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NTNU – Trondheim
Norwegian University of
Science and Technology

Exercise 5
TTK4130 Modeling and Simulation

Problem 1 (Coupled drives)

The purpose of the coupled drives model can be to model the process of winding some material from one spool to another, where the material might be wire, plastic strip, paper, textile yarn, or magnetic tape. The model, with some small modifications, may also represent two drives driving a conveyor belt/assembly line.

The system consists of two (electric) drives and a material belt (the material being wound). In addition to controlling the speed of the belt, it is usually necessary to control the tension in the belt. To these ends, one may use a spring mounted pulley such that the deflection of the spring indicates tension, and the pulley speed indicates belt speed. See Figure 1, which is the setup we will model in this problem.

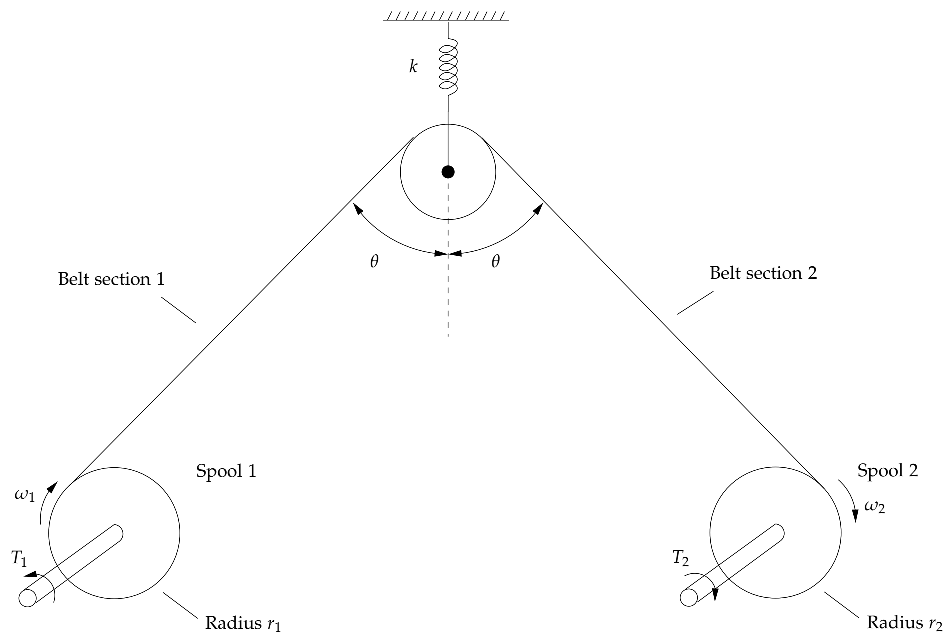


Figure 1: Coupled drives setup

We will implement the model as consisting of five parts, two spools driven by torques T_1 and T_2 (from some motors, for instance DC motors, that we will model only as a torque source), two elastic belt sections, and one "jockey pulley".

We will implement this model in Dymola. Create a package called CoupledDrives. In this package, create a model also called CoupledDrives, that will be the total model of the coupled drives system. In addition, we will create models for the drives, the belts and the pulley in the package, which we will use to put together the total model.

To help the modeling, we have drawn a "free body diagram" in Figure 2.

(a) We start be modelling the drives. In addition to the torque input and the belt force, we assume there is "viscous friction" of the form $B_i\omega$. Let the force from the belt be denoted F_i. Write up the torque balance for Spool 1 ($J_1\dot{\omega}_1 = \ldots$) and 2 ($J_2\dot{\omega}_2 = \ldots$). Note how the driving torques has opposite positive directions, and how the belt forces work in opposite directions compared to the rotation.

Solution:

$$J_1\dot{\omega}_1 = -T_1 - B_1\omega_1 + F_1 r_1,$$
$$J_2\dot{\omega}_2 = T_2 - B_2\omega_2 - F_2 r_2.$$

We will use the Modelica Standard Library, Modelica.Mechanics.Rotational to model the drives. By switching positive direction on the torque on Spool 1, we will use the same model for both drives/spools. It might seem that we then will get a problem with the sign of the belt force on Spool 2, but we will see later in the problem how this fixes itself.

(b) Create a model called Drive in the package you created, and drag and drop components Torque, Inertia, Damper and IdealGearR2T. Relate each component to a term in the torque balance. Insert parameters $J = 1 \cdot 10^{-3}\text{kg} \cdot \text{m}^2$, $B = 0.1\text{kg} \cdot \text{m}^2/\text{s}$, $r = 0.1\text{m}$ (same for both drives[1]).

We will use this model as a sub-model, so we need inputs and outputs: Use Modelica.Blocks.Interfaces.RealInput as input, and connect to the Torque-block. Use Modelica.Mechanics.Translational.Interfaces.Flange_b as output, and connect to the IdealGearR2T. If you like, open the Icon-view of the model and draw for example a circle and rectangle to represent the drive. The drive model is illustrated in Figure 3. (A possible icon for the model can be seen in Figure 4.)

Solution: See Figure 3.

(c) We model each belt section as a massless spring + damper, see Figure 2c. Write down the equation for the force, and find a component in Modelica.Mechanics.Translational.Components that can be used for this. Let the belt stiffness be $K = 50\text{kg}/\text{s}^2$ and belt viscous friction/damping be $B = 0.5\text{kg}/\text{s}$.

Solution: The force can be written

$$F = K(x_1 - x_2) + B(v_1 - v_2).$$

An appropriate model component is Modelica.Mechanics.Translational.Components.SpringDamper.

(d) Lastly, we want to model the pulley, see Figure 2d. We assume the pulley is without mass. A power balance can be written

$$F_1 v_1 - F_2 v_2 = F_k v_k.$$

The spring force can be written

$$F_k = k x_k, \quad \dot{x}_k = v_k.$$

Write down a force balance in horizontal and vertical direction, and show that the pulley can be described by

$$\begin{aligned} v_1 - v_2 &= 2 v_k \cos\theta, \\ \dot{x}_k &= v_k, \\ F_k &= k x_k, \\ F_1 = F_2 &= \frac{F_k}{2\cos\theta}. \end{aligned}$$

Assume the spring stiffness is $k = 200\text{kg}/\text{s}^2$, and $\theta = 45°$. (The above equation set is not minimal – several variables can be eliminated. However, Dymola can do this for us.)

[1] Radius r means a ratio $1/r$ when converting to translational motion.

Solution: The force balance in horizontal direction is

$$F_1 \sin\theta = F_2 \sin\theta \quad \Rightarrow \quad F_1 = F_2.$$

The force balance in vertical direction is

$$F_k = F_1 \cos\theta + F_2 \cos\theta \quad \Rightarrow \quad F_k = 2F_1 \cos\theta$$

This together with the two equations in the problem can be rearranged to give the model.

(e) To implement the pulley model, we cannot use elements from the standard library. Implement instead the model above in the following skeleton:

```
model Pulley
  extends Modelica.Mechanics.Translational.Interfaces.PartialTwoFlanges;
  import SI = Modelica.SIunits;
  SI.Velocity v1 "Velocity section belt 1";
  SI.Velocity v2 "Velocity section belt 2";
  SI.Velocity vk "Vertical velocity pulley";
  SI.Position xk "Spring deflection/vertical position pulley";
  SI.Force Fk "Spring force";
  parameter Real k "Spring constant";
  parameter Real theta "Angle";
equation
  // Implement Pulley model here
end Pulley;
```

The forces F_1 and F_2 can be taken from the connectors flange_a and flange_b, but note that the force in flange_b has opposite positive direction compared to Figure 2d. The corresponding velocities can be found by differentiating the positions in these connectors.

Solution:

```
model Pulley
  extends Modelica.Mechanics.Translational.Interfaces.PartialTwoFlanges;
  import SI = Modelica.SIunits;
  SI.Velocity v1 "Velocity section belt 1";
  SI.Velocity v2 "Velocity section belt 2";
  SI.Velocity vk "Vertical velocity pulley";
  SI.Position xk "Spring deflection/vertical position pulley";
  SI.Force Fk "Spring force";
  parameter Real k "Spring constant";
  parameter Real theta "Angle";
equation
   v1 = der(flange_a.s);
   v2 = der(flange_b.s);
   2*vk*cos(theta) = v1-v2;
   der(xk) = vk;
   Fk = k*xk;
   flange_a.f = Fk/(2*cos(theta));
   flange_b.f = -flange_a.f;
end Pulley;
```

(f) Put together the entire model to get something like Figure 4. Note how the force on Spool 2 now has the right "sign" due to Modelica sign conventions.

(g) Finally, we want to plot a Bode plot of the transfer-function from the two torques to the deflection of the pulley spring (a measurement related to belt tension).

- Add an output to the pulley model. A simple way to do this is to drag-and-drop a Modelica.Blocks.Interfaces.RealOutput into the pulley model, and add the equation `y = xk;`
- Add two Modelica.Blocks.Interfaces.Realinput to the CoupledDrives-model and connect to the torque inputs.

You should also add a sign reversal for the torque input to spool 1, cf. discussion before (b). See Figure 5. Linearize (in the simulation menu) and create a Bode-plot in Matlab, using the code

```
% load output from Dymola linearize
load dslin

% ABCD is A, B, C and D matrix stacked into one matrix
% nx is number of states (dimension of the A matrix)
A = ABCD(1:nx,1:nx); B = ABCD(1:nx,nx+1:end);
C = ABCD(nx+1:end,1:nx); D = ABCD(nx+1:end,nx+1:end);
sys = ss(A,B,C,D);

% Plot Bode response
w = logspace(-1,4,50);
bode(sys,w)
```

Comment.

Solution: See Figure 6.

- Since the input is torque (force) and the output is position, we expect the phase to start at -180°. That the phase in the figure starts at 180° is due to conventions in Matlab for plotting Bode plots for unstable systems (if you check the eigenvalues of the A-matrix, you might see that the system has some slightly unstable eigenvalues, that are not physical). Remember that phase can be added or subtracted 360°.

- The elasticity in the belt sections (the load) lead to a resonance (at frequency $w = \sqrt{\frac{K}{J/r^2}} \approx$ 22rad/s), which is an effective upper bound on the bandwidth.

Numerical inaccuracies in the linearization procedure may give some strange results for very small frequencies.

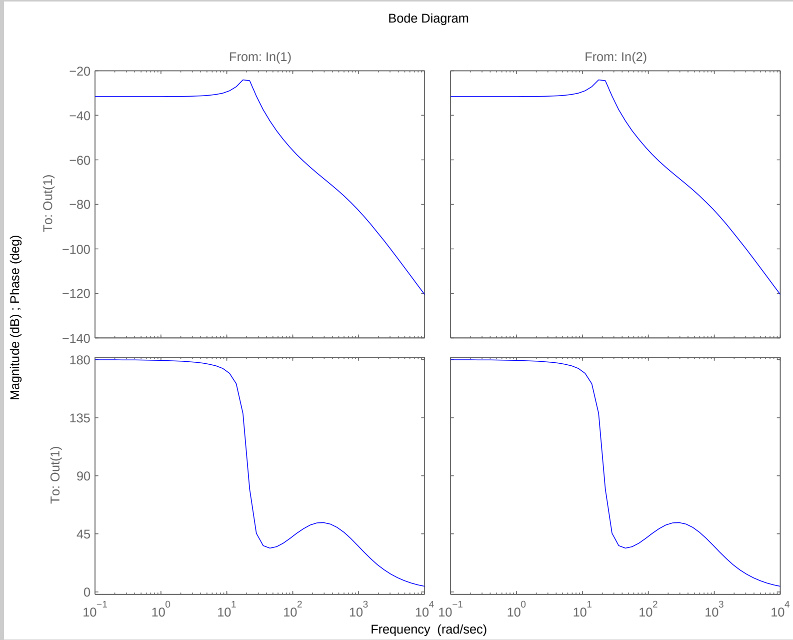
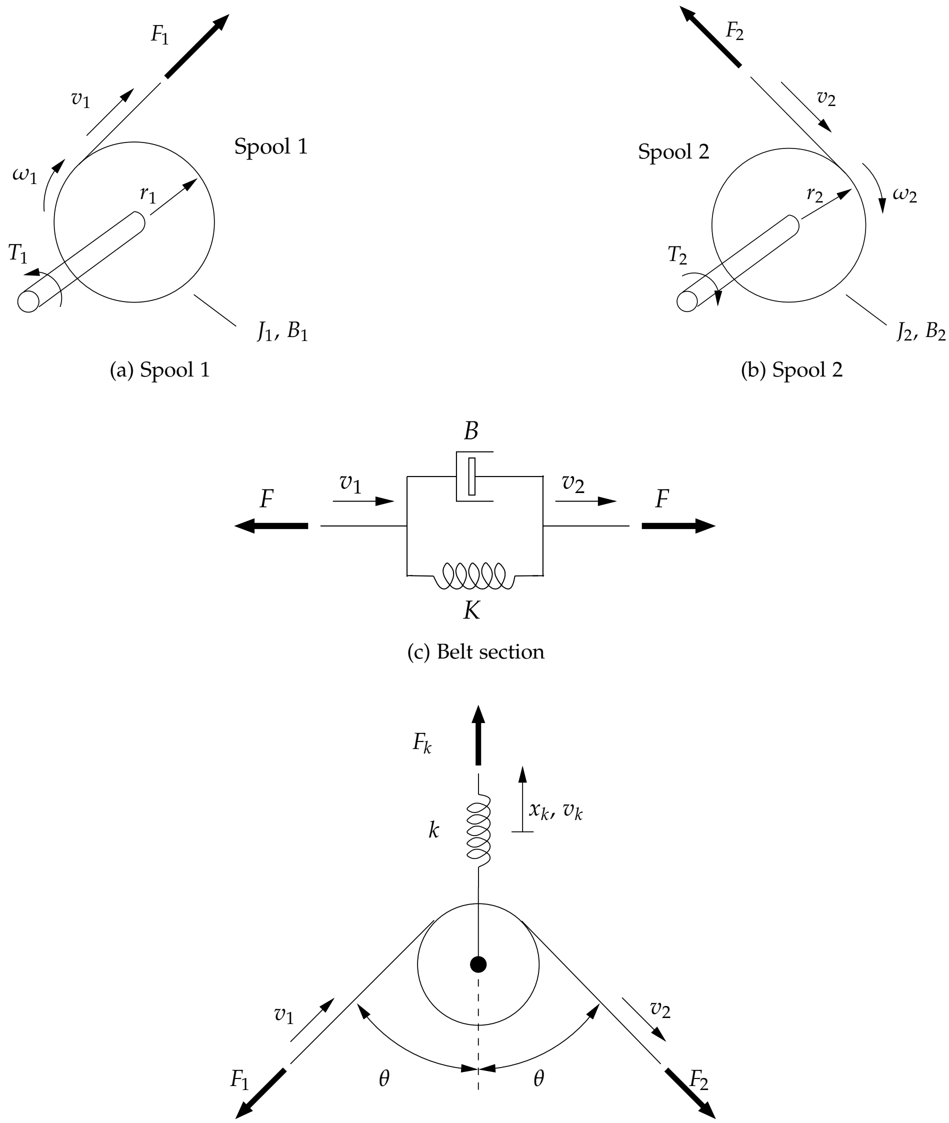


Figure 6: Bode plot from motor torque to pulley spring deflection

(h) (Optional!) Experiment with simulations of the model, for instance by adding a step torque to the torque input of Spool 2. Try to add PI controllers for controlling belt speed and belt tension (as measured by pulley speed and spring deflection, respectively). As this is a highly coupled two-by-two system, this might be somewhat challenging[2].

[2] Adding a "decoupler" by making a input coordinate change may help.



(a) Spool 1

(b) Spool 2

(c) Belt section

(d) Pulley

Figure 2: "Free body diagrams" illustrating forces and positive directions

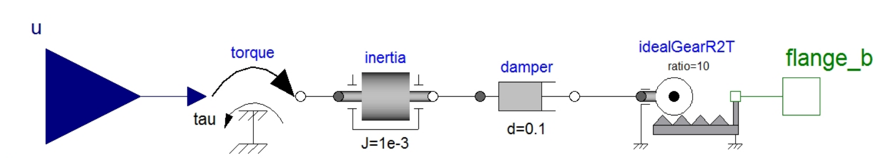


Figure 3: Dymola model of a drive/spool.

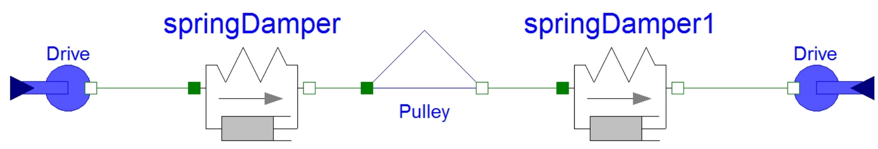


Figure 4: Dymola model of coupled drives

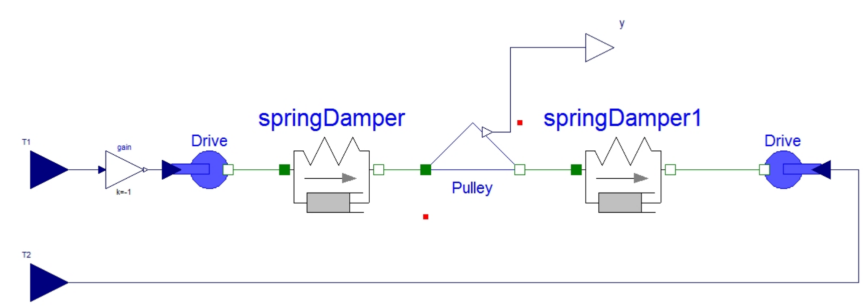


Figure 5: Dymola model of coupled drives with inputs and outputs