TTK4195
Modeling and Control
of Robots
Spring 2015

Exercise set 4

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The following problems are found in the physical book: "Robot Modeling and Control" (2006) by Spong et al.

1. Problem 4-3

Prove the assertion given in Equation (4.9) on page 123 that $R(a \times b) = Ra \times b$, for $R \in SO(3)$. 1.5 Points.

$$R(a \times b) = Ra \times Rb \tag{4.9}$$

Hint: Use $R = (r1, r2, r3)$ where $r1, r2, r3$ column vectors, and use rotation matrix properties.

2. Problem 4-4

Verify Equation (4.10) on page 123,

$$X^T SX = 0 \tag{4.10}$$

for an $n \times n$ skew symmetric matrix S and any vector $X \in \mathbb{R}^n$.
1.5 Points.

3. Problem 4-5

Verify Equation (4.17) from Example 4.2 by direct calculation.

$$\frac{d}{d\theta} R_{y,\theta} = S(j)R_{y,\theta} \quad \text{and} \quad \frac{d}{d\theta} R_{z,\theta} = S(k)R_{z,\theta} \tag{4.17}$$

1.5 Points.

4. Problem 4-6

Suppose that $a = [1, -1, 2]^T$ and that $R = R_{x,90}$. Show by direct calculation that $RS(a)R^T = S(Ra)$. 1.5 Points.

5. Problem 4-8

Use Equation (2.43) on page 58

$$R_{k,\theta} = \begin{bmatrix} k_x^2 v_\theta + c_\theta & k_x k_y v_\theta - k_z s_\theta & k_x k_z v_\theta + k_y s_\theta \\ k_x k_y v_\theta + k_z s_\theta & k_y^2 v_\theta + c_\theta & k_y k_z v_\theta - k_x s_\theta \\ k_x k_z v_\theta - k_y s_\theta & k_y k_z v_\theta - k_x s_\theta & k_z^2 v_\theta + c_\theta \end{bmatrix} \tag{2.43}$$

to show that

$$R_{k,\theta} = I + S(k)\sin(\theta) + S^2(k)\mathrm{vers}(\theta)$$

1 Point.

6. Problem 4-13

Given the Euler angle transformation $R = R_{z,\phi}R_{y,\theta}R_{z,\psi}$. Show that $dR/dt = S(\omega)R$ where

$$\omega = \{c_\psi s_\theta \dot{\phi} - s_\psi \dot{\theta}\}i + \{s_\psi s_\theta \dot{\phi} + c_\psi \dot{\theta}\}j + \{\dot{\psi} + c_\theta \dot{\phi}\}k$$

The components of i, j, k, respectively, are called the **nutation, spin** and **precession**.
2 Points.

7. Problem 4-15

Two frames $o_0x_0y_0z_0$ and $o_1x_1y_1z_1$ are related by the homogeneous transformation

$$H = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A particle has velocity $v_1(t) = [3, 1, 0]^T$ relative to frame $o_1x_1y_1z_1$. What is the velocity of the particle in frame $o_0x_0y_0z_0$?
1 Point.