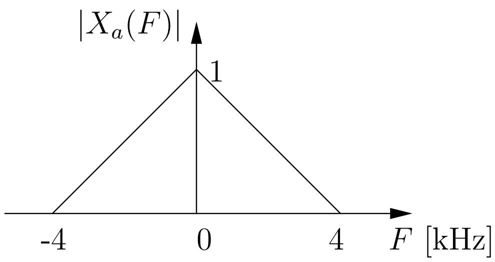
Norwegian University of Science and Technology
Department of Electronics and Telecommunications

TTT4120 Digital Signal Processing
Problem Set 11

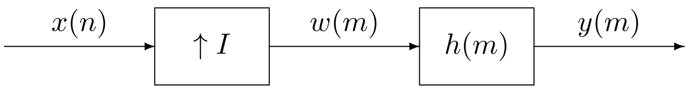
The main topic for this problem set is multirate signal processing. Relevant chapters from the textbook are 11.1-11.4. You will need **headsets** in order to do Problem 3. The maximum score for each problem is given in parentheses.

Problem 1 (3 points)

An analog signal $x_a(t)$ is given by the following magnitude spectrum.



A signal $x(n)$ has been generated by sampling $x_a(t)$ using the sampling frequency $F_{sx} = 8$kHz. We wish to increase the sampling frequency of the signal to $F_{sy} = 24$kHz using an intepolator as shown in the following block diagram.



(a) Explain the purpose of each block in the diagram.

(b) Show that

$$W\left(\frac{F}{F_{sy}}\right) = X\left(\frac{F}{F_{sx}}\right)$$

where $W(\cdot)$ and $X(\cdot)$ are spectra of the signals $w(m)$ and $x(n)$, respectively.

(c) Sketch the magnitude spectra $|X(F/F_{sx})|$, $|W(F/F_{sy})|$, $|Y(F/F_{sy})|$, and the magnitude response of the filter, $|H(F/F_{sy})|$. Comment the sketches.

Problem 2 (3 points)

Let $x(n)$ be the time-discrete signal with sampling frequency $F_{sx} = 8$ kHz from Problem 1. We wish to design a digital system that reduces the sampling frequency of the signal to $F_{sy} = 6$ kHz such that aliasing does not appear. Let $y(m)$ be the resulting output signal.

(a) Sketch the block diagram of the system and explain the function of each component.

(b) State the necessary specifications of the system components.

(c) Will the reduction in the sampling frequency cause any information loss, i.e. is it possible to reconstruct the original analog signal from the output signal $y(m)$? Justify your answer.

(d) Sketch the magnitude spectra of all the signals in the system (i.e. $|X(F/F_{sx})|$, $|Y(F/F_{sy})|$ and possible intermediate signals). State the value of the corresponding sampling frequency for each graph.

(e) *Optional:* The sampling rate of the signal $x(n)$ should now be increased to $F_{sy} = 12$kHz. Repeat (b)-(d) for this case.

Problem 3 (4 points)

Given the following discrete-time signals

- $x_1(n) = \cos(2\pi f_1 n)$
- $x_2(n) = \cos(2\pi f_2 n)$
- $x(n) = x_1(n) + x_2(n)$,

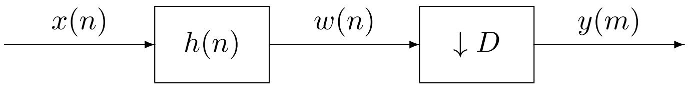
where f_1 and f_2 correspond to analog frequencies $F_1 = 900$Hz and $F_2 = 2000$Hz, and the sampling frequency is $F_s = 6000$Hz.

(a) Show that the spectrum of $x_1(n)$ is given by

$$X_1(f) = \frac{1}{2}(\delta(f - f_1) + \delta(f + f_1)).$$

(Hint: Show that the IDTFT of $X_1(f)$ is equal to $x_1(n)$.)

(b) The sampling frequency of the signal $x(n)$ should be decreased by a factor $D = 2$ using the system shown in the following figure. The filter $h(n)$ is designed such that no aliasing appears.



- Sketch the magntiude spectra of the signals $x(n)$, $w(n)$ and $y(m)$.
- Sketch the magnitude spectrum of the output signal $y(m)$ when the filter $h(n)$ is removed.

(c) i) Use Matlab to generate a segment of length 1s of the signal $x(n)$.

ii) Listen to the signal and its downsampled version both when the filter $h(n)$ is used in downsampling and when it is not used.

iii) Explain the differences based on the sketches in (b).

iv) Repeat (ii) and (iii) on the music signal “Dolly.wav” that can be downloaded from the course homepage (under Problem Sets).

v) Write down your observations.

Useful Matlab functions are `sound`, `decimate`, `downsample`.