Department of Engineering Cybernetics

TTK4215 System Identification and Adaptive Control
Solution 13

Problem 1
a) From the transfer function of the plant we get,

$$s^2 y + 2\xi\omega_n s y + \omega_n^2 y = \omega_n^2 u. \tag{1}$$

Therefore, the parametric model of the system is $z = \theta^* \phi$, where

$$z = \frac{s^2}{\Lambda_0(s)} y + \frac{\omega_n^2}{\Lambda_0(s)} y - \frac{\omega_n^2}{\Lambda_0(s)} u,\ \theta^* = \xi,\ \phi = \frac{2\omega_n s}{\Lambda_0(s)} y,\ \Lambda_0(s) = (s+\lambda_0)^2,\ \lambda_0 > 0. \tag{2}$$

If we choose the gradient algorithm, then

$$\dot{\theta} = \gamma\epsilon\phi,\ \epsilon = \frac{z - \theta^T\phi}{m_s^2},\ m_s^2 = 1 + \phi^T\phi,\ \gamma > 0,\ \xi(0) = \xi_0. \tag{3}$$

b) Designing the controller: Since $y_m = c$, we select $Q_m = s$, so that $q = 1$. Since $n = 2$ and $q = 1$, P, L and Λ are in the form

$$L = s + l_0,\ P = p_2 s^2 + p_1 s + p_0,\ \Lambda = (s+\lambda_1)^2 \tag{4}$$

where $\lambda_1 > 0$ and p_2, p_1, p_0 and l_0 are calculated by solving

$$(s + l_0)\, s \left(s^2 + 2\xi\omega_n s + \omega_n^2\right) + \left(p_2 s^2 + p_1 s + p_0\right)\omega_n^2 = A^*(s) \tag{5}$$

where the roots of $A^*(s) = 0$ are the desired closed-loop poles. From the overshoot and settling time requirements of the closed-loop system we can find its transfer function poles. The characteristic equation of a 2nd order system is

$$s^2 + 2\xi_c\omega_{nc} s + \omega_{nc}^2 = 0. \tag{6}$$

The overshoot and settle time of this system are $y(t_{overshoot})/y(\infty) = 1 + \exp\left(-\frac{\pi\xi_c}{\sqrt{1-\xi_c^2}}\right)$, $t_s \cong \frac{3.2}{\xi_c\omega_{nc}}$. Thus, we have

$$y(t_{overshoot})/y(\infty) = \frac{1.05c}{c} = 1 + \exp\left(-\frac{\pi\xi_c}{\sqrt{1-\xi_c^2}}\right) \tag{7}$$

which gives

$$\xi_c = \frac{-\ln 0.05}{\sqrt{\pi^2 + (-\ln 0.05)^2}} = 0.69, \tag{8}$$

and

$$2=\frac{3.2}{\xi_c\omega_{nc}}, \tag{9}$$

which gives

$$\omega_{nc}=\frac{3.2}{2\xi_c}=2.32. \tag{10}$$

The characteristic equation of the system is $s^2+3.2s+5.4=0$. Now, we can choose $A^*(s)$ as

$$A^*(s)=\left(s^2+3.2s+5.4\right)(s+30)^2. \tag{11}$$

The last factor is added to get $A^*(s)$ to the required degree of $2n+q-1=4$. Since the factor represents a much faster mode than the first part, the dominant poles of $A^*(s)$ are the poles of $s^2+3.2s+5.4=0$. After equating powers of s in (5), we find L and P, and the control law is given by

$$u_p=\frac{\Lambda-LQ_m}{\Lambda}u_p-\frac{P}{\Lambda}(y_p-y_m). \tag{12}$$

The APPC scheme now appears by solving for $\hat{p}_2$, $\hat{p}_1$, $\hat{p}_0$ and $\hat{l}_0$ by replacing ξ with θ in the Diophantine equation (5).

Problem 7.5 in I&S

a) From the transfer function we have $s^2y+asy=su+bu$. This gives us the parametric model $z=\theta^{*^T}\phi$ with

$$z=\frac{s^2}{\Lambda_0(s)}y-\frac{s}{\Lambda_0(s)}u,\ \theta^*=\begin{bmatrix} a & b\end{bmatrix}^T,\ \phi=\begin{bmatrix} -\frac{s}{\Lambda_0(s)}y & \frac{1}{\Lambda_0(s)}u\end{bmatrix}^T, \tag{13}$$

$\Lambda_0=(s+\lambda_0)^2$ and $\lambda_0>0$. Using the gradient algorithm, we have

$$\dot{\theta}=\Gamma\epsilon\phi,\ \epsilon=\frac{z-\theta^T\phi}{m_s^2},\ m_s^2=1+\phi^T\phi,\ \Gamma=\Gamma^T>0 \tag{14}$$

where $\theta=\begin{bmatrix}\hat{a} & \hat{b}\end{bmatrix}^T$.

b) Since we are regulating to zero, $Q_m=1$. Thus, we have $q=0$, and from $R_p=s^2+as$, $n=2$. Thus, L has degree 1, P has degree 1, and A^* has degree 3. So,

$$L=s+l_0,\ P=p_1s+p_0, \tag{15}$$

and we select

$$A^*=(s+1)^2. \tag{16}$$

To solve the Diophantine equation, $LQ_mR_p+PZ_p=A^*$, we must require that R_p and Z_p are co-prime. That is, $b\neq 0$ and $a\neq b$. Under these conditions, we can solve

$$(s+l_0)\left(s^2+as\right)+(s+b)(p_1s+p_0)=(s+1)^3, \tag{17}$$

to obtain

$$l_0 = \frac{3-3b+ab+1/b}{a-b}, \tag{18}$$

$$p_1 = 3-a-\frac{3-3b+ab+1/b}{a-b}, \tag{19}$$

$$p_0 = 1/b. \tag{20}$$

Using the estimates from a), we get estimates of the controller parameters as follows

$$\hat{l}_0 = \frac{3 - 3\hat{b} + \hat{a}\hat{b} + 1/\hat{b}}{\hat{a} - \hat{b}}, \tag{21}$$

$$\hat{p}_1 = 3 - \hat{a} - \frac{3 - 3\hat{b} + \hat{a}\hat{b} + 1/\hat{b}}{\hat{a} - \hat{b}}, \tag{22}$$

$$\hat{p}_0 = 1/\hat{b}. \tag{23}$$

The control law becomes

$$u_p = \frac{\lambda - \hat{l}_0}{s + \lambda} u_p - \frac{\hat{p}_1 s + \hat{p}_0}{s + \lambda} y_p, \tag{24}$$

where $\lambda > 0$.

c) Loss of stabilizability occurs if $\hat{b} = 0$ or $\hat{a} - \hat{b} = 0$. Therefore, the stabilizability conditions require that $\hat{b} \neq 0$ or $\hat{a} - \hat{b} \neq 0$ at all times, avoiding zero-pole cancellations in the estimated plant $(s+\hat{b})/(s(s+\hat{a}))$, or equivalently, ensuring that the estimated plant is controllable and observable at all times.

d) The problem can be solved by using projection. This requires that b be bounded away from 0 and that $a - b$ be bounded away from zero. This in turn, requires some a priori knowledge about the real parameters a and b, so that the initial parameter estimate is on the "right side" of these constraints.