



TTT4120 Digital Signal Processing Problem Set 3

Problem 1 (2.5 points)

Consider the two causal analog filters shown in Figure 1.

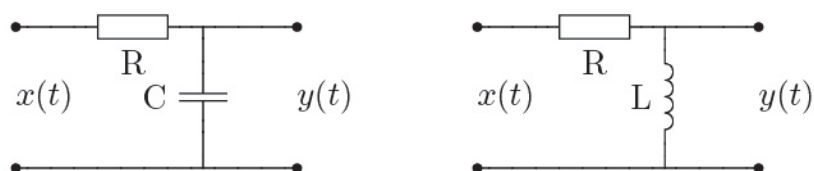


Figure 1: Analog filters

- (a) Derive the differential equations and transfer functions for the two filters.
- (b) Determine frequency responses and filter types (i.e. lowpass, highpass, bandpass, bandstop) for the filters.
- (c) Derive the unit pulse responses for the filters.

Problem 2 (2.5 points)

Find the region of convergence and unit pulse response $h(n)$ for the following digital filters.

- (a) A causal filter with transfer function

$$H(z) = \frac{1}{1 - \frac{2}{3}z^{-1}}$$

- (b) A causal filters with transfer function

$$H(z) = \frac{1}{(1 + \frac{1}{2}z^{-1})(1 - z^{-1})}$$

- (c) An anti-causal filter with transfer function

$$H(z) = \frac{z^{-1}}{(1 + \frac{3}{2}z^{-1})(1 - 3z^{-1})}$$

- (d) Which of the three systems are stable? Justify your answer.

Problem 3 (3 points)

Consider a linear time-invariant (LTI) system with unit pulse response $h(n)$ and input signal $x(n)$.

$$h(n) = \begin{cases} \frac{1}{2^n} & n \geq 0 \\ 0 & n < 0, \end{cases}$$
$$x(n) = \begin{cases} 1 & n \geq 2 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Determine the z-transform of the unit pulse response $h(n)$ and input signal $x(n)$ and their respective ROCs.
- (b) Derive an expression for the output signal $y(n)$ by performing the convolution in the time domain.
- (c) Derive an expression for the output signal $y(n)$ using the z-transform.

Problem 4 (2 points)

A digital filter is given by the following difference equation

$$y(n) = x(n) - x(n-2) - \frac{1}{4}y(n-2).$$

- (a) Find the transfer function of the filter.
- (b) Find the poles and zeros of the filter and sketch them in the z-plane.
- (c) Is the filter stable? Justify your answer based on the pole-zero plot.
- (d) Determine the filter type (i.e. HP, LP, BP or BS) based on the pole-zero plot.