Norwegian University of Science and Technology
Department of Engineering Cybernetics

TTK4195
Modeling and Control of Robots
Spring 2015

Exercise set 9

Hand-in date: March 24th, 23:00

Simulation Problem

Given a 2-DOF planar elbow manipulator with the following dynamic equation of motion:

$$M(q)\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + C(q,\dot{q})\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} + G(q) = B(q)u \tag{1}$$

with the inertia matrix

$$M(q) = \begin{bmatrix} p_1 + p_2 + 2p_3\cos(q_2) & p_2 + p_3\cos(q_2) \\ p_2 + p_3\cos(q_2) & p_2 \end{bmatrix},$$

the matrix corresponding to Coriolis and centrifugal forces

$$C(q,\dot{q}) = \begin{bmatrix} -p_3\sin(q_2)\dot{q}_2 & -p_3\sin(q_2)(\dot{q}_1+\dot{q}_2) \\ p_3\sin(q_2)\dot{q}_1 & 0 \end{bmatrix},$$

the gravitational torque vector

$$G(q) = \begin{bmatrix} p_4\cos(q_1) + p_5\cos(q_1+q_2) \\ p_5\cos(q_1+q_2) \end{bmatrix},$$

and the input matrix

$$B(q) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Here $q = [q_1, q_2] = T$ represent the joint variables and u is a scalar control input, which in this case only affects the first link directly, whereas the second link is not actuated. This underactuated system is also known as Pendubot. The physical model parameters are combined to

$$\begin{aligned}
p_1 &= m_1 r_1^2 + m_2 l_1^2 + J_{c1} = 0.0319\ \mathrm{kgm}^2\\
p_2 &= m_2 r_2^2 + J_{c2} = 0.0092\ \mathrm{kgm}^2\\
p_3 &= m_2 l_1 r_2 = 0.01\ \mathrm{kgm}^2\\
p_4 &= (m_1 r_1 + m_2 l_1)g = 1.2954\ \mathrm{Nm}\\
p_5 &= m_2 r_2 g = 0.3915\ \mathrm{Nm}
\end{aligned}$$

Tasks:

1. Write a state space representation of the equations of motion (1) in standard form

$$\dot{x} = f(x) + g(x)u$$
$$y = h(x)$$

   where $y = h(x)$ shall be defined as an output function that only depends on the position variables q_1, q_2.
   (1 Point)

2. Find all equilibrium points of the unforced system (1), i.e. $u = 0$.
   (1 Point)

3. Linearize the state equation for the upright-upright equilibrium assuming that the input u can fluctate around zero.
   (2 Points)

4. Check controllability and provide a control law that ensures local stability of the closed-loop system around the upright-upright equilibrium point.
   (2 Points)

5. Is it possible to apply feedback lineariztion to the system (1)?
   (2 Points)
   Hint: Take as many time derivatives of the output function $y = h(x)$ as required for the control variable u to show up on the right hand side.
   Note that $\frac{d}{dy} y = \frac{\partial}{\partial x} h(x)[f(x) + g(x)u]$. In that way one can check the possibility of linearizing the input-output map.

6. Suppose there exists a motion in which the first link is synchronized linearly to the behavior of the second link. That is, the following functions

$$q_1 = kq_2$$
$$\dot{q}_1 = k\dot{q}_2$$
$$\ddot{q}_1 = k\ddot{q}_2$$

   are kept invariant by a suitable control law along solutions of system (1). Substituting the above relations into (1), how many state variables are left? How can we see that the motion of q_2, and inherently q_1, is constrained for such a choice of synchronization functions between the links?
   (2 Points)

Use MATLAB/Simulink if appropriate.