Department of Engineering Cybernetics

Examination paper for TTK4195-Modeling and Control of Robots

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Examination time (from-to): 09.00-13.00

Other information: This exam counts for 100% of the final grade. It consists of 3 exercises each comprising a number of questions. Each question gives a number of points and the sum of points is 100.

Language: English

Number of pages (front page excluded): 3

Number of pages enclosed: 4

Informasjon om trykking av eksamensoppgave

Originalen er:

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Students will find the examination results in Studentweb. Please contact the department if you have questions about your results. The Examinations Office will not be able to answer this.

1. Suppose a rigid body with a fixed point rotates in the inertial frame such that its vector of angular velocity is aligned all the time with the vector

$$\vec{e}(t) = \left[e_x(t);\ e_y(t);\ e_z(t)\right] = \frac{1}{2}\left[\cos t;\ \sin t;\ \sqrt{2}\right]$$

and has a constant amplitude ω_0. Tasks:

   (a) Find the corresponding time evolution of Euler angles, which are used for the ZYZ-**parametrization**

   $$R(t) = R_{\phi(t),z} \cdot R_{\theta(t),y} \cdot R_{\psi(t),z}$$

   of this rotation. Comment the result. **(10)**

   (b) Find the corresponding time evolution of Euler angles, which are used for the ZXZ-**parametrization** (observe the change of parametrization)

   $$R(t) = R_{\phi(t),z} \cdot \underline{R_{\theta(t),x}} \cdot R_{\psi(t),z}$$

   of this rotation. Comment the result. **(5)**

   (c) Find the coordinates of the corresponding vector of the angular velocity of the rotation written in the body frame, i.e. in the frame firmly attached to the rigid body. Comment the result. **(5)**

2. The robot depicted on Fig. 1 consists of a puck (modeled as a point mass

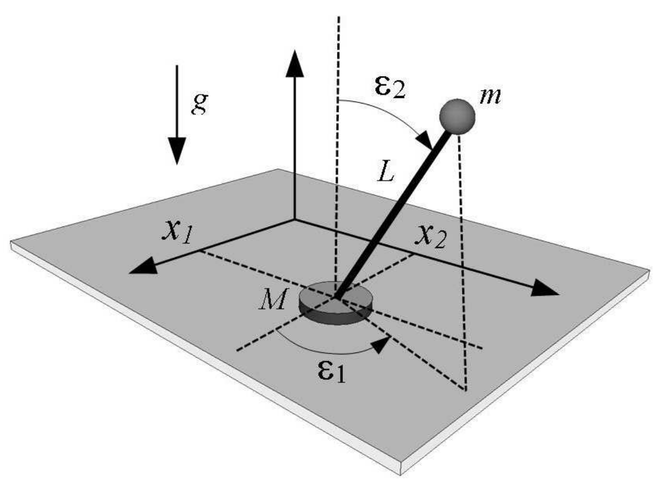

Figure 1: A spherical pendulum on a puck: robot from the problem 2.

of M [kg]), a spherical pendulum (modeled as a point mass of m [kg] on a distance L [m] from the suspension point), which is put on the puck. By assumption, the puck can slide on the horizontal (x_1, x_2)-plane without friction. By assumption, the system has two actuators that form external forces u_1 and u_2 – control inputs – each acting on the puck along the corresponding axes x_1 and x_2. Meanwhile, the dynamics of the pendulum angles ε_1 and ε_2 are assumed to be passive, and the behavior of the pendulum is only defined by acceleration of the pendulum point on the puck and the force due to gravity.

Given such description of the system, you are requested to complete the following tasks:

(a) Find the potential energy $\mathcal{P}(\cdot)$ of the robot. **(10)**

(b) Find the kinetic energy $\mathcal{K}(\cdot)$ of the robot. **(15)**

(c) Obtain the Euler-Lagrange equations of the system dynamics. **(15)**

(d) Derive the linearization of the system dynamics of the robot around the upright equilibrium of the pendulum when the cart is put in the origin of the (x_1, x_2)-plane. Check whether the resulted linear control system is controllable. Comment the arguments used in the derivation of the answer. **(20)**

3. Consider a coin rolling without slipping on the horizontal plane depicted on Fig. 2. Assume that when rolling, the coin is always in contact with the plane at single point (point contact). Then the non-slipping condition for the case study literally means that the instantaneous velocity of the point of the coin's rim, which is in contact with the plane, is zero. Hence, along a motion the projections of the velocity vector of this point on the x-and y-axes satisfy the identities

$$\dot{x}_c(t) - R_c \cdot \dot{\phi}(t) \cdot \cos\theta(t) = 0, \qquad \dot{y}_c(t) - R_c \cdot \dot{\phi}(t) \cdot \sin\theta(t) = 0, \qquad (1)$$

where R_c is the radius of the coin; $x_c(\cdot)$ and $y_c(\cdot)$ are coordinates of the center of the symmetry of the coin. You are requested to complete the following tasks:

(a) Find the basis for the distribution Δ of velocity vectors

$$\left[\dot{x}; \dot{y}; \dot{\theta}; \dot{\phi}\right]$$

defined by the non-slipping constraints written as Eqn. (1). **(5)**;

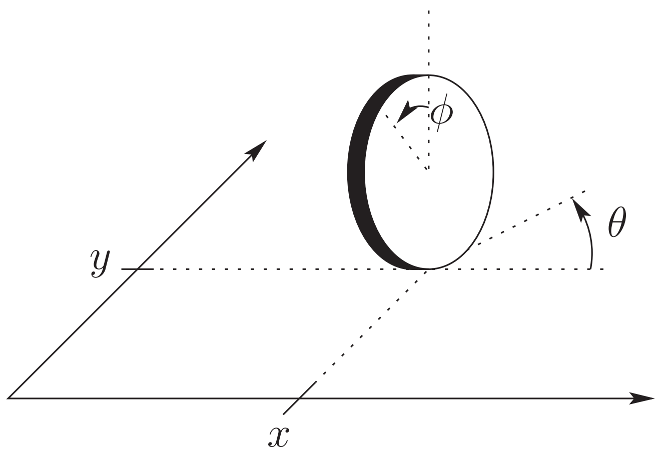

Figure 2: A coin rolling without slipping on the horizontal plane.

(b) Check the integrability conditions of the distribution Δ given by Frobenius theorem, i.e. check whether the distribution has a constant rank and whether the distribution is involutive. **(5)**

(c) Derive the kinetic and potential energies of the coin rolling without slipping on the horizontal plane, assuming that the mass of the coin is equal to m_c, and J_ϕ, J_θ are moments of inertia of the coin about the corresponding axes **(5)**;

(d) Derive the equations of motion of the coin, which rolls without slipping on the horizontal plane **(5)**.