Department of Engineering Cybernetics

TTK4215 System Identification and Adaptive Control

Solution 11

Problem 6.1 from I&S

Consider the control law

$$u = -k^* y + l^* r. \tag{1}$$

If b is known, $k^* = 3/b$ and $l^* = 2/b$. However, since b is unkown we use the estimate of k^* and l^*. Letting $e = y - y_m$, the dynamics of the tracking error is given by

$$\dot{e} = -2e + b\left(-\tilde{k}y + \tilde{l}r\right), \tag{2}$$

where $\tilde{k} = k - k^*$ and $\tilde{l} = l - l^*$. Similar to the analysis in Section 6.2.2 of I&S it can be shown that the following adaptive law guarantees $e, \dot{e}, \tilde{k}, \tilde{l}, y, u \in \mathcal{L}_\infty$, $e \in \mathcal{L}_2$, and that $e \to 0$ as $t \to \infty$,

$$\begin{aligned} \dot{k} &= \gamma_1 e y, \;\; k(0) = k_0, &(3)\\ \dot{l} &= -\gamma_2 e r, \;\; l(0) = l_0, &(4) \end{aligned}$$

where $\gamma_1, \gamma_2 > 0$. It should be mentioned that sufficiently richness of the reference input is a sufficient condition for convergence of k and l to k^* and l^*, respectively. Run simulations to investigate this issue.

Problem 6.2 from I&S

a) The dynamics of the system is represented by the following differential equation

$$\dot{V} = -aV + b\theta + ad, \tag{5}$$

and that of the reference model is given by

$$\dot{V}_m = -0.5V_m + 0.5V_s. \tag{6}$$

Let $\theta = -k_1^* V + k_2^* V_s - k_3^*$, then

$$\dot{V} = -\left(a + bk_1^*\right)V + bk_2^* V_s - bk_3^* + ad. \tag{7}$$

Thus, for model-plant transfer function matching, we have

$$\begin{aligned} k_1^* &= (0.5 - a)/b, &(8)\\ k_2^* &= 0.5/b, &(9)\\ k_3^* &= ad/b, &(10) \end{aligned}$$

where $b > 0$.

b) Let $e = V - V_m$ and $\theta = -k_1 V + k_2 V_s - k_3$, then

$$\begin{aligned} \dot{e} &= -0.5e + b\left(k_1^* V - k_2^* V_s + k_3^* + \theta\right) \\ &= -0.5e + b\left(-\tilde{k}_1 V + \tilde{k}_2 V_s - \tilde{k}_3\right), \end{aligned}$$

where $\tilde{k}_1 = k_1 - k_1^*$, $\tilde{k}_2 = k_2 - k_2^*$, and $\tilde{k}_3 = k_3 - k_3^*$. Consider the following Lyapunov-like function

$$E = \frac{1}{2}e^2 + \frac{b}{2\gamma_1}\tilde{k}_1^2 + \frac{b}{2\gamma_2}\tilde{k}_2^2 + \frac{b}{2\gamma_3}\tilde{k}_3^2,$$

then

$$\dot{E} = -0.5e^2 - be\tilde{k}_1 V + be\tilde{k}_2 V_s - be\tilde{k}_3 + \frac{b}{\gamma_1}\tilde{k}_1\dot{\tilde{k}}_1 + \frac{b}{\gamma_2}\tilde{k}_2\dot{\tilde{k}}_2 + \frac{b}{\gamma_3}\tilde{k}_3\dot{\tilde{k}}_3. \tag{11}$$

By choosing $\dot{\tilde{k}}_1 = \dot{k}_1 = \gamma_1 eV$, $\dot{\tilde{k}}_2 = \dot{k}_2 = -\gamma_2 eV_s$, $\dot{\tilde{k}}_3 = \dot{k}_3 = \gamma_3 e$, we have $\dot{E} = -0.5e^2 \leq 0$ which implies the boundedness of E and therefore $e, \tilde{k}_1, \tilde{k}_2, \tilde{k}_3$. Also, from $E > 0$ and $\dot{E} \leq 0$, E has a limit, i.e. $\lim_{t\to\infty} E(t) = E_\infty$. Therefore,

$$0.5\int_0^\infty e^2(\tau)\, d\tau \leq E(0) - E_\infty,$$

which implies $e \in \mathcal{L}_2$. Since $e, \tilde{k}_1, \tilde{k}_2, \tilde{k}_3 \in \mathcal{L}_\infty$, we have $\dot{e} \in \mathcal{L}_\infty$ which together with $e \in \mathcal{L}_2$ implies that $e(t) \to 0$ as $t \to \infty$.