Department of Engineering Cybernetics

TTK4215 System Identification and Adaptive Control
Solution 9

Problem 4.10 from I&S

c) Recall the last assignment. If we select

$$\Gamma_1 = \begin{bmatrix} \gamma_{11} & 0 \\ 0 & \gamma_{22} \end{bmatrix}, \tag{1}$$

we have the update laws

$$\dot{m} = \gamma_{11}\epsilon_2 \left(\frac{s^2}{\Lambda(s)} y_2 \right), \tag{2}$$

$$\dot{\beta} = \gamma_{22}\epsilon_2 \left(\frac{s}{\Lambda(s)} y_2 \right), \tag{3}$$

$$\dot{k} = \gamma_1\epsilon_1 (y_1 - y_2). \tag{4}$$

To enforce $m \geq 10$, we have

$$S_m = \{ m \in R \,|10 - m \leq 0 \}, \tag{5}$$

so $g(m) = 10 - m$, and the gradient is -1. The projection algorithm then gives

$$\dot{m} = \begin{cases} \gamma_{11}\epsilon_2 \left(\frac{s^2}{\Lambda(s)} y_2 \right) & \text{if } (m > 10) \text{ or } (m = 10 \text{ and } \epsilon_2 \left(\frac{s^2}{\Lambda(s)} y_2 \right) \geq 0) \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

To enforce $0 \leq \beta \leq 1$, we have

$$S_\beta = \{ \beta \in R \,|\max\{-\beta, \beta - 1\} \leq 0 \},$$

so $g(\beta) = \max\{-\beta, \beta - 1\}$, and the gradient is -1 when $\beta = 0$, and 1 when $\beta = 1$. The projection algorithm then gives

$$\dot{\beta} = \begin{cases} \gamma_{22}\epsilon_2 \left(\frac{s}{\Lambda(s)} y_2 \right) & \begin{array}{c} \text{if } (0 < \beta < 1) \text{ or } (\beta = 0 \text{ and } \epsilon_2 \left(\frac{s}{\Lambda(s)} y_2 \right) \geq 0) \\ \text{or } (\beta = 1 \text{ and } \epsilon_2 \left(\frac{s}{\Lambda(s)} y_2 \right) \leq 0) \end{array} \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

To enforce $k \geq 0.1$, we have

$$S_k = \{ k \in R \,|0.1 - k \leq 0 \},$$

so $g(k) = 0.1 - k$, and the gradient is -1. The projection algorithm then gives

$$\dot{k} = \begin{cases} \gamma_1 \epsilon_1 (y_1 - y_2) & \text{if } (k > 0.1) \text{ or } (k = 0.1 \text{ and } \epsilon_1 (y_1 - y_2) \geq 0) \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

The initial values must satisfy $m(0) \geq 10$, $0 \leq \beta(0) \leq 1$, and $k(0) \geq 0.1$.
d) Simulation.

Problem 4.11 from I&S
a) We have

$$\theta_p = \frac{k_0 \omega_0^2}{s^2 + 2\xi_0 \omega_0 s + \omega_0^2 (1 - k_0)} (r - \theta_p), \tag{9}$$

so that

$$\theta_p = \frac{k_0 \omega_0^2}{s^2 + 2\xi_0 \omega_0 s + \omega_0^2} r, \tag{10}$$

and

$$\dot{\theta} = \frac{k_1 \omega_1^2}{s^2 + 2\xi_1 \omega_1 s + \omega_1^2} \theta_p. \tag{11}$$

so that

$$\left(s^2 + 2\xi_0 \omega_0 s + \omega_0^2 (1 - k_0)\right) \theta_p = k_0 \omega_0^2 (r - \theta_p). \tag{12}$$

Let $\Lambda(s) = (s+1)^2$ and filter the equation by $1/\Lambda(s)$, obtaining

$$\frac{s^2}{\Lambda(s)} \theta_p = \omega_0^2 \left(-\frac{1}{\Lambda(s)} \theta_p\right) + \xi_0 \omega_0 \left(-\frac{2s}{\Lambda(s)} \theta_p\right) + k_0 \omega_0^2 \left(\frac{1}{\Lambda(s)} r\right). \tag{13}$$

So, we obtain

$$z_1 = \theta_1^{*T} \phi_1, \tag{14}$$

with

$$\begin{aligned} z_1 &= \frac{s^2}{\Lambda(s)} \theta_p, && (15) \\ \theta_1^* &= \begin{bmatrix} \omega_0^2 & \xi_0 \omega_0 & k_0 \omega_0^2 \end{bmatrix}^T, && (16) \\ \phi_1 &= \begin{bmatrix} -\frac{1}{\Lambda(s)} \theta_p & -\frac{2s}{\Lambda(s)} \theta_p & \frac{1}{\Lambda(s)} r \end{bmatrix}^T. && (17) \end{aligned}$$

Similarly, we have

$$\left(s^2 + 2\xi_1 \omega_1 s + \omega_1^2\right) \dot{\theta} = k_1 \omega_1^2 \theta_p, \tag{18}$$

which leads to

$$z_2 = \theta_2^{*T} \phi_2, \tag{19}$$

with

$$\begin{aligned} z_2 &= \frac{s^2}{\Lambda(s)} \dot{\theta}, && (20) \\ \theta_2^* &= \begin{bmatrix} \omega_1^2 & \xi_1 \omega_1 & k_1 \omega_1^2 \end{bmatrix}^T, && (21) \\ \phi_1 &= \begin{bmatrix} -\frac{1}{\Lambda(s)} \dot{\theta} & -\frac{2s}{\Lambda(s)} \dot{\theta} & \frac{1}{\Lambda(s)} \theta_p \end{bmatrix}^T. && (22) \end{aligned}$$

One may use the recursive LS method given by

$$\dot{\theta}_i = P_i \epsilon_i \phi_i, \ \theta_i(0) = \theta_{i,0}, \tag{23}$$

$$\dot{P}_i = \beta_i P_i - P_i \frac{\phi_i \phi_i^T}{m_{s_i}} P_i, \ P_i(0) = P_{i,0}, \tag{24}$$

for $i = 1, 2$ to estimate θ_1^* and θ_2^*.
b) and c) Simulation.