{1}{RC}}{s^2+s\frac{R}{L}+\frac{1}{LC}}$$

$$s_{1,2}=-\frac{R}{2L}\pm\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}} \qquad R>2\sqrt{\frac{L}{C}} \quad\rightarrow\quad \textit{overdempet}$$

$$V_C(s)=I_{inn}R\cdot\frac{\frac{1}{RC}}{(s-s_1)(s-s_2)}$$

Delbrøksoppspaltning: $$V_C(s)=\frac{V_{C,1}}{s-s_1}+\frac{V_{C,2}}{s-s_2}$$

$$V_{C,1} = I_{inn} R \cdot \frac{\frac{1}{RC}}{s_1 - s_2} \qquad V_{C,2} = I_{inn} R \cdot \frac{\frac{1}{RC}}{s_2 - s_1}$$

$$v_C(t) = \left(V_{C,1} \cdot e^{s_1 t} + V_{C,2} \cdot e^{s_2 t}\right) \cdot u(t)$$

32.4)

$$v_C(t)=\left(V_{C,1}\cdot e^{s_1 t}+V_{C,2}\cdot e^{s_2 t}\right)\cdot u(t)$$

$$V_{C,2}=-V_{C,1}$$

$$v_C(t)=V_{C,1}\left(e^{s_1 t}-e^{s_2 t}\right)\cdot u(t)$$

$$\frac{d\,v_C(t)}{d\,t}=V_{C,1}\left(s_1\cdot e^{s_1 t}-s_2\cdot e^{s_2 t}\right) \qquad t>0$$

$$V_{C,1}\left(s_1\cdot e^{s_1 t_{max}}-s_2\cdot e^{s_2 t_{max}}\right)=0 \qquad t_{max}>0$$

$$s_1\cdot e^{s_1 t_{max}}=s_2\cdot e^{s_2 t_{max}}$$

$$1=\frac{s_2}{s_1}e^{(s_2-s_1)t_{max}}$$

$$\ln\left(\frac{s_1}{s_2}\right)=(s_2-s_1)t_{max}$$

$$t_{max}=\frac{\ln\left(\frac{s_1}{s_2}\right)}{s_2-s_1}$$

$$v_C(t)=V_{C,1}\left(e^{s_1 t}-e^{s_2 t}\right)\cdot u(t)$$

$$v_C(t_{max})=V_{C,1}\left(e^{s_1 t_{max}}-e^{s_2 t_{max}}\right)$$

$$v_C(t_{max})=V_{C,1}\left(e^{s_1\frac{\ln\left(\frac{s_1}{s_2}\right)}{s_2-s_1}}-e^{s_2\frac{\ln\left(\frac{s_1}{s_2}\right)}{s_2-s_1}}\right)$$

$$v_C(t_{max})=V_{C,1}\left(\left(\frac{s_1}{s_2}\right)^{\left(\frac{s_1}{s_2-s_1}\right)}-\left(\frac{s_1}{s_2}\right)^{\left(\frac{s_2}{s_2-s_1}\right)}\right)$$

33) LaPlace-Analyse 8.B

33.1) $V_C(s)=\frac{1}{sC}I_C(s)$ $I_L(s)=\frac{1}{sL}V_C(s)$

$$\frac{1}{sC}I_C(s)=sLI_L(s)$$

$$I_C(s)=s^2LCI_L(s)$$

$$I_R(s)=I_C(s)+I_L(s)$$

$$I_R(s)=\left(1+s^2LC\right)I_L(s)=\frac{1+s^2LC}{sL}V_C(s)$$

$$I_R(s)=\frac{V_s(s)-V_C(s)}{R}$$

$$\frac{V_s(s)-V_C(s)}{R}=\frac{1+s^2LC}{sL}V_C(s)$$

$$\frac{V_s(s)}{R}=\left(\frac{1+s^2LC}{sL}+\frac{1}{R}\right)V_C(s)$$

$$\frac{V_C(s)}{V_s(s)}=\frac{\frac{1}{R}}{\frac{1+s^2LC}{sL}+\frac{1}{R}}$$

$$\frac{V_C(s)}{V_s(s)}=\frac{s\frac{1}{RC}}{s^2+s\frac{1}{RC}+\frac{1}{LC}}$$

33.2)

$$I_R(s)=\frac{V_s(s)-V_C(s)}{R} \qquad \frac{V_C(s)}{V_s(s)}=\frac{s\frac{1}{RC}}{s^2+s\frac{1}{RC}+\frac{1}{LC}}$$

$$I_R(s)=\frac{V_s(s)-\frac{s\frac{1}{RC}}{s^2+s\frac{1}{RC}+\frac{1}{LC}}\cdot V_s(s)}{R}$$

$$\frac{I_R(s)}{V_s(s)}=\frac{1}{R}\left(1-\frac{s\frac{1}{RC}}{s^2+s\frac{1}{RC}+\frac{1}{LC}}\right)$$

$$\frac{I_R(s)}{V_s(s)}=\frac{1}{R}\cdot\frac{s^2+s\frac{1}{RC}+\frac{1}{LC}-s\frac{1}{RC}}{s^2+s\frac{1}{RC}+\frac{1}{LC}}$$

$$\frac{I_R(s)}{V_s(s)}=\frac{1}{R}\cdot\frac{s^2+\frac{1}{LC}}{s^2+s\frac{1}{RC}+\frac{1}{LC}}$$

33.3) $$\frac{V_C(s)}{V_s(s)}=\frac{s\frac{1}{RC}}{s^2+s\frac{1}{RC}+\frac{1}{LC}} \qquad V_s(s)=\frac{V_s}{s}$$

$$V_C(s)=V_s\cdot\frac{\frac{1}{RC}}{s^2+s\frac{1}{RC}+\frac{1}{LC}}=2V_s\cdot\frac{\frac{1}{2RC}}{\left(s+\frac{1}{2RC}\right)^2+\left(\sqrt{\frac{1}{LC}-\frac{1}{4R^2C^2}}\right)^2}$$

$$\alpha=\frac{1}{2RC} \qquad \omega_d=\sqrt{\frac{1}{LC}-\frac{1}{4R^2C^2}}$$

$$V_C(s)=2V_s\cdot\frac{\alpha}{(s+\alpha)^2+\omega_d{}^2}=\frac{2V_s\alpha}{\omega_d}\cdot\frac{\omega_d}{(s+\alpha)^2+\omega_d{}^2}$$

$$V_C=\frac{2V_s\alpha}{\omega_d}$$

$$V_C(s)=V_C\cdot\frac{\omega_d}{(s+\alpha)^2+\omega_d{}^2}$$

$$v_C(t)=V_C\cdot e^{-\alpha t}\cdot\sin(\omega_d t)\cdot u(t)$$

33.4) $$\frac{I_R(s)}{V_s(s)}=\frac{1}{R}\cdot\frac{s^2+\frac{1}{LC}}{s^2+s\frac{1}{RC}+\frac{1}{LC}} \qquad V_s(s)=\frac{V_s}{s}$$

$$I_R(s)=\frac{V_s}{R}\cdot\frac{s^2+\frac{1}{LC}}{s\left(s^2+s\frac{1}{RC}+\frac{1}{LC}\right)}=\frac{V_s}{R}\cdot\frac{s^2+\frac{1}{LC}}{s\left((s+\alpha)^2+{\omega_d}^2\right)}$$

$$\frac{1}{LC}=\alpha^2+\omega_d^2$$

$$I_R(s)=\frac{V_s}{R}\cdot\frac{s^2+\alpha^2+\omega_d^2}{s\left(s-(-\alpha+j\omega_d)\right)\left(s-(-\alpha-j\omega_d)\right)}$$

Delbrøksoppspaltning: $I_R(s)=\frac{I_{R,stat}}{s}+\frac{0{,}5\,\boldsymbol{I}_{\boldsymbol{R,dyn}}}{s-(-\alpha+j\omega_d)}+\frac{0{,}5\,\boldsymbol{I}^*_{\boldsymbol{R,dyn}}}{s-(-\alpha-j\omega_d)}$

har dermed:
$$\frac{V_s}{R}\cdot\left(s^2+\alpha^2+\omega_d^2\right)=I_{R,stat}(s+\alpha-j\omega_d)(s+\alpha+j\omega_d)$$
$$+0{,}5\,\boldsymbol{I}_{\boldsymbol{R,dyn}}\,s(s+\alpha+j\omega_d)$$
$$+0{,}5\,\boldsymbol{I}^*_{\boldsymbol{R,dyn}}\,s(s+\alpha-j\omega_d)$$

Setter inn $s=0$:

$$\frac{V_s}{R}\cdot\left(\alpha^2+\omega_d^2\right)=I_{R,stat}(\alpha-j\omega_d)(\alpha+j\omega_d)$$

$$I_{R,stat}=\frac{V_s}{R}\cdot\frac{\alpha^2+\omega_d^2}{(\alpha-j\omega_d)(\alpha+j\omega_d)}=\frac{V_s}{R}$$

Setter inn $s=-\alpha+j\omega_d$:

$$\frac{V_s}{R}\cdot\left((-\alpha+j\omega_d)^2+\alpha^2+\omega_d^2\right)=0{,}5\,\boldsymbol{I}_{\boldsymbol{R,dyn}}(-\alpha+j\omega_d)\,j2\omega_d$$

$$0{,}5\,\boldsymbol{I}_{\boldsymbol{R,dyn}}=\frac{V_s}{R}\cdot\frac{(-\alpha+j\omega_d)^2+\alpha^2+\omega_d^2}{(-\alpha+j\omega_d)\,j2\omega_d}=j\frac{\alpha}{\omega_d}\frac{V_s}{R}$$

$$\boldsymbol{I}_{\boldsymbol{R,dyn}}=-\boldsymbol{I}^{*}_{\boldsymbol{R,dyn}}=j\,I_{R,dyn}$$

$$I_R(s)=\frac{I_{R,stat}}{s}+\frac{j0{,}5\,I_{R,dyn}}{s-(-\alpha+j\omega_d)}-\frac{j0{,}5\,I_{R,dyn}}{s-(-\alpha-j\omega_d)}$$

$$i_R(t)=(I_{R,stat}+j0{,}5\,I_{R,dyn}e^{(-\alpha+j\omega_d)t}-j0{,}5\,I_{R,dyn}e^{(-\alpha-j\omega_d)t})u(t)$$

$$i_R(t)=(I_{R,stat}+j0{,}5\,I_{R,dyn}e^{-\alpha t}\cdot e^{j\omega_d t}-j0{,}5\,I_{R,dyn}e^{-\alpha t}\cdot e^{-j\omega_d t})u(t)$$

$$i_R(t)=\left(I_{R,stat}+j0{,}5\,I_{R,dyn}e^{-\alpha t}\left(e^{j\omega_d t}-e^{-j\omega_d t}\right)\right)u(t)$$

$$i_R(t)=\left(I_{R,stat}+j0{,}5\,I_{R,dyn}e^{-\alpha t}\left(\cos(\omega_d t)+j\sin(\omega_d t)-\cos(\omega_d t)+j\sin(\omega_d t)\right)\right)u(t)$$

$$i_R(t)=\left(I_{R,stat}-I_{R,dyn}e^{-\alpha t}\sin(\omega_d t)\right)u(t)$$

Alternativ Løsning:

$$i_R(t)=\frac{v_s(t)-v_C(t)}{R}\qquad v_s(t)=V_s\cdot u(t)\qquad v_C(t)=V_C\cdot e^{-\alpha t}\cdot\sin(\omega_d t)\cdot u(t)$$

$$i_R(t)=\frac{V_s\cdot u(t)-V_C\cdot e^{-\alpha t}\cdot\sin(\omega_d t)\cdot u(t)}{R}$$

$$I_{R,stat}=\frac{V_s}{R}\qquad I_{R,dyn}=\frac{V_C}{R}$$

$$i_R(t)=\left(I_{R,stat}-I_{R,dyn}\cdot e^{-\alpha t}\sin(\omega_d t)\right)u(t)$$

34) LaPlace-Analyse 8.C

34.1) Startbetingelse: $v_C(0) = -I_S \cdot (R \| G \| R_s) = -\dfrac{I_s}{\dfrac{1}{R} + \dfrac{1}{R_s} + G}$ $\quad i_L(0) = -v_C(0) \cdot G$

Bryteren i posisjon b = Kortslutning på R ---> $V_L = V_G = V_C$

$$I_C(s) = s C V_C(s) - C \cdot v_C(0)$$

$$I_L(s) = \frac{1}{sL} V_C(s) + \frac{1}{s} i_L(0) = \frac{1}{sL} V_C(s) - \frac{G}{s} v_C(0)$$

$$I_G(s) = G V_C(s)$$

$$I_C(s) + I_L(s) + I_G(s) = 0$$

$$s C V_C(s) + \frac{1}{sL} V_C(s) + G V_C(s) - \frac{G}{s} v_C(0) - C \cdot v_C(0) = 0$$

$$V_C(s) = v_C(0) \frac{s + \dfrac{G}{C}}{s^2 + s\dfrac{G}{C} + \dfrac{1}{LC}}$$

34.2) $I_L(s) = \dfrac{1}{sL} V_C(s) - \dfrac{G}{s} v_C(0) \qquad V_C(s) = v_C(0) \dfrac{s + \dfrac{G}{C}}{s^2 + s\dfrac{G}{C} + \dfrac{1}{LC}}$

$$I_L(s) = \frac{1}{sL} v_C(0) \frac{s + \dfrac{G}{C}}{s^2 + s\dfrac{G}{C} + \dfrac{1}{LC}} - \frac{G}{s} v_C(0)$$

$$I_L(s) = -G v_C(0) \frac{s + \dfrac{G}{C} - \dfrac{1}{LG}}{s^2 + s\dfrac{G}{C} + \dfrac{1}{LC}}$$

34.3) $$V_C(s)=v_C(0)\frac{s+\frac{G}{C}}{s^2+s\frac{G}{C}+\frac{1}{LC}}$$

$$G<2\sqrt{\frac{C}{L}} \quad \rightarrow \quad underdempet$$

$$V_C(s)=v_C(0)\frac{s+\left(\frac{G}{2C}+\frac{G}{2C}\right)}{\left(s+\frac{G}{2C}\right)^2+\left(\sqrt{\frac{1}{LC}-\frac{G^2}{4C^2}}\right)^2}$$

$$\alpha=\frac{G}{2C} \qquad \omega_d=\sqrt{\frac{1}{LC}-\frac{G^2}{4C^2}} \qquad K\cdot\omega_d=\frac{G}{2C}$$

$$K=\frac{\alpha}{\omega_d}$$

$$V_C(s)=v_C(0)\frac{(s+\alpha)+K\cdot\omega_d}{(s+\alpha)^2+\omega_d{}^2}$$

$$V_C(s)=v_C(0)\cdot\left(\frac{s+\alpha}{(s+\alpha)^2+\omega_d{}^2}+K\frac{\omega_d}{(s+\alpha)^2+\omega_d{}^2}\right)$$

$$v_C(t)=v_C(0)\cdot\left(\cos(\omega_d t)+K\cdot\sin(\omega_d t)\right)e^{-\alpha t}u(t)$$

34.4) $$I_L(s) = -G v_C(0) \frac{s + \frac{G}{C} - \frac{1}{LG}}{s^2 + s\frac{G}{C} + \frac{1}{LC}}$$

$$I_L(s) = -G v_C(0) \frac{s + \left(\frac{G}{2\mathrm{C}} + \left(\frac{G}{2\mathrm{C}} - \frac{1}{LG}\right)\right)}{\left(s + \frac{G}{2C}\right)^2 + \left(\sqrt{\frac{1}{LC} - \frac{G^2}{4C^2}}\right)^2}$$

$$\alpha = \frac{G}{2C} \qquad \omega_d = \sqrt{\frac{1}{LC} - \frac{G^2}{4C^2}} \qquad K \cdot \omega_d = \frac{G}{2\mathrm{C}} - \frac{1}{LG}$$

$K = \frac{\alpha^2 - \omega_d^2}{2\alpha\omega_d}$ litt vanskelig å finne algebraisk

$$I_L(s) = -G v_C(0) \frac{(s+\alpha) + K \cdot \omega_d}{(s+\alpha)^2 + \omega_d^2}$$

$$I_L(s) = -G v_C(0) \cdot \left(\frac{s+\alpha}{(s+\alpha)^2 + \omega_d^2} + K \frac{\omega_d}{(s+\alpha)^2 + \omega_d^2}\right)$$

$$i_L(t) = -G v_C(0) \cdot \left(\cos(\omega_d t) + K \cdot \sin(\omega_d t)\right) e^{-\alpha t} u(t)$$

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35) LaPlace-Analyse 8.D

35.1) $M=k\cdot\sqrt{L_1\cdot L_2}$ $\quad L_1=L_2$ $\quad k=\frac{M}{L}$

35.2) Høyre sløyfe: $-R\cdot I_2(s)-sL\cdot I_2(s)-sM\cdot I_1(s)=0$

$$I_1(s)=-\frac{R+sL}{sM}I_2(s)$$

Venstre sløyfe: $V_g(s)-R\cdot I_1(s)-sL\cdot I_1(s)-sM\cdot I_2(s)=0$

$$I_2(s)=\frac{V_g(s)}{sM}-\frac{R+sL}{sM}\cdot I_1(s)$$

$$I_2(s)=\frac{V_g(s)}{sM}+\left(\frac{R+sL}{sM}\right)^2 I_2(s)$$

$$I_2(s)=\frac{V_g(s)}{sM}\cdot\frac{(sM)^2}{(sM)^2-(R+sL)^2}$$

$$I_2(s)=\frac{V_0(s)}{R}$$

$$\frac{V_0(s)}{R}=\frac{V_g(s)}{sM}\cdot\frac{(sM)^2}{(sM)^2-(R+sL)^2}$$

$$\frac{V_0(s)}{V_g(s)}=\frac{sRM}{(sM)^2-(R+sL)^2}$$

$$\frac{V_0(s)}{V_g(s)}=\frac{sRM}{s^2(M^2-L^2)-s2RL-R^2}$$

$$\frac{V_0(s)}{V_g(s)}=-\frac{sRM}{s^2(L^2-M^2)+s2RL+R^2}$$

$$\frac{V_0(s)}{V_g(s)}=-\frac{s\frac{RM}{L^2-M^2}}{s^2+s\frac{2RL}{L^2-M^2}+\frac{R^2}{L^2-M^2}}$$

35.3) $v_g(t)=V_g\cdot\cos(\omega_f t)\cdot u(t)$

$$V_g(s)=V_g\cdot\frac{s}{s^2+\omega_f^2}$$

$$V_0(s)=-\frac{s\frac{RM}{L^2-M^2}}{s^2+s\frac{2RL}{L^2-M^2}+\frac{R^2}{L^2-M^2}}\cdot V_g\cdot\frac{s}{s^2+\omega_f^2}$$

$$V_0(s)=-V_g\cdot\frac{s^2\frac{RM}{L^2-M^2}}{\left(s^2+s\frac{2RL}{L^2-M^2}+\frac{R^2}{L^2-M^2}\right)\cdot\left(s^2+\omega_f^2\right)}$$

35.4) $0=s^2+s\frac{2RL}{L^2-M^2}+\frac{R^2}{L^2-M^2}$

$$s_{1,2}=-\frac{RL}{L^2-M^2}\pm\sqrt{\left(\frac{RL}{L^2-M^2}\right)^2-\frac{R^2}{L^2-M^2}}$$

$$\left(\frac{RL}{L^2-M^2}\right)^2-\frac{R^2}{L^2-M^2}>0 \quad\rightarrow\quad overdempet$$

NB: Overdemping er logisk for det finnes ingen kapasitans i kretsen.

$$V_0(s)=-V_g\frac{s^2\frac{RM}{L^2-M^2}}{\left(s-s_1\right)\cdot\left(s-s_2\right)\cdot\left(s^2+\omega_f^2\right)}$$

$$\omega_x=\frac{RM}{L^2-M^2}$$

$$V_0(s)=-V_g\frac{s^2\omega_x}{\left(s-s_1\right)\cdot\left(s-s_2\right)\cdot\left(s+j\omega_f\right)\left(s-j\omega_f\right)}$$

$$V_0(s)=-V_g\frac{s^2\omega_x}{(s-s_1)\cdot(s-s_2)\cdot(s+j\omega_f)(s-j\omega_f)}$$

Delbrøksoppspaltning: $V_0(s)=\frac{V_{dyn,1}}{s-s_1}+\frac{V_{dyn,2}}{s-s_2}+\frac{0{,}5\boldsymbol{V}_{\boldsymbol{stat}}}{s+j\omega_f}+\frac{0{,}5\boldsymbol{V}^*_{\boldsymbol{stat}}}{s-j\omega_f}$

$$\begin{aligned}-V_g\omega_x s^2=&V_{dyn,1}(s-s_2)(s+j\omega_f)(s-j\omega_f)+V_{dyn,2}(s-s_1)(s+j\omega_f)(s-j\omega_f)\\&+0{,}5\boldsymbol{V}_{\boldsymbol{stat}}(s-s_1)(s-s_2)(s-j\omega_f)+0{,}5\boldsymbol{V}^*_{\boldsymbol{stat}}(s-s_1)(s-s_2)(s+j\omega_f)\end{aligned}$$

$$s=s_1$$

$$-V_g\omega_x s_1^2=V_{dyn,1}(s_1-s_2)(s_1+j\omega_f)(s_1-j\omega_f)$$

$$V_{dyn,1}=-V_g\frac{s_1^2\omega_x}{(s_1-s_2)\cdot(s_1^2+\omega_f^2)}$$

$$s=s_2$$

$$-V_g\omega_x s_2^2=V_{dyn,2}(s_2-s_1)(s_2+j\omega_f)(s_2-j\omega_f)$$

$$V_{dyn,2}=-V_g\frac{s_2^2\omega_x}{(s_2-s_1)\cdot(s_2^2+\omega_f^2)}$$

$$s=-j\omega_f$$

$$-V_g\omega_x(-j\omega_f)^2=0{,}5\boldsymbol{V}_{\boldsymbol{stat}}((-j\omega_f)-s_1)((-j\omega_f)-s_2)((-j\omega_f)-j\omega_f)$$

$$0{,}5\boldsymbol{V}_{\boldsymbol{stat}}=0{,}5V_g\frac{j\omega_f\omega_x}{s_1s_2-\omega_f^2+j\omega_f(s_1+s_2)}$$

Kartesisk Løsning:

$$V_0(s)=\frac{V_{dyn,1}}{s-s_1}+\frac{V_{dyn,2}}{s-s_2}+\frac{0{,}5\boldsymbol{V_{stat}}}{s+j\omega_f}+\frac{0{,}5\boldsymbol{V^*_{stat}}}{s-j\omega_f}$$

$$\boldsymbol{V_{stat}}=V_{stat,\Re}+jV_{stat,\Im}$$

$$V_0(s)=\frac{V_{dyn,1}}{s-s_1}+\frac{V_{dyn,2}}{s-s_2}+0{,}5\left(\frac{V_{stat,\Re}+jV_{stat,\Im}}{s+j\omega_f}+\frac{V_{stat,\Re}-jV_{stat,\Im}}{s-j\omega_f}\right)$$

$$V_0(s)=\frac{V_{dyn,1}}{s-s_1}+\frac{V_{dyn,2}}{s-s_2}+\frac{sV_{stat,\Re}+\omega_f V_{stat,\Im}}{s^2+\omega_f^2}$$

$$V_0(s)=\frac{V_{dyn,1}}{s-s_1}+\frac{V_{dyn,2}}{s-s_2}+V_{stat,\Re}\frac{s}{s^2+\omega_f^2}+V_{stat,\Im}\frac{\omega_f}{s^2+\omega_f^2}$$

$$v_0(t)=V_{dyn,1}e^{s_1 t}+V_{dyn,2}e^{s_2 t}+V_{stat,\Re}\cos\left(\omega_f t\right)+V_{stat,\Im}\sin\left(\omega_f t\right)$$

Polar Løsning:

$$V_0(s)=\frac{V_{dyn,1}}{s-s_1}+\frac{V_{dyn,2}}{s-s_2}+\frac{0{,}5\boldsymbol{V_{stat}}}{s+j\omega_f}+\frac{0{,}5\boldsymbol{V^*_{stat}}}{s-j\omega_f}$$

$$\boldsymbol{V_{stat}}=V_{stat}\measuredangle\phi_{stat}$$

$$V_0(s)=\frac{V_{dyn,1}}{s-s_1}+\frac{V_{dyn,2}}{s-s_2}+0{,}5\left(\frac{V_{stat}\measuredangle\phi_{stat}}{s+j\omega_f}+\frac{V_{stat}\measuredangle-\phi_{stat}}{s-j\omega_f}\right)$$

$$v_0(t)=V_{dyn,1}e^{s_1 t}+V_{dyn,2}e^{s_2 t}+V_{stat}\cos\left(\omega_f t-\phi_{stat}\right)$$