Norwegian University of Science and Technology
Department of Electronics and Telecommunications

TTT4120 Digital Signal Processing
Problem Set 2

The main topics for this problem set are representation of signals and systems in the frequency domain, and the sampling theorem. Relevant chapters from the text book are 4.2.3, 6.1, 4.3, 4.4, 5.1.1, 5.1.4, and 5.4.1. You will need headphones for Problem 4. The maximum score for each problem is given in parentheses.

Problem 1 (3 points)

Two signals $x(n)$ and $y(n)$ are given by

$$x(n) = \begin{cases} 2 & n = 0 \\ 1 & n = \pm 1 \\ 0 & \text{otherwise,} \end{cases} \qquad y(n) = \begin{cases} 1 & -M \le n \le M \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that the Fourier transform of $x(n)$ is given by

$$X(\omega) = 2 + 2\cos\omega$$

and sketch it for $\omega \in [-\pi, \pi]$.

(b) Show that the Fourier transform of $y(n)$ is given by

$$Y(\omega) = \frac{\sin\left(\omega(M + \frac{1}{2})\right)}{\sin(\frac{\omega}{2})}$$

and sketch it for M=10 and $\omega \in [-\pi, \pi]$ (you may use Matlab).

(c) Explain why the signals $x(n)$ and $y(n)$ have real valued spectra.

(d) Let the signal

$$z(n) = \sum_{l=-\infty}^{\infty} x(n - lN)$$

be the periodic extension of $x(n)$. Assume that N is greater than the length of the signal, i.e. $N > 3$.

Sketch the signal $z(n)$.

Find the Fourier series coefficients $\{c_k\}$ of $z(n)$.

Sketch $\{c_k\}$ as a function of $\omega = 2\pi \frac{k}{N} \in [-\pi, \pi]$ for $N = 10$.

(e) Compare the spectra of $x(n)$ and $z(n)$, i.e. $X(\omega)$ and $\{c_k\}$. What is the relationship between the spectra?

Problem 2 (1.5 points)

Let $x(n)$ be a signal with the Fourier transform $X(w)$. Find the fourier tranforms of the following signals:

(a) $x_1(n) = x(n+3)$

(b) $x_2(n) = x(-n)$

(c) $x_3(n) = x(3-n)$

(d) $x_4(n) = x(n) * x(n)$

Problem 3 (3 points)

Two systems (from Problem Set 1) are given by the following difference equations

$$y(n) = x(n) + 2x(n-1) + x(n-2) \tag{1}$$
$$y(n) = -0.9y(n-1) + x(n). \tag{2}$$

(a) Find the frequency responses of these two systems.

(b) Find the magnitude and phase responses of the two systems. Are they even or odd functions?

(c) Use Matlab (the functions `freqz`, `abs` and `angle`) to find and plot the magnitude and phase responses of the systems.

(d) Determine whether each system represents a lowpass, bandpass, bandstop or highpass filter. Justify your answers.

(e) The signal $x(n) = \frac{1}{2}\cos(\frac{\pi}{2}n + \frac{\pi}{4})$ is passed through the two systems. Find the frequency, amplitude and phase of the corresponding output signals.

Problem 4 (2 points)

The analog signal $x_a(t)$ is given by

$$x_a(t) = \cos(2000\pi t).$$

The spectrum of $x_a(t)$ is shown in Fig. 1.

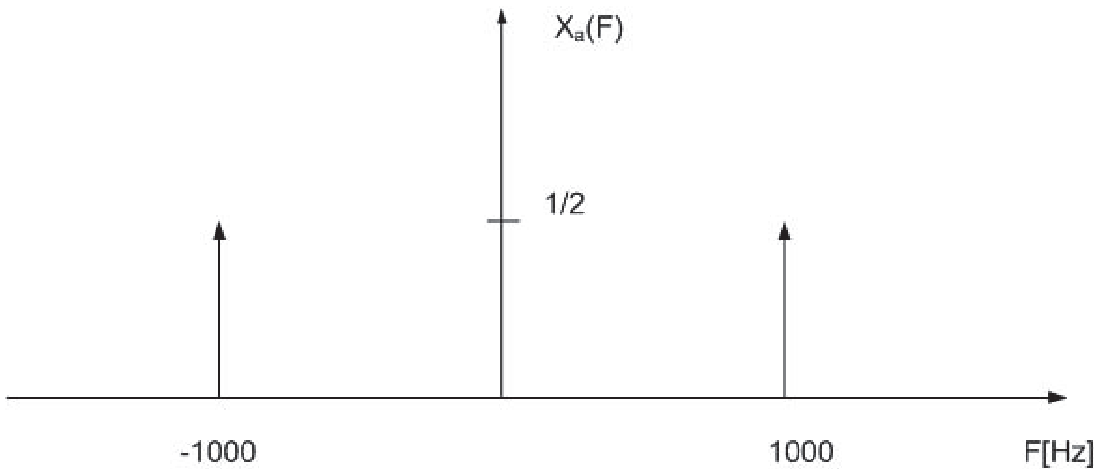

Figure 1: Spectrum of the signal $x_a(t)$

(a) Two time-discrete signals, $x_1(n)$ and $x_2(n)$, are generated by sampling the signal $x_a(n)$ using the sampling frequencies $F_s = 4000\text{Hz}$ and $F_s = 1500\text{Hz}$, respectively.

Sketch the spectra of the two sampled signals, $X_1(f)$ and $X_2(f)$, for $f \in [-\frac{1}{2}, \frac{1}{2}]$.

(b) Use Matlab to generate a segment of both time-discrete signals corresponding to the signal duration of $1s$.

Play the two signal segments using Matlab function `sound` with the corresponding sampling frequencies.

Why do they sound differently when they both represent the same analog signal?