Department of Engineering Cybernetics

TTK4215 System Identification and Adaptive Control
Solution of Assignment 7

Problem 4.8 from I&S

(a) suppose $\omega:\mathcal{R}^{+}\to\mathcal{R}^{n}$, $\omega,\dot{\omega}\in\mathcal{L}_{\infty}$ and ω is PE that is

$$\alpha_0 I \le \frac{1}{T_0}\int_t^{t+T_0}\omega(\tau)\omega^T(\tau)d\tau \le \alpha_1 I \qquad \forall t\ge 0 \tag{1}$$

Then for $n\in\Re$

$$\alpha_0 nT_0 I \le \int_t^{t+nT_0}\omega(\tau)\omega^T(\tau)d\tau \le \alpha_1 nT_0 I \qquad \forall t\ge 0 \tag{2}$$

and $e\in\mathcal{L}_2$ that is $\int_0^t |e(\tau)|d\tau<\infty$. To study PE condition of $\omega_0=\omega+e$, we should form the integral

$$\int_t^{t+nT_0}\left(\omega(\tau)+e(\tau)\right)\left(\omega^T(\tau)+e^T(\tau)\right)d\tau=\int_t^{t+nT_0}\omega(\tau)\omega^T(\tau)d\tau+\int_t^{t+nT_0}e(\tau)e^T(\tau)d\tau+2\int_t^{t+nT_0}e(\tau)\omega^T(\tau)d\tau \tag{3}$$

The first integral is PE condition of ω. For the second integral, we have

$$0\le\int_t^{t+nT_0}e(\tau)e^T(\tau)d\tau\le\gamma I, \tag{4}$$

since $e\in\mathcal{L}_2$. Because Holder's Inequality holds for truncated functions, it is obtained

$$\left|\int_t^{t+nT_0}e(\tau)\omega^T(\tau)d\tau\right|\le\left|\int_t^{t+nT_0}\omega(\tau)\omega^T(\tau)d\tau\right|^{\frac{1}{2}}.\left|\int_t^{t+nT_0}e(\tau)e^T(\tau)d\tau\right|^{\frac{1}{2}}\le\left(\alpha_1 nT_0\gamma I\right)^{\frac{1}{2}} \tag{5}$$

so (3) is bounded from above, what about the lower bound? we have

$$\alpha_0 I-\frac{2}{nT_0}\left(\alpha_1 nT_0\gamma I\right)^{\frac{1}{2}}\le\frac{1}{nT_0}\int_t^{t+nT_0}\omega_0(\tau)\omega_0^T(\tau)d\tau\le\alpha_1 I+\frac{1}{nT_0}\gamma I+\frac{2}{nT_0}\left(\alpha_1 nT_0\gamma I\right)^{\frac{1}{2}} \qquad \forall t\ge 0 \tag{6}$$

The lower bound is

$$\alpha_0 I-\frac{2}{nT_0}\left(\alpha_1 nT_0\gamma I\right)^{\frac{1}{2}} \tag{7}$$

which must be positive. We choose n such that

$$\alpha_0 nT_0 > 2\left(\alpha_1 nT_0\gamma\right)^{\frac{1}{2}} \tag{8}$$

Thus

$$nT_0 = \{4\frac{\alpha_1\sqrt{\gamma}}{\alpha_0^2}\} + \epsilon \tag{9}$$

(b) When $e \in \mathcal{L}_\infty$, we have $\|e\|_\infty = m$; thus

$$0 \le \frac{1}{nT_0}\int_t^{t+nT_0} e(\tau)e^T(\tau)d\tau \le c^2 \tag{10}$$

Since $e \to 0$ as $t \to \infty$, we can choose n in a way that c becomes arbitrary small

$$\left|\int_t^{t+nT_0} e(\tau)\omega^T(\tau)d\tau\right| \le \left|\int_t^{t+nT_0} \omega(\tau)\omega^T(\tau)d\tau\right|^{\frac{1}{2}} . \left|\int_t^{t+nT_0} e(\tau)e^T(\tau)d\tau\right|^{\frac{1}{2}} \le nT_0\left(\alpha_1 c^2 I\right)^{\frac{1}{2}} \tag{11}$$

Therefore,

$$\int_0^t\left(\omega(\tau)+e(\tau)\right)\left(\omega^T(\tau)+e^T(\tau)\right)d\tau = \int_0^t \omega(\tau)\omega^T(\tau)d\tau + \int_0^t e(\tau)e^T(\tau)d\tau + 2\int_0^t e(\tau)\omega^T(\tau)d\tau \tag{12}$$

Similar to previous case, we have

$$\alpha_0 I - 2c\sqrt{\alpha_1} I \le \frac{1}{nT_0}\int_t^{t+nT_0} \omega_0(\tau)\omega_0{}^T(\tau)d\tau \le \alpha_1 I + c^2 I + 2c\sqrt{\alpha_1} I \qquad \forall t \ge 0 \tag{13}$$

if n is chosen large enough then

$$\frac{\alpha_0}{2\sqrt{\alpha_1}} > c \tag{14}$$

Problem 4.9

a) We know the force imposed by a damper with the damping coefficient β is $\beta\dot{y}$ and the force of a spring with the spring constant k is ky, in which y is displacement. According to Newton's second law we have

$$m\ddot{y} = u - ky - \beta\dot{y} \tag{15}$$

b) The parametric model is

$$\begin{aligned} &u = \theta\psi \\ &\theta^T = [m, \beta, k] \\ &\psi^T = [\ddot{y}, \dot{y}, y] \end{aligned} \tag{16}$$

filtering both sides

$$\begin{aligned} &z = Wu = W\theta\psi \\ &\theta^T = [m, \beta, k] \\ &\Phi = W\psi \\ &\rightarrow z = \theta^T\Phi \end{aligned} \tag{17}$$

The update law is (4.3.52) according to instantaneous cost function, that is

$$\begin{aligned} &\dot{\hat{\theta}} = \Gamma\epsilon\Phi \\ &\Gamma = \Gamma^T > 0 \\ &\epsilon = z - \hat{z} \end{aligned} \tag{18}$$

and the signals with \hat\ represent the estimates.

For c and d)

Look at MATLAB files.

The input signal is chosen PRBS. To read about PRBS signals, type "doc idinput" in MATLAB help or visit

http://www.mathworks.com/help/toolbox/ident/ref/idinput.html

At initialization of the SIMULINK files, three commands are called to generate the PRBS input signals. To see them, right click on "System" block , go to "Model

Properties”, select the tab “Callbacks”. Then choose “InitFcn” to see the command lies.

In fact, since the system is underdamped and the response is very oscillatory, you may be able to identify the system with a single step with amplitude 1. However, if the system was overdamped and for example $\beta = 10$ then a single step signal is not rich enough.

You can do the same for Integral cost function, as well.