TET4100 Kretsanalyse – Løsning

Institutt: Elkraftteknikk Dato: 2012.08.15

Øving 2

4) RLC-Krets 2.A

4.1) $v_0(0)=0 \quad i(0)=0 \quad \frac{d\,v_0(0^+)}{dt}=\frac{i(0^+)}{C}=0 \quad v_0(\infty)=V_s$ = 80

$$0=s^2+s\frac{R}{L}+\frac{1}{LC} \qquad s_{1,2}=-\frac{R}{2\mathrm{L}}\pm\sqrt{\left(\frac{R}{2\mathrm{L}}\right)^2-\frac{1}{LC}}$$

$$R<2\sqrt{\frac{L}{C}} \quad \rightarrow \quad s_{1,2}=-\frac{R}{2\mathrm{L}}\pm j\sqrt{\frac{1}{LC}-\left(\frac{R}{2\mathrm{L}}\right)^2}$$

$\alpha = 4000$, $\omega_d = 2000$

$$\alpha=\frac{R}{2\mathrm{L}} \qquad \omega_d=\sqrt{\frac{1}{LC}-\left(\frac{R}{2\mathrm{L}}\right)^2}$$

standard ligning: $v_0(t)=v_0(\infty)+\left(B_1\cdot\cos(\omega_d t)+B_2\cdot\sin(\omega_d t)\right)e^{-\alpha t}$

$$v_0(0)=V_s+\left(B_1\cdot\cos(0)+B_2\cdot\sin(0)\right)e^0=V_s+B_1$$

$$\frac{d\,v_0(0^+)}{dt}=\left(B_1\left(\omega_d(-\sin(0))+(-\alpha)\cos(0)\right)+B_2\left(\omega_d\cos(0)+(-\alpha)\sin(0)\right)\right)e^0$$

$$\frac{d\,v_0(0^+)}{dt}=B_2\cdot\omega_d-B_1\cdot\alpha$$

$V_s+B_1=0 \qquad B_1=-V_s$ = −80

$B_2\cdot\omega_d-B_1\cdot\alpha=0 \qquad B_2=B_1\cdot\frac{\alpha}{\omega_d}=-V_s\cdot\frac{\alpha}{\omega_d}$ = −160

$$v_0(t)=v_0(\infty)+\left(B_1\cdot\cos(\omega_d t)+B_2\cdot\sin(\omega_d t)\right)e^{-\alpha t}$$

4.2) $$v_0(t)=v_0(\infty)+\left(B_1\cdot\cos(\omega_d t)+B_2\cdot\sin(\omega_d t)\right)e^{-\alpha t}$$

$$Finn\ \ max\ \ \rightarrow\ \ \frac{d\,v_0(t_{max})}{d\,t_{max}}=0$$

$$\frac{d\,v_0(t_{max})}{d\,t_{max}}=\left(-B_1\alpha\cos(\omega_d t_{max})-B_1\omega_d\sin(\omega_d t_{max})\right)e^{-\alpha t_{max}}$$

$$+\left(-B_2\alpha\sin(\omega_d t_{max})+B_2\omega_d\cos(\omega_d t_{max})\right)e^{-\alpha t_{max}}=0$$

$$-B_1\alpha\cos(\omega_d t_{max})-B_1\omega_d\sin(\omega_d t_{max})-B_2\alpha\sin(\omega_d t_{max})+B_2\omega_d\cos(\omega_d t_{max})=0$$

$$B_2=B_1\cdot\frac{\alpha}{\omega_d}$$

$$-B_1\alpha\cos(\omega_d t_{max})-B_1\omega_d\sin(\omega_d t_{max})-B_1\cdot\frac{\alpha^2}{\omega_d}\sin(\omega_d t_{max})+B_1\cdot\alpha\cos(\omega_d t_{max})=0$$

$$\alpha\cos(\omega_d t_{max})+\omega_d\sin(\omega_d t_{max})+\frac{\alpha^2}{\omega_d}\sin(\omega_d t_{max})-\alpha\cos(\omega_d t_{max})=0$$

$$\omega_d\sin(\omega_d t_{max})+\frac{\alpha^2}{\omega_d}\sin(\omega_d t_{max})=0$$

$$(\omega_d^2+\alpha^2)\sin(\omega_d t_{max})=0$$

$$\sin(\omega_d t_{max})=0$$

$$t\geq 0^+$$

$$t_{max}=0\quad ingen\,løsning$$

$$t_{max}=\frac{\pi}{\omega_d}=157\,ms$$

$$v_0(t_{max})=v_0(\infty)+\left(B_1\cdot\cos(\omega_d t_{max})+B_2\cdot\sin(\omega_d t_{max})\right)e^{-\alpha t_{max}}$$

$$v_0(t_{max})=V_s\left(1+e^{-\alpha t_{max}}\right)\approx 80{,}15\text{ V}$$

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5) RLC-Krets 2.B

5.1) $G=\frac{1}{R}$

Parallell RLC $\rightarrow$ $s_{1,2}=-\frac{G}{2C}\pm\sqrt{\left(\frac{G}{2C}\right)^2-\frac{1}{LC}}$

$G=2\sqrt{\frac{C}{L}}$ $\rightarrow$ $s_1=s_2=-\frac{G}{2C}$ $= -10^4$

$\rightarrow$ *Kritisk dempet* $\rightarrow v_C(t)=(D_1\cdot t+D_2)e^{-\alpha t}$ $\alpha=-s$ $= 10^4$

$v_C(0^+)=v_C(0)$ $i_L(0^+)=i_L(0)$

$$\frac{d\,v_C(0^+)}{dt}=\frac{i_C(0^+)}{C}=\frac{I_{inn}-i_L(0^+)-i_R(0^+)}{C}=\frac{I_{inn}-i_L(0^+)-G\cdot v_C(0^+)}{C}$$

$v_C(0^+)=(D_1\cdot 0+D_2)e^0=D_2$ $= 15\text{ V}$

$$\frac{d\,v_C(0^+)}{dt}=\left(D_1(0\cdot(-\alpha)+1)+D_2\cdot(-\alpha)\right)e^0=D_1-\alpha D_2$$

$$D_2=v_C(0) \qquad D_1=\frac{I_{inn}-i_L(0)-G\cdot v_C(0)}{C}+\alpha\cdot v_C(0) = 37{,}5\cdot 10^4$$

$$i_R(t)=\frac{v_C(t)}{R}=\left(\frac{D_1}{R}\cdot t+\frac{D_2}{R}\right)e^{-\alpha t}$$

5.2) $i_C(t)=C\cdot\frac{d\,v_C}{dt}$

$$v_C(t)=(D_1\cdot t+D_2)e^{-\alpha t}$$

$$\frac{d\,v_C}{dt}=\left((-\alpha D_1)t+(D_1-\alpha D_2)\right)e^{-\alpha t}$$

$$i_C(t)=\left((-\alpha C D_1)t+(C D_1-\alpha C D_2)\right)\cdot e^{-\alpha t}$$

5.3) $i_L(t)=I_{inn}-i_C(t)-i_R(t)=I_{inn}-\left(\left(\frac{D_1}{R}-\alpha C D_1\right)t+\left(C D_1-\alpha C D_2+\frac{D_2}{R}\right)\right)e^{-\alpha t}$

5.4) $v_C(t)=\left(D_1\cdot t+D_2\right)e^{-\alpha t}$

$$Finn\ \ max \rightarrow \frac{d\,v_C(t_{max})}{d\,t_{max}}=0$$

$$\frac{d\,v_C(t_{max})}{d\,t_{max}}=\left(\left(D_1-\alpha D_2\right)-\left(\alpha D_1\right)t_{max}\right)e^{-\alpha t_{max}}=0$$

$$\left(D_1-\alpha D_2\right)-\left(\alpha D_1\right)t_{max}=0$$

$$t_{max}=\frac{D_1-\alpha D_2}{\alpha D_1} = 60\ \mu s$$

$$v_C(t)=\left(D_1\cdot t+D_2\right)e^{-\alpha t}$$

$$v_C(t_{max})=\left(D_1\frac{D_1-\alpha D_2}{\alpha D_1}+D_2\right)e^{-\alpha\frac{D_1-\alpha D_2}{\alpha D_1}}$$

$$v_C(t_{max})=\frac{D_1}{\alpha}e^{-\frac{D_1-\alpha D_2}{D_1}} = 20{,}6\ V$$

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6) RLC-Krets 2.C

6.1) $v_C(0^-)=V_s \quad i_C(0^-)=0 \quad i_L(0^-)=0 \quad i_R(0^-)=0$

6.2) $v_C(0^+)=V_s \quad i_R(0^+)=\dfrac{v_C(0^+)}{R} \quad i_L(0^+)=0 \quad i_C(0^+)=-i_R(0^+)$

6.3) $G=\dfrac{1}{R}$

$$\text{Parallell } RLC \rightarrow s_{1,2}=\underbrace{-\frac{G}{2C}}_{=-5000}\pm\underbrace{\sqrt{\left(\frac{G}{2C}\right)^2-\frac{1}{LC}}}_{=\pm 3000}$$

$s_1=-2000$

$s_2=-8000$

$$G>2\sqrt{\frac{C}{L}} \quad \rightarrow Overdempet \rightarrow v_C(t)=A_1\cdot e^{s_1 t}+A_2\cdot e^{s_2 t}$$

$$v_C(0^+)=V_s \qquad \frac{d\,v_C(0^+)}{dt}=\frac{i_C(0^+)}{C}=\frac{-V_s}{RC}$$

$$v_C(0^+)=A_1\cdot e^0+A_2\cdot e^0=A_1+A_2$$

$$\frac{d\,v_C(0^+)}{dt}=A_1\cdot s_1\cdot e^0+A_2\cdot s_2\cdot e^0=A_1\cdot s_1+A_2\cdot s_2$$

$$A_1+A_2=V_s \qquad A_1\cdot s_1+A_2\cdot s_2=\frac{-V_s}{RC}$$

$$A_1\cdot s_1+(V_s-A_1)\cdot s_2=\frac{-V_s}{RC} \qquad A_1=-V_s\cdot\frac{\frac{1}{RC}+s_2}{s_1-s_2} = -33{,}33\ V$$

$$A_2=V_s-A_1=V_s\cdot\left(1+\frac{\frac{1}{RC}+s_2}{s_1-s_2}\right) = 133{,}33\ V$$

$$v_C(t)=A_1\cdot e^{s_1 t}+A_2\cdot e^{s_2 t} = -33{,}33\,e^{-2000t}+133{,}33\,e^{-8000t}$$

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6.4) $v_C(t)=A_1\cdot e^{s_1 t}+A_2\cdot e^{s_2 t}$

$$Finn\ \ min \quad\rightarrow\quad \frac{d\,v_C(t_{min})}{d\,t_{min}}=0$$

$$\frac{d\,v_C(t_{min})}{d\,t_{min}}=A_1 s_1 e^{s_1 t_{min}}+A_2 s_2 e^{s_2 t_{min}}=0$$

$$A_1 s_1 e^{s_1 t_{min}}=-A_2 s_2 e^{s_2 t_{min}}$$

$$\frac{A_1 s_1 e^{s_1 t_{min}}}{A_2 s_2 e^{s_2 t_{min}}}=-1$$

$$\frac{A_1 s_1}{A_2 s_2}e^{(s_1-s_2)t_{min}}=-1$$

$$e^{(s_1-s_2)t_{min}}=-\frac{A_2 s_2}{A_1 s_1}$$

$$(s_1-s_2)t_{min}=\ln\left(-\frac{A_2 s_2}{A_1 s_1}\right)$$

$$t_{min}=\frac{1}{(s_1-s_2)}\ln\left(-\frac{A_2 s_2}{A_1 s_1}\right) = 462\ \mu s$$

$$v_C(t_{min})=A_1\cdot e^{s_1 t_{min}}+A_2\cdot e^{s_2 t_{min}}$$

$$v_C(t_{min})=A_1\cdot e^{s_1\left(\frac{1}{(s_1-s_2)}\ln\left(-\frac{A_2 s_2}{A_1 s_1}\right)\right)}+A_2\cdot e^{s_2\left(\frac{1}{(s_1-s_2)}\ln\left(-\frac{A_2 s_2}{A_1 s_1}\right)\right)}$$

$$v_C(t_{min})=A_1\cdot\left(-\frac{A_2 s_2}{A_1 s_1}\right)^{\left(\frac{s_1}{s_1-s_2}\right)}+A_2\cdot\left(-\frac{A_2 s_2}{A_1 s_1}\right)^{\left(\frac{s_2}{s_1-s_2}\right)} \approx 10\ V$$

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7) RLC-Krets 2.D

7.1) Seriel $RLC \rightarrow s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$$R < 2\sqrt{\frac{L}{C}} \quad \rightarrow \quad \textit{Underdempet}$$

$$s_{1,2} = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$\alpha = 6000$, $\omega_d = 8000$

$$\alpha = \frac{R}{2L} \qquad \omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$i(t) = \left(B_1 \cdot \cos(\omega_d t) + B_2 \cdot \sin(\omega_d t)\right) e^{-\alpha t}$$

$$i(0^+) = 0 \qquad \frac{d\,i(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{v_C(0^+)}{L} = \frac{V_s}{L}$$

$$i(0^+) = \left(B_1 \cdot \cos(0) + B_2 \cdot \sin(0)\right) e^0 = B_1 = 0$$

$$\rightarrow i(t) = B_2 \cdot e^{-\alpha t} \cdot \sin(\omega_d t)$$

$$\frac{d\,i(0^+)}{dt} = B_2\left(e^0 \cdot \omega_d \cdot \cos(0) - \alpha \cdot e^0 \cdot \sin(0)\right) = B_2 \cdot \omega_d$$

$$B_2 = \frac{1}{\omega_d} \cdot \frac{d\,i(0^+)}{dt} = \frac{V_s}{\omega_d L} \quad = 0{,}75$$

$$i(t) = B_2 \cdot e^{-\alpha t} \cdot \sin(\omega_d t)$$

$$v_L(t) = L\frac{d\,i(t)}{dt} = \left(\omega_d \cdot \cos(\omega_d t) - \alpha \cdot \sin(\omega_d t)\right) L \cdot B_2 \cdot e^{-\alpha t}$$

$$v_C(t) = v_R(t) + v_L(t) = R \cdot i_R(t) + v_L(t)$$

$$v_C(t) = \left((R \cdot B_2 - L \cdot B_2 \cdot \alpha)\sin(\omega_d t) + L \cdot B_2 \cdot \omega_d \cdot \cos(\omega_d t)\right) e^{-\alpha t}$$

7.2) *Kritisk dempet* $\rightarrow \qquad R = 2\sqrt{\frac{L_{Krit}}{C}}$

$$L_{Krit} = \frac{R^2 C}{4}$$

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7.3) $i_{Krit}(t)=(D_1\cdot t+D_2)e^{-\alpha_{Krit}t}$

$$\alpha_{Krit}=\frac{R}{2L_{Krit}} = 1{,}67\cdot 10^4$$

$$i_{Krit}(0^+)=0 \qquad \frac{d\,i_{Krit}(0^+)}{dt}=\frac{v_L(0^+)}{L_{Krit}}=\frac{v_C(0^+)}{L_{Krit}}=\frac{V_s}{L_{Krit}}$$

$$i_{Krit}(0^+)=D_1\cdot 0\cdot e^0+D_2\cdot e^0=D_2=0$$

$$\rightarrow i_{Krit}(t)=D_1\cdot t\cdot e^{-\alpha_{Krit}t}$$

$$\frac{d\,i_{Krit}(0^+)}{dt}=D_1\left(0\cdot(-\alpha_{Krit})\cdot e^0+1\cdot e^0\right)=D_1=\frac{V_s}{L_{Krit}} = 1{,}67\cdot 10^4$$

$$i_{Krit}(t)=D_1\cdot t\cdot e^{-\alpha_{Krit}t} = 1{,}67\cdot 10^4\cdot t\cdot e^{-1{,}67\cdot 10^4 t}$$