Hand-in date: April 14th, 23:00

Ball-pitching Challenge

Problem Description

Figure 1 shows a two-link underactuated robot that can be used to simulate a dynamic throwing motion. The robot has actuation in the shoulder via a control torque τ_1, while the elbow is a passive, spring-articulated joint with stiffness k_2. Both joints are revolute and motion is restricted to the xy-plane. The joint angles q_1, q_2 are measured with respect to the x-axis as indicated in the figure.

The robot is fitted with a gripping tool that secures a ball with mass m_b. When the end effector of the robot crosses the vertical line defined by the y-axis ($x_b = 0$), the ball is assumed to experience a friction-less release from the tool followed by an unconstrained ballistic flight phase. The initial velocity v_e of the ball at release is considered to be orthogonal to the second link of the robot, as shown in Figure 2.

Tasks

1. Derive the Euler-Lagrange equations for the underactuated ball-pitching robot depicted in Figure 1. **(4 Points)**

   Hints:

   - The system consists of two separate phases; a ball-pitching phase and a ballistic flight phase. The ball-pitching phase can be modeled according to the Euler-Lagrange method introduced in the textbook (Spong, 2006) with some minor adjustments due to the torsional spring between joints.
   - The torsional spring stores potential energy according to Hooke's law, there are thus two potential fields acting on the system. There is no tension in the spring when the first and second link are parallel ($q_1 = q_2$) as indicated by the red, dashed line in Figure 1.
   - During the ballistic flight phase, assume that no external forces other than gravity acts on the ball. The ball can be considered a point mass and will follow a standard parabolic trajectory.

2. Use the Euler-Lagrange equations of motions derived in the previous task and create a MATLAB/Simulink model that simulates the ball-pitching challenge. **(4 Points)**

   Your model should contain an instantaneous transition between the previously discussed ball-pitching phase and ballistic flight phase such that a complete throw can be simulated. Physical parameters can be found in Table 1.

3. Search for a control torque $\tau_1(t)$ such that the ball is launched as far as possible (measured along the x-axis). Design and implementation of a controller for this purpose is not needed, and your chosen control torque can be applied in open-loop. The control torque must obey the limitations of the robot stated in Table 2, and should ensure that q_1 is monotonically increasing along the motion. Use the following initial conditions

$$q_1 = \frac{5\pi}{6}, \quad q_2 = \pi + \arcsin\left(\frac{l_1}{2l_2}\right), \quad \dot{q}_1 = \dot{q}_2 = 0$$

   (2 Points) The strategies for finding control torques are many and will not be discussed here. The goal of this task is to encourage the student to experiment with the system and try to shape interesting trajectories.

   The student that it is capable of reaching the greatest distance with his/her throw, will be given a pizza as a prize! To participate in this competition, you must submit executable simulation files and a screenshot of the distance of your throw.

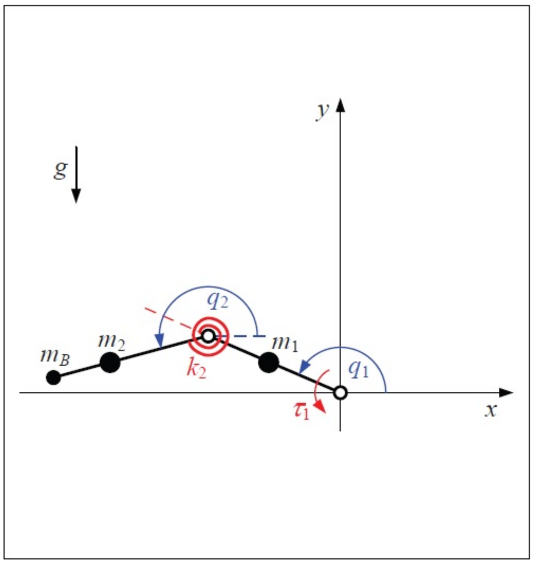

Figure 1: Underactuated two-link robot arm with a ball secured at the end of the second link.

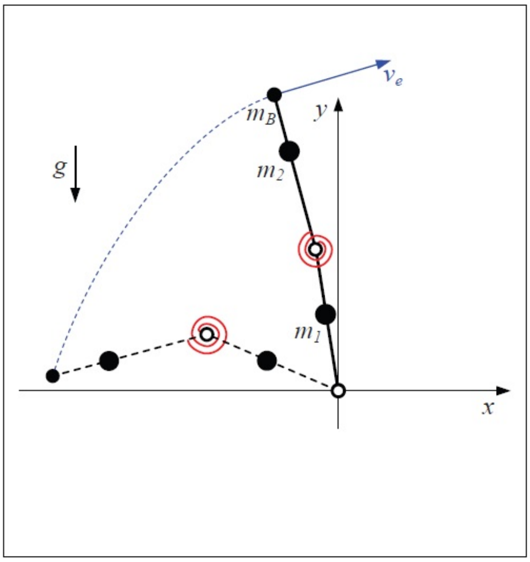

Figure 2: Example trajectory from initial configuration towards ball release. Note that the velocity of the ball v_e is orthogonal to the second link throughout the pitching motion.

Parameter	First Link	Second Link
Length [m]	$l_1 = 0.3$	$l_2 = 0.542$
Mass [kg]	$m_1 = 2.934$	$m_2 = 1.1022$
Distance to CoM [m]	$l_{1c} = 0.2071$	$l_{2c} = 0.2717$
Inertia about joint [kg m^2]	$I_1 = 0.2067$	$I_2 = 0.1362$
Elbow spring constant	$k_2 = 14.1543\text{Nm/rad}$	
Ball mass	$m_B = 0.064\text{kg}$	
Gravitational constant	$g = 9.81\text{m/s}^2$	

Table 1: Parameters of the two-link robot arm with spring-articulated elbow joint. The parameters belong to an experimental setup at the German Aerospace Center.

Quantity	Limitation
Torque [Nm]	$\lvert\tau_1\rvert \leq 180$
Velocity [rad/s]	$\lvert\dot{q}_1\rvert \leq 3.787$
Power [Nm/s]	$\lvert P_1\rvert = \lvert\tau_1 \dot{q}_1\rvert \leq 270$

Table 2: Actuation constraints at the shoulder joint. The parameters belong to an experimental setup at the German Aerospace Center.