NORGES TEKNISK-
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EXAM FOR COURSE TTK 4195
Robots Modeling and Control

Tuesday, May 20, 2008
Time: 09.00-13.00

Allowable aids: D - No printed or written material allowed. NTNU type approved calculator with an empty memory allowed. Language: English. Number of Pages: 4 (+ 4 formula sheet). This exam counts for 100% of the final grade.

This exam consists of 3 exercises each consisting of a number of questions. Each question gives a number of points and a sum of points is 100.

1. Given three orthogonal frames with the same origin $Ox_1y_1z_1$, $Ox_2y_2z_2$, $Ox_3y_3z_3$, suppose rotation matrices R_2^1 and R_3^1 are

$$R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, \qquad R_3^1 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find the rotation matrix R_3^2. **(10)**

2. Consider a robot depicted on Fig. 1

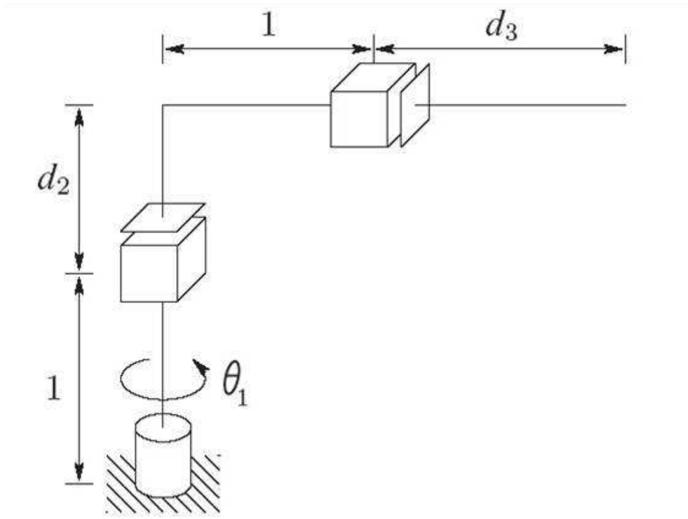

Figure 1: A robot from the problem 2.

(a) For each link introduce the frame following DH-convention and derive forward kinematics equations **(10)**

(b) Solve the inverse position kinematics problem, i.e. given coordinates (position) of the tool frame of the cylindrical robot in the base frame $p_e = (x_e, y_e, z_e) \in \mathbb{R}^3$ find corresponding values for angle θ_1 and extensions d_2, d_3, with which the tool frame is in the requested position p_e. **(10)**

(c) Compute the manipulator Jacobian for representation of linear and angular velocity of the origin of the tool frame, which is located at the end of the second prismatic link of the robot, see Fig. 1 **(10)**

(d) Compute the total velocity of the origin of the tool frame when the variables θ_1, d_2 and d_3 are changing with time as follows

$$\theta_1(t) = \sin(t), \quad d_2(t) = \cos(2t), \quad d_3(t) = \sin(3t)$$

Computation can be based on the Jacobian computed on the previous step or the vector of velocity of this point can be computed directly. **(10)**

3. The two link planar robot – the so-called inertia wheel pendulum – depicted on Fig. 2. It has two revolute joints, while the center of mass of

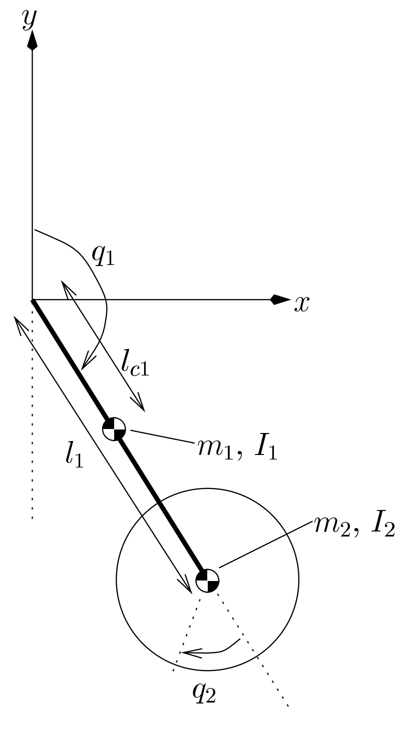

Figure 2: A robot from the problem 3: inertia wheel pendulum

the second link coincide with its suspension point.

Given physical parameters of the system (masses of the links m_1, m_2; inertias I_1, I_2; the length of the first link l_1 and the distance to its center of mass l_{c1}), you are requested to implement the following tasks:

(a) Find the potential energy $\mathcal{P}(q_1, q_2)$ of the robot **(10)**

(b) Find the kinetic energy $\mathcal{K}(q_1, q_2, \dot{q}_1, \dot{q}_2)$ of the robot **(10)**

(c) Obtain the Euler-Lagrange equations of the system dynamics **(15)**

(d) Assume that both links are actuated and compute the linearization of the system dynamics around the upright equilibrium $q_{1e} = q_{2e} = 0$. Is the resulted linear control system controllable? **(15)**