TTK4195
Modeling and Control of Robots
Assignment 8

Solution

March 26, 2014

Problem 8-12

a)

The state space is of dimension 4.

b)

Choose state and control variables

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} y_1 \\ \dot{y}_1 \\ y_2 \\ \dot{y}_2 \end{bmatrix}$$
$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

then

$$\begin{aligned} \dot{\mathbf{x}} &= \begin{bmatrix} x_2 \\ -3x_1x_3 - x_3^2 \\ x_4 \\ -x_4\cos(x_1) - 3(x_1 - x_3) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & x_3 \\ 0 & 0 \\ -3x_3\cos^2(x_1) & 1 \end{bmatrix} \mathbf{u} \\ &= \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \end{aligned}$$

c)

The inverse dynamics control must be

$$\mathbf{u} = \mathbf{g}^{-1}(\mathbf{x})[\mathbf{v} - \mathbf{f}(\mathbf{x})]$$

where $\mathbf{g^{-1}}(\mathbf{x})$ is the pseudo inverse of $\mathbf{g}(\mathbf{x})$ and $\mathbf{v}$ is the desired closed loop linear system. In order to decouple the resulting system, we must find a suitable control signal $\mathbf{v}$. Following the symbolic multiplication for $\mathbf{u}$, we can choose $\mathbf{v}$ so that

$$\mathbf{u} = \frac{1}{1 + x_3^2 \cos^2(x_1)} \begin{bmatrix} 3x_1x_3 + x_3^2 - x_3x_4\cos(x_1) - 3x_1x_3 + 3x_3^2 + \nu_1 - x_3\nu_2 \\ 3x_1x_3^2\cos^2(x_1) + x_3^2\cos^2(x_1) + x_4\cos(x_1) + 3(x_1 - x_3) + x_3\cos^2(x_1)\nu_1 + \nu_2 \end{bmatrix}$$

where

$$\begin{aligned} \nu_1 &= -10x_2 - 100x_1 + r_1 \\ \nu_2 &= -10x_4 - 100x_3 + r_2 \end{aligned}$$

d)

The system can be linearized by a first order Taylor approximation as follows

$$\dot{\mathbf{x}} \approx \mathbf{f}(\mathbf{x}^*) + \mathbf{g}(\mathbf{x}^*)\mathbf{u}^* + \frac{\partial(\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u})}{\partial \mathbf{x}}|_{x=x_d,u=u_d} \cdot (x - x_d) + \frac{\partial(\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u})}{\partial \mathbf{u}}|_{x=x_d,u=u_d} \cdot (u - u_d)$$

$$\Downarrow$$

$$\dot{\mathbf{z}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -3x_{3d} & 0 & -3x_{1d} - 2x_{3d} + u_{2d} & 0 \\ 0 & 0 & 0 & 1 \\ x_{4d}\sin(x_{1d}) + 3\sin^2(x_{1d})u_{1d} & 0 & 3 & -\cos(x_{1d}) \end{bmatrix} \mathbf{z} + \begin{bmatrix} 0 & 0 \\ 1 & x_{3d} \\ 0 & 0 \\ -3x_{3d}\cos^2(x_{1d}) & 0 \end{bmatrix} \omega$$

Simulation Problem

See simulation files provided together with this solution set.