Friction

Sébastien Gros
NTNU, Eng. Cybernetics

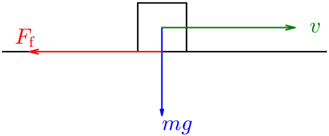
Slides for TTK4130
2021

Modeling Friction

- Friction phenomena are very complex and difficult to model
- Most models are ad-hoc and empirical: they aim at capturing complex phenomena using simple equations rather than genuinely modelling them
- “Science” of friction is called **Tribology**
- Simulating (& controlling) systems with friction can be difficult

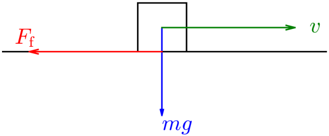
Coulomb Friction model

- Contact force (gravity here): $F_{\mathrm{N}} = mg$
- Velocity v
- Friction F_{f}



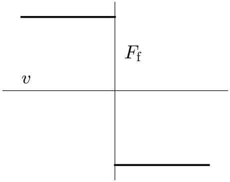
Coulomb Friction model

- Contact force (gravity here): $F_{\mathrm{N}} = mg$
- Velocity v
- Friction F_{f}



Coulomb model:

$$F_{\mathrm{f}} = -\mu F_{\mathrm{N}} \mathrm{sgn}\,(v)$$



Remarks:

- For $v = 0$, friction is $F_{\mathrm{f}} = 0$
- Nonsmooth model: F_{f} "jumps" at $v = 0$

Simulation with Coulomb Friction model

Consider a unitary mass (position p velocity v)

$$\mathbf{x} = \begin{bmatrix} p \\ v \end{bmatrix}, \qquad \dot{\mathbf{x}} = \begin{bmatrix} v \\ u + F_{\mathrm{f}} \end{bmatrix}$$

subject to Coulomb friction

$$F_{\mathrm{f}} = -\mu F_{\mathrm{N}} \mathrm{sgn}\left(v\right)$$

i.e.

$$\dot{\mathbf{x}} = \begin{bmatrix} v \\ u - \mu F_{\mathrm{N}} \mathrm{sgn}\left(v\right) \end{bmatrix}$$

Simulation with Coulomb Friction model

Consider a unitary mass (position p velocity v)

$$\mathbf{x} = \begin{bmatrix} p \\ v \end{bmatrix}, \qquad \dot{\mathbf{x}} = \begin{bmatrix} v \\ u + F_{\mathrm{f}} \end{bmatrix}$$

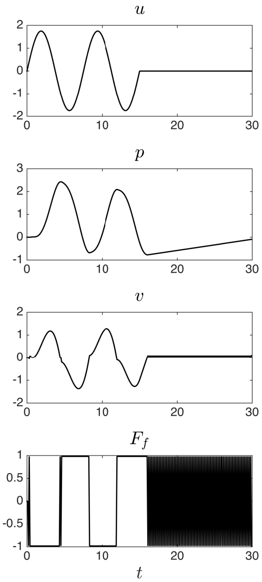
subject to Coulomb friction

$$F_{\mathrm{f}} = -\mu F_{\mathrm{N}} \mathrm{sgn}\left(v\right)$$

i.e.

$$\dot{\mathbf{x}} = \begin{bmatrix} v \\ u - \mu F_{\mathrm{N}} \mathrm{sgn}\left(v\right) \end{bmatrix}$$

Use an explicit Euler integrator



Simulation with Coulomb Friction model

Consider a unitary mass (position p velocity v)

$$\mathbf{x} = \begin{bmatrix} p \\ v \end{bmatrix}, \qquad \dot{\mathbf{x}} = \begin{bmatrix} v \\ u + F_{\mathrm{f}} \end{bmatrix}$$

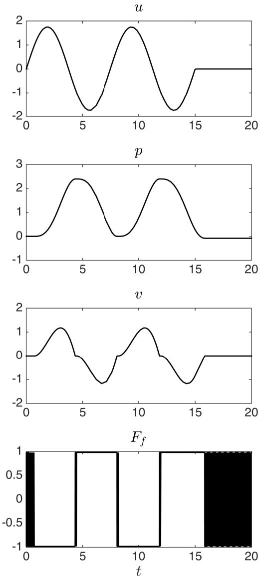
subject to Coulomb friction

$$F_{\mathrm{f}} = -\mu F_{\mathrm{N}} \mathrm{sgn}\left(v\right)$$

i.e.

$$\dot{\mathbf{x}} = \begin{bmatrix} v \\ u - \mu F_{\mathrm{N}} \mathrm{sgn}\left(v\right) \end{bmatrix}$$

Use Matlab ODE45



Simulation with Coulomb Friction model

Consider a unitary mass (position p velocity v)

$$\mathbf{x} = \begin{bmatrix} p \\ v \end{bmatrix}, \qquad \dot{\mathbf{x}} = \begin{bmatrix} v \\ u + F_{\mathrm{f}} \end{bmatrix}$$

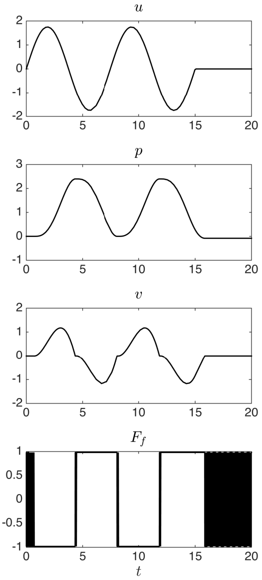
subject to Coulomb friction

$$F_{\mathrm{f}} = -\mu F_{\mathrm{N}} \mathrm{sgn}\,(v)$$

i.e.

$$\dot{\mathbf{x}} = \begin{bmatrix} v \\ u - \mu F_{\mathrm{N}} \mathrm{sgn}\,(v) \end{bmatrix}$$

Use Matlab ODE45

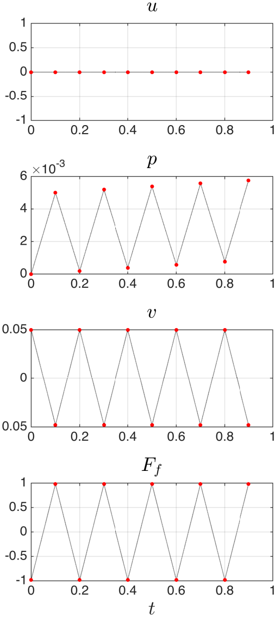


What's going on?

Simulation with Coulomb Friction model - What's going on?

$$\dot{\mathbf{x}} = \begin{bmatrix} v \\ u - \mu F_{\mathrm{N}} \mathrm{sgn}(v) \end{bmatrix}$$

- E.g. simulate with $u = 0$, and $v(0) = 5 \cdot 10^{-2}$
- F_{f} is "jumping" from minus to plus when v changes sign
- Physically, $|v|$ should decrease until $v = 0$, such that $F_{\mathrm{f}} = 0$, where the system should stop moving
- But Euler "misses" the exact time where $v = 0$
- A solution is to use "event-based" integrators, detecting that time



Viscous friction

Viscous model:

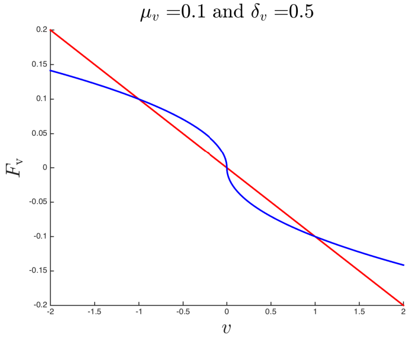
- Most friction phenomena involve some viscous friction, i.e. speed dependent
- Viscous friction is typically due to lubricants
- Models e.g.

$$F_{\mathrm{v}} = -\mu_{\mathrm{v}} v$$

or

$$F_{\mathrm{v}} = -\mu_{\mathrm{v}} |v|^{\gamma_{\mathrm{v}}} \mathrm{sgn}\,(v)$$

with $\gamma \in [0, 1]$



Viscous friction

Viscous model:

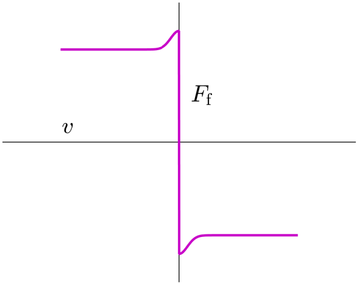
- Most friction phenomena involve some viscous friction, i.e. speed dependent
- Viscous friction is typically due to lubricants
- Models e.g.

$$F_{\mathrm{v}} = -\mu_{\mathrm{v}} v$$

or

$$F_{\mathrm{v}} = -\mu_{\mathrm{v}} |v|^{\gamma_{\mathrm{v}}} \operatorname{sgn}(v)$$

with $\gamma \in [0, 1]$



Stribeck model: lubrification thickness increases with v and reduces the friction, model:

$$F_{\mathrm{v}} = -\left[F_{\mathrm{c}} + (F_{\mathrm{s}} - F_{\mathrm{c}})\, e^{-\left(\frac{v}{v_{\mathrm{s}}}\right)^2}\right] \operatorname{sgn}(v)$$

where F_{c}, F_{s} and v_{s} are constants.

Static friction

- Coulomb model: **Coulomb model:**

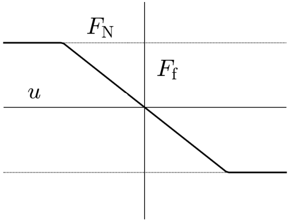
$$F_{\mathrm{f}} = -\mu F_{\mathrm{N}} \mathrm{sgn}\left(v\right)$$

  delivers no friction when $v = 0$.

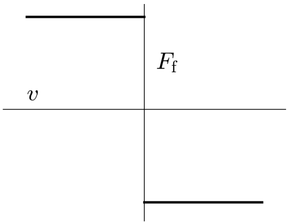
- Static friction: the system "sticks" until a certain force threshold is reached.
- Karnopp+Coulomb model:

$$F_{\mathrm{f}} = \begin{cases} \mathrm{sat}\left(-u, F_{\mathrm{N}}\right) & \text{if} \quad v = 0 \\ -\mu F_{\mathrm{N}} \mathrm{sgn}\left(v\right) & \text{if} \quad v \neq 0 \end{cases}$$

If $v = 0$



If $v \neq 0$



Model smoothing

Coulomb model:

$$F_{\mathrm{f}} = -\mu F_{\mathrm{N}} \mathrm{sgn}\left(v\right)$$

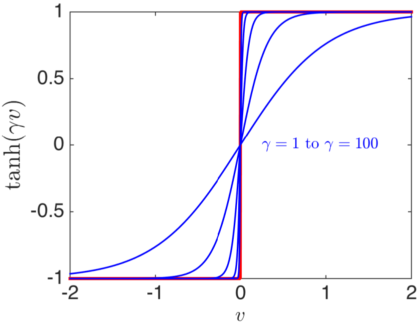
is problematic to simulate because of the discontinuity of the sgn function

Model smoothing

Coulomb model:

$$F_{\mathrm{f}} = -\mu F_{\mathrm{N}} \operatorname{sgn}(v)$$

is problematic to simulate because of the discontinuity of the sgn function



Smooth model approximation:

$$\operatorname{sgn}(v) \approx \tanh(\gamma v)$$

for γ large.

Model smoothing

Coulomb model:

$$F_{\mathrm{f}} = -\mu F_{\mathrm{N}} \mathrm{sgn}\,(v)$$

is problematic to simulate because of the discontinuity of the sgn function