TTK4195
Modeling and Control of Robots
Assignment 5

Department of Engineering Cybernetics
NTNU

February 27, 2014

Solution

Problem 4-18

Compute the Jacobian J_{11} for the 3-link spherical manipulator of Example 4.10.

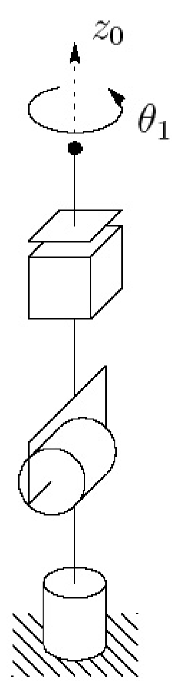

Figure 4.9: Singularity of spherical manipulator with no offsets.

First we need the forward kinematics for the manipulator.

Link	a_i	α_i	d_i	θ_i
1	0	$-\frac{\pi}{2}$	d_1	θ_1
2	0	$\frac{\pi}{2}$	0	θ_2
3	0	0	d_3	0

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \; A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^0 = A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1 A_2 = \begin{bmatrix} c_1 c_2 & -s_1 & c_1 s_2 & 0 \\ s_1 c_2 & c_1 & s_1 s_2 & 0 \\ -s_2 & 0 & c_2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 c_2 & -s_1 & c_1 s_2 & d_3 c_1 s_2 \\ s_1 c_2 & c_1 & s_1 s_2 & d_3 s_1 s_2 \\ -s_2 & 0 & c_2 & d_1 + d_3 c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From the forward kinematics, we can find the needed vectors to form the geometric Jacobian.

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \; o_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, \; o_2 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, \; o_3 = \begin{bmatrix} d_3 c_1 s_2 \\ d_3 s_1 s_2 \\ d_1 + d_3 c_2 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; z_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}, \; z_2 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix}$$

We were asked to find the upper left part of the Jacobian. According to the procedure on page 133, this will be:

$$J_{11} = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 \end{bmatrix} = \begin{bmatrix} -d_3 s_1 s_2 & -d_3 c_1 c_2 & c_1 s_2 \\ d_3 c_1 s_2 & d_3 s_1 c_2 & s_1 s_2 \\ 0 & -d_3 s_2 & c_2 \end{bmatrix}$$

Problem 4-19

Use Equation (4.102) to show that the singularities of the SCARA manipulator are given by Equation (4.104).

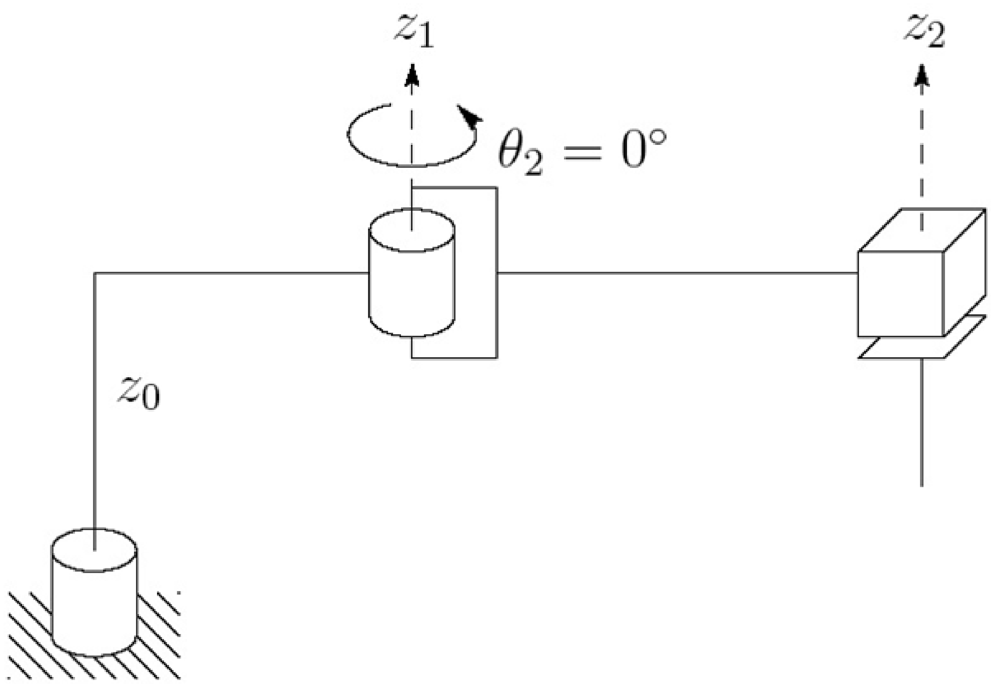

Figure 4.10: SCARA manipulator singularity.

The singularities are given by $\alpha_1\alpha_4 - \alpha_2\alpha_3 = 0$

For this particular manipulator, we have

$\alpha_1\alpha_4 - \alpha_2\alpha_3 = (-a_1s_1 - a_2s_{12})(a_1c_{12}) + (a_1s_{12})(a_1c_1 + a_2c_{12}) = a_1^2s_2$

It follows that s_2 must be 0. Hence, θ_2 must be either 0 or π.

Problem 4-20

Find the 6×3 Jacobian for the 3 links of the cylindrical manipulator of Figure 3.7. Find the singular configurations for this arm.

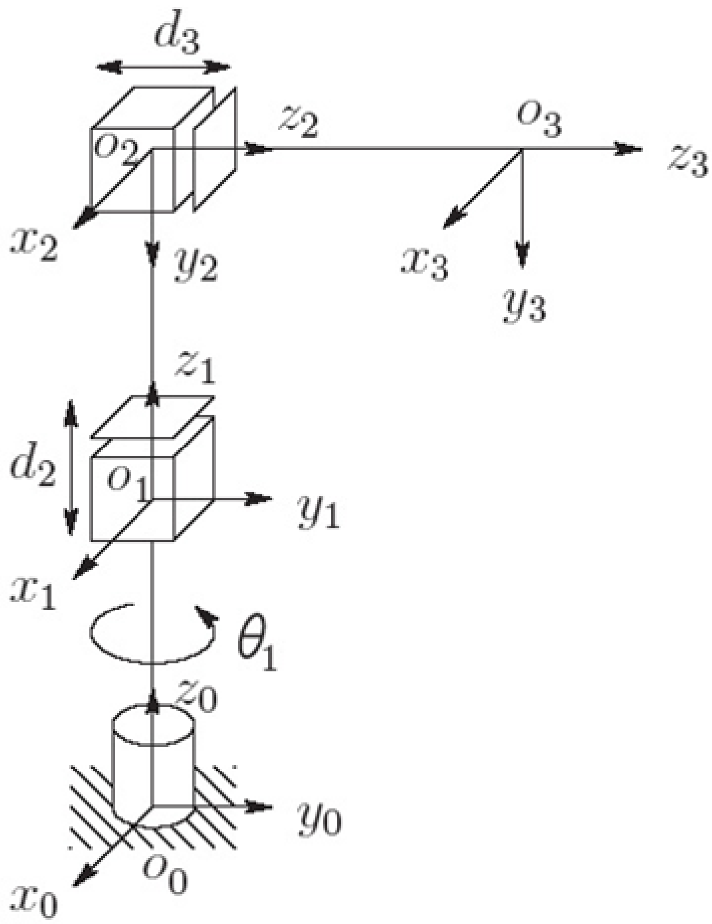

Figure 3.7: Three-link cylindrical manipulator.

The forward kinematics for the cylindrical manipulator is given on page 86 in the textbook.

$$T_1^0 = A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; T_2^0 = A_1 A_2 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1 A_2 A_3 = \begin{bmatrix} c_1 & 0 & -s_1 & -d_3 s_1 \\ s_1 & 0 & c_1 & d_3 c_1 \\ 0 & -1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \; o_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}, \; o_2 = \begin{bmatrix} 0 \\ 0 \\ d_1 + d_2 \end{bmatrix}, \; o_3 = \begin{bmatrix} -d_3 s_1 \\ d_3 c_1 \\ d_1 + d_2 \end{bmatrix}$$

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; z_2 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}$$

From page 133 in the textbook, we know how to find the geometric Jacobian.

$$J = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 & z_2 \\ z_0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -d_3 c_1 & 0 & -s_1 \\ -d_3 s_1 & 0 & c_1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

The rank of J will at most be rank $J = \min(6, 3) = 3$.

We have $\det J_v = d_3 {s_1}^2 + d_3 {c_1}^2 = d_3$. As long as $d_3 \neq 0$ we can achieve arbitrary linear velocity. Since $\det J_\omega = 0$, arbitrary angular velocity is not possible. This can be seen from:

$$\xi = \begin{bmatrix} v_3^0 \\ \omega_3^0 \end{bmatrix} = J\dot{q} = \begin{bmatrix} -d_3 c_1 \dot{\theta}_1 - \dot{d}_3 s_1 \\ -d_3 s_1 \dot{\theta}_1 + \dot{d}_3 c_1 \\ \dot{d}_2 \\ 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

Only angular velocity in the $x_0 y_0$-plane is possible.

Problem 4-25

Suppose that $\dot{q}$ is a solution to Equation (4.110) for $m < n$.

1. Show that $\dot{q} + (I - J^+J)b$ is also a solution to Equation (4.110) for any $b \in \mathbb{R}^n$.

$$\begin{aligned}
\xi &= J\dot{q} \\
&= J(J^+\xi + (I - J^+J)b) \\
&= JJ^+\xi + (J - JJ^+J)b \\
&= I\xi + (J - IJ)b \\
&= \xi
\end{aligned}$$

2. Show that $b = 0$ gives the solution that minimizes the resulting joint velocities.

$$\|\dot{q}\| = \|J^+\xi + (I - J^+J)b\|$$

By the triangle inequality, we have

$$\begin{aligned}
\|\dot{q}\| &\leq \|J^+\xi\| + \|(I - J^+J)b\| \\
&= \|J^+\xi\| + \|(I - J^+J)\|\|b\|
\end{aligned}$$

Since $\|(I - J^+J)\| \geq 0$, choosing $b = 0$ minimizes $\|\dot{q}\|$.