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# EXAM FOR COURSE TTK 4195
# Robots Modeling and Control

Thursday, May 18, 2017
Time: 09.00-13.00

Allowable aids: D - No printed or written material allowed.
NTNU type approved calculator with an empty memory allowed.
Language: English. Number of Pages: 4 (+ 3 formula sheet). This exam counts for 100% of the final grade.

This exam consists of 4 exercises each consisting of a number of questions. Each question gives a number of points and a sum of points is 100.

1. Given three orthogonal frames $\mathcal{N}$, $\mathcal{B}$ and $\mathcal{F}$ with the same origin, the coordinates of the orthonormal unit base vectors of the frame $\mathcal{B}$ written in the frame $\mathcal{N}$ are

$$\vec{b}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{b}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{b}_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix},$$

the coordinates of the orthonormal unit base vectors of the frame $\mathcal{F}$ written in the frame $\mathcal{N}$ are

$$\vec{f}_1 = \begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \\ 0 \end{bmatrix}, \quad \vec{f}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{f}_3 = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \\ 0 \end{bmatrix}.$$

(a) Consider a vector $\vec{a}$, which coordinates in the frame $\mathcal{F}$ are $(a_1,\ a_2,\ a_3)_f^T$, i.e.

$$\vec{a}_f = a_1\vec{f}_1 + a_2\vec{f}_2 + a_3\vec{f}_3$$

Find the coordinates of the vector $\vec{a}$ in the frames $\mathcal{N}$ and $\mathcal{B}$. **(10)**

(b) If the coordinates of the vector $\vec{a}$ in the frame $\mathcal{F}$ are $(a_1,\ a_2,\ a_3)_f = (0,\ 1,\ 0)$, what are the coordinates of this vector in the frames $\mathcal{N}$ and $\mathcal{B}$? **(5)**

2. Suppose that a rigid body with a fixed point in the inertial frame rotates around the fixed axis with the coordinates

$$\vec{e} = \frac{1}{2}\left(1,\ 1,\ \sqrt{2}\right)^T$$

and with a constant angular velocity.

(a) Find the corresponding time evolution of of Euler angles – $\phi(t)$-precession, $\theta$-nutation, $\psi$-internal rotation of the rigid body – which serve for the $ZXZ$-parametrization

$$R(t) = R_{\phi(t),z} \cdot R_{\theta(t),x} \cdot R_{\psi(t),z}$$

of this rotation. **(10)**

(b) Find the coordinates of the corresponding vector of the angular velocity of the rotation written in the body frame, i.e. in the frame firmly attached to the rigid body. **(10)**

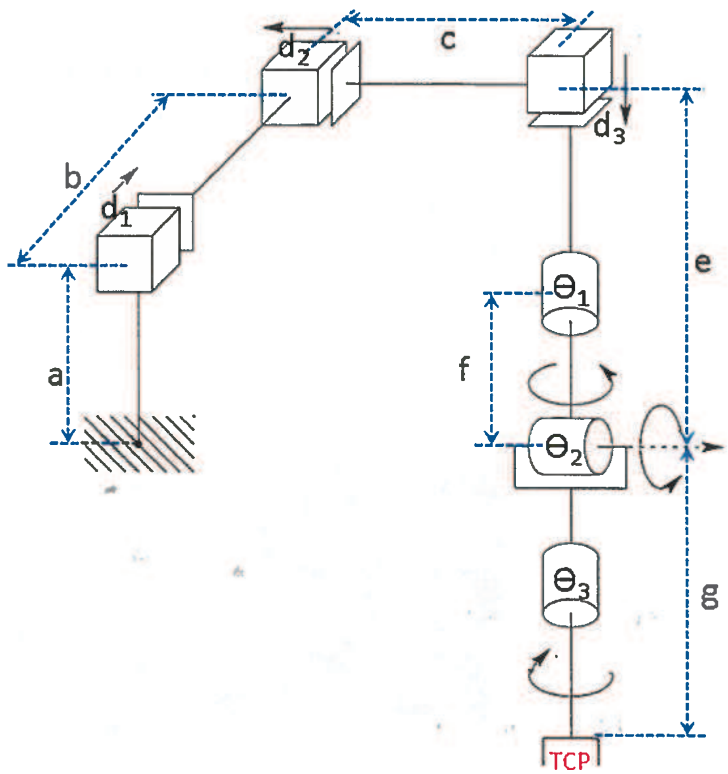


Figure 1: A robot from the problem 3.

3. Consider a robot depicted on Fig. 1 with first three prismatic and last three revolute joints. The axes of rotation of the revolute joints by design intersect in one point.

   (a) For each link introduce the frame following DH-convention and derive forward kinematics equations **(15)**

   (b) Solve the inverse position kinematics problem for positioning the origin of the end-effector frame, i.e. given coordinates (position) of the tool frame of the robot in the base frame $p_e = (x_e, y_e, z_e) \in \mathbb{R}^3$ find corresponding values for the robot joint variables, with which the tool frame origin is put in the requested position $p_e$. **(10)**

4. The planar robot depicted on Fig. 2. consists of a wheel, which can roll on

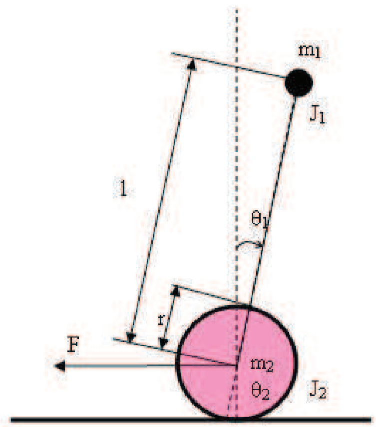


Figure 2: A robot from the problem 4: the pendulum on the rolling base

the horizontal floor, and the pendulum suspended on the axis of rotation of the wheel. The pendulum can rotate freely about this moving axis.

Given physical parameters of the system (the radius of the wheel $r$, the masses of the pendulum and the wheel $m_1$, $m_2$; the inertia of the pendulum and the wheel $J_1$, $J_2$ around their centers of masses; the distance between the suspension point and the center of mass point of the pendulum 1 [m]), suppose **the robot is located in the vertical plane, i.e. its dynamics are affected by the gravity** and there is no slipping of the wheel when it rolls on the floor, you are requested to implement the following tasks:

(a) Find the potential energy $\mathcal{P}(\theta_1, \theta_2)$ of the robot **(5)**

(b) Find the kinetic energy $\mathcal{K}(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2)$ of the robot **(10)**

(c) Obtain the Euler-Lagrange equations of the system dynamics **(10)**

(d) Assume that the controller applies the force $u = F$ to the axis of the rotation of the wheel as shown on the picture, compute the linearization of the system dynamics around the upright equilibrium $\theta_{1e} = \theta_{2e} = 0$. Is the resulted linear control system controllable? **(10)**

(e) If the wheel slips on the floor when rolling, how many variables are needed for describing the configuration of the robot? Give an example of such variables and give comments on the origin of slippage of the wheel if it is observed. **(5)**