Department of Engineering Cybernetics

TTK4215 System Identification and Adaptive Control

Solution 12

Problem 1

a) We write

$$
\begin{aligned}
y_p &= k_p \frac{Z_p}{R_p} u_p, && (1)\\
y_m &= k_m \frac{Z_m}{R_m} r, && (2)
\end{aligned}
$$

where $k_p = b_1$, $Z_p = s + b_0/b_1$, $R_p = s^2 + a_1 s + a_0$, $k_m = 4$, $Z_m = 1$, and $R_m = s + 5$. Consider the control law

$$
u_p = \frac{\theta_1^*}{\Lambda} u_p + \frac{\theta_2^*}{\Lambda} y_p + \theta_3^* y_p + c_0^* r, \qquad (3)
$$

where $\Lambda = s + 1$. This control law can be implemented as

$$
\begin{aligned}
\dot{\omega}_1 &= -\omega_1 + u_p, && (4)\\
\dot{\omega}_2 &= -\omega_2 + y_p, && (5)\\
u_p &= \theta_1^* \omega_1 + \theta_2^* \omega_2 + \theta_3^* y_p + c_0^* r, && (6)
\end{aligned}
$$

where $\theta_1^* = (b_1 - b_0)/b_1$, $\theta_2^* = (a_0 - a_1 + 1)/b_1$, $\theta_3^* = (a_1 - 6)/b_1$, and $c_0^* = 4/b_1$ are obtained using the relations (6.3.12), (6.3.16) and (6.3.17).

b) Let $e = y_p - y_m$, $\theta = [\begin{array}{cccc} \theta_1 & \theta_2 & \theta_3 & c_0 \end{array}]^T$, and $\omega = [\begin{array}{cccc} \omega_1 & \omega_2 & y_p & r \end{array}]^T$. The control law is designed as (Table 6.1)

$$
\begin{aligned}
\dot{\omega}_1 &= -\omega_1 + u_p, && (7)\\
\dot{\omega}_2 &= -\omega_2 + y_p, && (8)\\
u_p &= \theta^T \omega, && (9)
\end{aligned}
$$

where θ is generated by the adaptive law

$$
\dot{\theta} = -\Gamma e \omega, \qquad (10)
$$

where $\Gamma = \Gamma^T > 0$.

c) When it is known that $a_0 = -1$, $a_1 = 0$, and $b_1 = 1$, we see that θ_2^*, θ_3^*, and c_0^* are known. Inserting this information into the control law, we get

$$
\begin{aligned}
u_p &= \theta^T \omega = \theta_1 \omega_1 + \left((a_0 - a_1 + 1)/b_1 \right) \omega_2 + (a_1 - 6)/b_1 y_p + 4/b_1 r \\
&= \theta_1 \omega_1 - 6 y_p + 4r. && (11)
\end{aligned}
$$

To implement this, we only need the estimate θ_1 and ω_1, which are obtained by

$$\begin{aligned} \dot{\omega}_1 &= -\omega_1 + u_p, && (12) \\ \dot{\theta}_1 &= -\gamma e \omega_1, && (13) \end{aligned}$$

where $\gamma > 0$.

Problem 2
a) The plant and the reference models are given by

$$y_p = \frac{b_0}{s^2 + a_1 s + a_0} u_p \text{ and } y_m = \frac{1}{s^2 + \sqrt{2}s + 1} r, \tag{14}$$

where $b_0 = 1/M$, $a_1 = f/M$, and $a_0 = k/M$. Using Table 6.2, the MRAC law is given by

$$\begin{aligned} \dot{\omega}_1 &= -\omega_1 + u_p, && (15) \\ \dot{\omega}_2 &= -\omega_2 + y_p, && (16) \\ \dot{\phi} &= -\phi + \omega, && (17) \\ u_p &= \theta^T \omega - \phi^T \Gamma \phi e, && (18) \\ \dot{\theta} &= -\Gamma e \phi, && (19) \end{aligned}$$

where $\Gamma = \Gamma^T > 0$, $\omega = \begin{bmatrix} \omega_1 & \omega_2 & y_p & r \end{bmatrix}^T$, $\theta = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & c_0 \end{bmatrix}^T$, $e = y_p - y_m$, and $\omega_1(0) = \omega_2(0) = \phi(0) = 0$.
b) Simulation.