NORGES TEKNISK-
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EXAM FOR COURSE TTK 4195
Robots Modeling and Control

Tuesday, May 25, 2009
Time: 09.00-13.00

Allowable aids: D - No printed or written material allowed. NTNU type approved calculator with an empty memory allowed. Language: English. Number of Pages: 4 (+ 4 formula sheet). This exam counts for 100% of the final grade.

This exam consists of 5 exercises each consisting of a number of questions. Each question gives a number of points and a sum of points is 100.

1. **Problem:** Consider the following sequence of rotations

   (a) Rotate by ϕ about the world axis x;

   (b) Rotate by θ about the world axis y;

   (c) Rotate by ψ about the current axis z;

   (d) Rotate by α about the world axis y

   Write the matrix product that will give the resulting rotation matrix (do not perform the matrix multiplication) **(5)**

2. **Problem:** Given three matrices

$$T_1 = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{7}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{7}}{2} & 0 & \frac{1}{2} \end{bmatrix}, \quad T_2 = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}, \quad T_3 = \begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & -1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

   (a) Which of these matrices are orthogonal? **(5)**

   (b) Which of these matrices do represent a rotation? **(5)**

3. **Problem:** A motion of the center of mass of a rigid body with respect to an inertia frame is a rotation

$$R_1(t) = \begin{bmatrix} \cos t & 0 & -\sin t \\ 0 & 1 & 0 \\ \sin t & 0 & \cos t \end{bmatrix}$$

   Find an angular velocity of the motion **(5)**. Compute the vector of angular velocity when the rotation is **(10)**

$$R_2(t) = \begin{bmatrix} \cos t & -(\sin t)\cdot(\cos 2t) & (\sin t)\cdot(\sin 2t) \\ \sin t & (\cos t)\cdot(\cos 2t) & -(\cos t)\cdot(\sin 2t) \\ 0 & \sin 2t & \cos 2t \end{bmatrix}$$

4. **Problem:** Consider a robot depicted on Fig. 1. Derive the forward kinematic equations for this manipulator. In particular, introduce a DH-frame for each of the links of the robots, compute DH-parameters of homogeneous transforms between consecutive frames, presenting the solution of the problem do not perform multiplication of such transforms. **(20)**

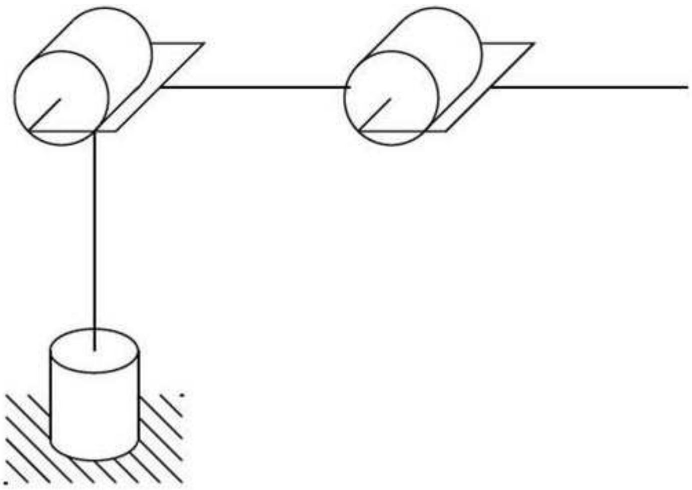

Figure 1: A robot from the problem 4: elbow manipulator

5. **Problem:** The pendulum on the cart system is depicted on Fig. 2. It has two degrees of freedom: the position x of center of mass of the cart on the horizontal and the angle θ of the pendulum with vertical counted anticlockwise. It is assumed that the rod of the pendulum is massless and the mass of the pendulum is all concentrated in the bob. While the only one control variable in the system is the force applied to the cart along the horizontal direction.

   Given physical parameters of the system (mass of the cart M, the mass of the bob m, the distance l from the suspension to the position of the bob), you are requested to implement the following tasks:

   (a) Find the potential energy $\mathcal{P}(x, \theta)$ of the robot **(5)**

   (b) Find the kinetic energy $\mathcal{K}(x, \theta, \dot{x}, \dot{\theta})$ of the robot **(10)**

   (c) Obtain the Euler-Lagrange equations of the system dynamics **(15)**

   (d) Assume that $M = m = 1$ [kg], $l = 1$ [m]. Compute the linearization of the system dynamics around the upright equilibrium $\{x_{ue} = 0, \theta_{ue} = 0\}$. Check if the resulted linear control system is controllable. **(20)**

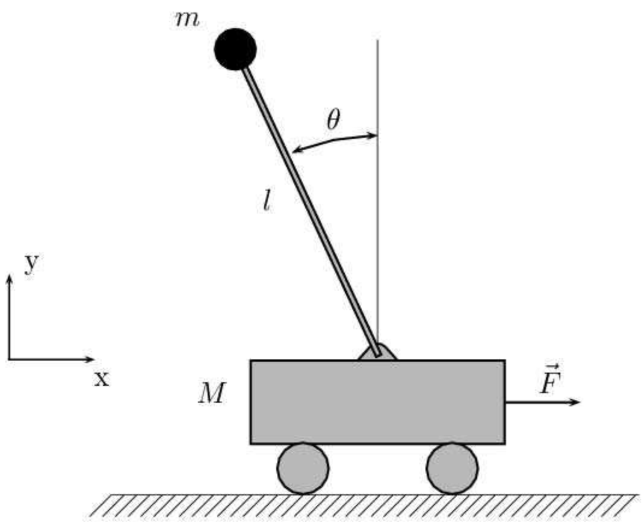

Figure 2: A robot from the problem 5: the cart-pendulum system