Department of Engineering Cybernetics

TTK4215 System Identification and Adaptive Control
Solution 8

Problem 4.9 from I&S
c) The recursive LS algorithm for generating θ is given by

$$\dot{\theta} = P\epsilon\phi,\ \theta(0) = \theta_0,\ \dot{P} = \beta P - P\frac{\phi\phi^T}{m^2}P,\ P(0) = P_0 = P_0^T > 0,$$

where $\beta \geq 0$ and P_0 are design constants.
d) and e) A SIMULINK example that uses LS is available on It's Learning.

Problem 4.10 from I&S
a) The equation

$$k(y_1 - y_2) = u \tag{1}$$

is Hook's law and the equation

$$m\ddot{y}_2 + \beta\dot{y}_2 + k(y_2 - y_1) = 0 \tag{2}$$

is Newton's second law applied to the mass m.
b) Notice that (2) can also be written as

$$m\ddot{y}_2 + \beta\dot{y}_2 = u, \tag{3}$$

which allows us to split our estimation scheme into two independent estimation problems. One estimates k based on equation (1), and the other estimates m and β based on equation (3). The linear parameterization for the first problem is simply

$$z_1 = \theta_1^*\phi_1, \tag{4}$$

where $z_1 = u$ and $\phi_1 = y_1 - y_2$. For the second problem, we filter equation (3) by $\Lambda(s) = (s+1)^2$ and obtain

$$m\frac{s^2}{\Lambda(s)}y_2 + \beta\frac{s}{\Lambda(s)}y_2 = \frac{1}{\Lambda(s)}u. \tag{5}$$

The linear parameterization is now obtained by defining

$$\theta_2^* = \begin{bmatrix} m & \beta \end{bmatrix}, \tag{6}$$

$$\phi_2 = \begin{bmatrix} \frac{s^2}{\Lambda(s)}y_2 & \frac{s}{\Lambda(s)}y_2 \end{bmatrix}, \tag{7}$$

and

$$z_2 = \frac{1}{\Lambda(s)}u. \tag{8}$$

If we pick the gradient algorithm, for instance, we get the update laws

$$\begin{aligned}
\dot{\theta}_1 &= \gamma_1\epsilon_1\phi_1,\ \epsilon_1 = \frac{z_1 - \theta_1\phi_1}{m_1^2},\ m_1^2 = 1 + \phi_1^2,\ \gamma_1 > 0, \\
\dot{\theta}_2 &= \Gamma_2\epsilon_2\phi_2,\ \epsilon_2 = \frac{z_2 - \theta_2^T\phi_2}{m_2^2},\ m_2^2 = 1 + \phi_2^T\phi_2,\ \Gamma_2 = \Gamma_2^T > 0.
\end{aligned}$$