NORGES TEKNISK-
NATURVITENSKAPELIGE UNIVERSITET
INSTITUTT FOR TEKNISK KYBERNETIKK

Contact person during the exam:
Institutt for teknisk kybernetikk, Gløshaugen
Professor Anton Shiriaev, phone: 92885339

EXAM FOR COURSE TTK 4195
Robots Modeling and Control

Thursday, June 2, 2016
Time: 09.00-13.00

Allowable aids: D - No printed or written material allowed.
NTNU type approved calculator with an empty memory allowed.
Language: English. Number of Pages: 4 (+ 3 formula sheet). This exam counts for 100% of the final grade.

This exam consists of 4 exercises each consisting of a number of questions. Each question gives a number of points and a sum of points is 100.

1. Given three orthogonal frames with the same origin $Ox_1y_1z_1$, $Ox_2y_2z_2$, $Ox_3y_3z_3$, suppose rotation matrices R_2^1 and R_3^1 are

$$R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, \qquad R_3^1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find the rotation matrix R_3^2. **(5)**

2. Consider a robot depicted on Fig. 1

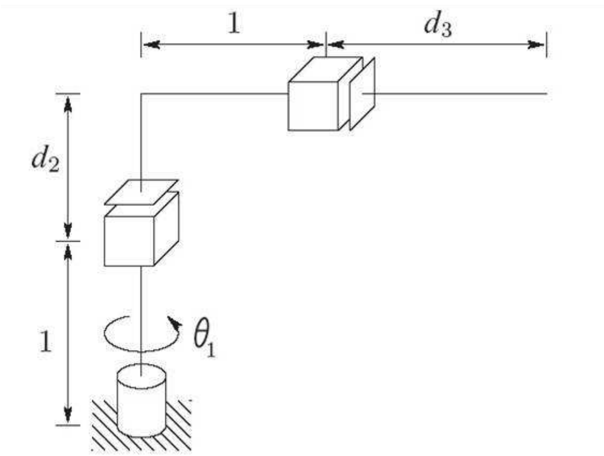

Figure 1: A robot from the problem 2.

(a) For each link introduce the frame following DH-convention and derive forward kinematics equations **(10)**

(b) Solve the inverse position kinematics problem, i.e. given coordinates (position) of the tool frame of the cylindrical robot in the base frame $p_e = (x_e, y_e, z_e) \in \mathbb{R}^3$ find corresponding values for angle θ_1 and extensions d_2, d_3, with which the tool frame is in the requested position p_e. **(10)**

(c) Compute the manipulator Jacobian for representation of linear and angular velocity of the origin of the tool frame, which is located at the end of the second prismatic link of the robot, see Fig. 1 **(10)**

(d) Compute the total velocity of the origin of the tool frame when the variables θ_1, d_2 and d_3 are changing with time as follows

$$\theta_1(t) = \cos(3t), \quad d_2(t) = \sin(2t), \quad d_3(t) = \cos(t)$$

Computation can be based on the Jacobian computed on the previous step or the vector of velocity of this point can be computed directly. **(15)**

3. The two link planar robot – the so-called inertia wheel pendulum – depicted on Fig. 2. It has two revolute joints, while the location of the center of mass

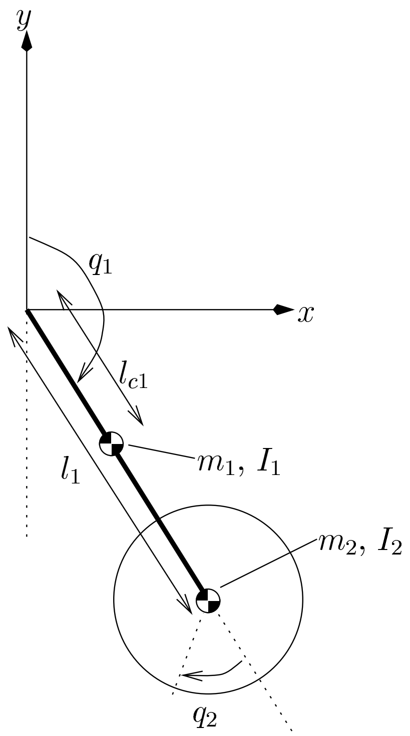

Figure 2: A robot from the problems 3 and 4: inertia wheel pendulum

of the second link coincides with its suspension point.

Given physical parameters of the system (masses of the links m_1, m_2; inertias I_1, I_2; the length of the first link l_1 and the distance to its center of mass l_{c1}), suppose **the robot is located in the vertical plane and its dynamics are affected by the gravity**, you are requested to implement the following tasks:

(a) Find the potential energy $\mathcal{P}(q_1, q_2)$ of the robot **(2)**

(b) Find the kinetic energy $\mathcal{K}(q_1, q_2, \dot{q}_1, \dot{q}_2)$ of the robot **(3)**

(c) Obtain the Euler-Lagrange equations of the system dynamics **(5)**

(d) Assume that both links are actuated and compute the linearization of the system dynamics around the upright equilibrium $q_{1e} = q_{2e} = 0$. Is the resulted linear control system controllable? **(5)**

(e) Assume that there is only one actuator that provides a control torque to change angular acceleration of the first link (the pendulum). Check whether for this case the system dynamics is feedback linearizable around the equilibrium $q_{1e} = q_{2e} = 0$. **(10)**

(f) Assume that there is only one actuator that provides a control torque to change angular acceleration of the second link (the inertia wheel). Check whether for this case the system dynamics is feedback linearizable around the equilibrium $q_{1e} = q_{2e} = 0$. **(10)**

4. Consider the inertia wheel pendulum robot depicted on Fig. 2 with masses of the links equal to m_1, m_2; inertias equal to I_1, I_2; the length of the first link equal to l_1 and the distance to its center of mass equal to l_{c1}). Suppose that the robot is put in the horizontal plane and its dynamics are **not** affected by the gravity, you are requested to implement the following tasks:

   (a) Find the potential energy $\mathcal{P}(q_1, q_2)$ of the robot **(1)**

   (b) Find the kinetic energy $\mathcal{K}(q_1, q_2, \dot{q}_1, \dot{q}_2)$ of the robot **(1)**

   (c) Obtain the Euler-Lagrange equations of the system dynamics **(1)**

   (d) Assume that both links are actuated and compute the linearization of the system dynamics around the equilibrium $q_{1e} = q_{2e} = 0$. Is the resulted linear control system controllable? **(2)**

   (e) Assume that there is only one actuator that provides a control torque to change angular acceleration of the first link (the pendulum). Check whether for this case the system dynamics is feedback linearizable around the equilibrium $q_{1e} = q_{2e} = 0$. **(5)**

   (f) Assume that there is only one actuator that provides a control torque to change angular acceleration of the second link (the inertia wheel). Check whether for this case the system dynamics is feedback linearizable around the equilibrium $q_{1e} = q_{2e} = 0$. **(5)**