TTK4195
Modeling and Control of Robots
Assignment 2

Solution

Problem 3-3

First, the coordinate frames need to be placed according to the DH convention. This is done as shown by Figure 1.

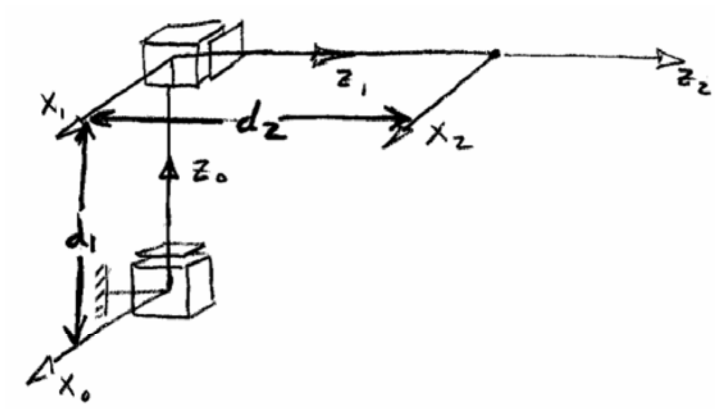

Figure 1: Coordinate frames placed according to the DH convention.

Second, the DH parameters must be found. These are listed in Table 1.

Link	θ_i	d_i	a_i	α_i
1	0	d_1^*	0	$-\frac{\pi}{2}$
2	0	d_2^*	0	0

Table 1: DH parameters for the coordinate frames in Figure 1.

Third, the homogeneous transformation matrices between coordinate frame $i-1$ and i need to be found. These are named A_i, and are found to be

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Last, the forward kinematics is found to be given as

$$T_2^0 = A_1 A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_2^* \\ 0 & -1 & 0 & d_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3-5

First, the coordinate frames need to be placed according to the DH convention. This is done as shown by Figure 2.

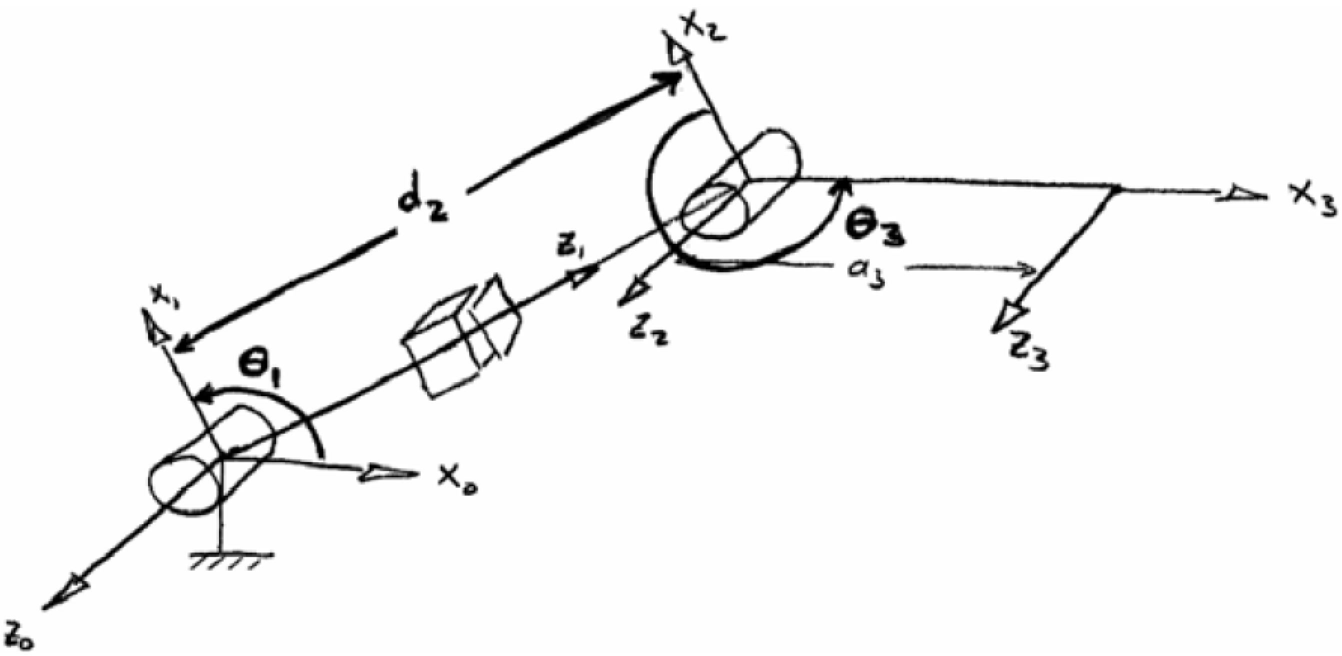

Figure 2: Coordinate frames placed according to the DH convention.

Second, the DH parameters must be found. These are listed in Table 2.

Link	θ	d	a	α
1	θ_1^*	0	0	$\frac{\pi}{2}$
2	0	d_2^*	0	$-\frac{\pi}{2}$
3	θ_3^*	0	a_3	0

Table 2: DH parameters for the coordinate frames in Figure 2.

Third, the homogeneous transformation matrices between coordinate frame $i-1$ and i need to be found. These are named A_i, and are found to be

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Last, the forward kinematics is found to be given as

$$T_0^3 = A_1A_2A_3 = \begin{bmatrix} c_{13} & -s_{13} & 0 & s_1d_2 + a_3c_{13} \\ s_{13} & c_{13} & 0 & -c_1d_2 + a_3s_{13} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3-7

By using the same strategy as in the previous problem, the coordinate frames are illustrated in Figure 3 with DH parameters given by Table 3.

Link	θ	d	a	α
1	0	d_1^*	0	$-\frac{\pi}{2}$
2	$\frac{\pi}{2}$	d_2^*	0	$-\frac{\pi}{2}$
3	0	d_3^*	0	0

Table 3: DH parameters for the coordinate frames in Figure 3.

The A_i matrices are then found to be

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

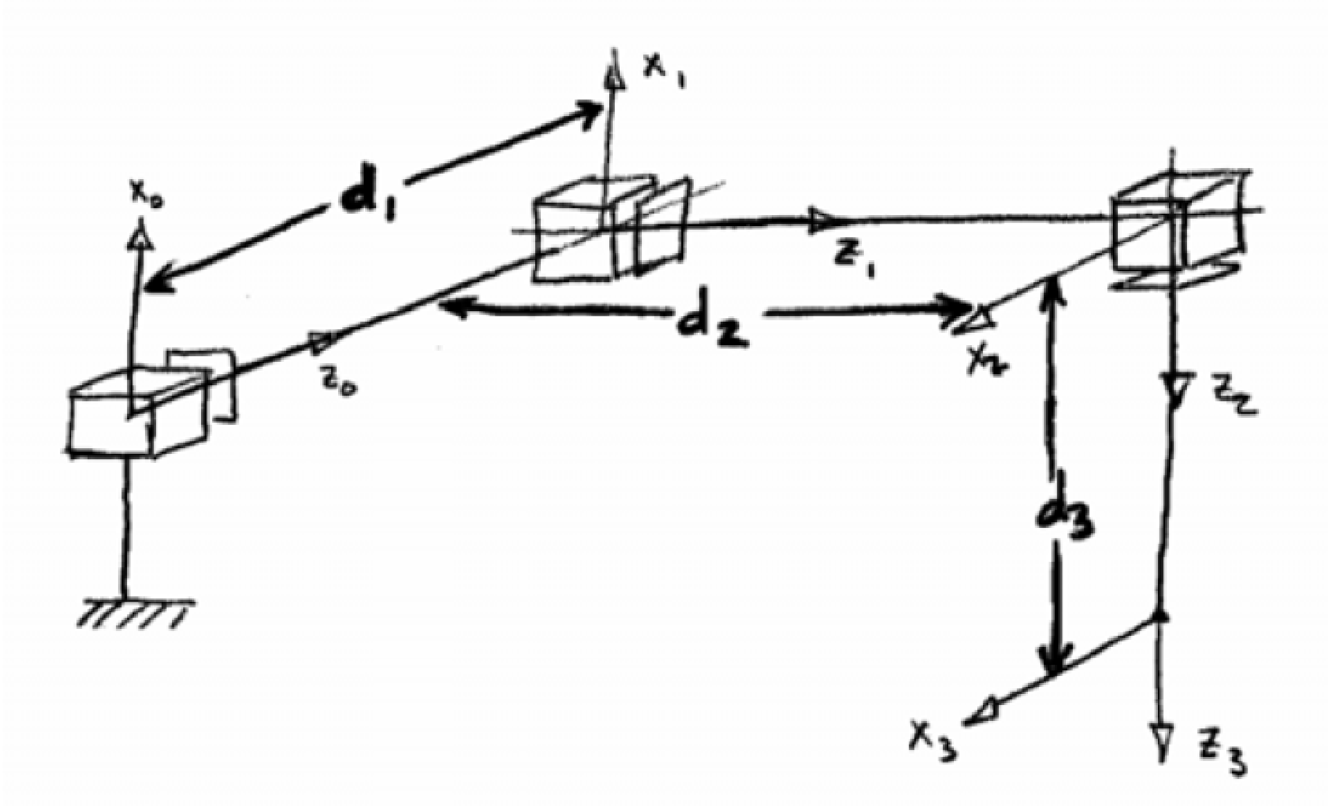

Figure 3: Coordinate frames placed according to the DH convention.

Which gives the following forward kinematics

$$T_0^3 = A_1 A_2 A_3 = \begin{bmatrix} 0 & 0 & -1 & -d_3 \\ 0 & -1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3-8

The DH parameters for the robot is given in Table 4. Note the difference between the wrist

Link	θ	d	a	α
1	θ_1^*	d_1	0	$-\frac{\pi}{2}$
2	θ_2^*	0	a_2	0
3	θ_3^*	0	0	$-\frac{\pi}{2}$
4	θ_3^*	d_4	0	$+\frac{\pi}{2}$
5	θ_5^*	0	0	$-\frac{\pi}{2}$
6	θ_6^*	d_6	0	0

Table 4: DH parameters for the coordinate frames in Figure 3.

parameters listed here, and Example 3.3 in the book. The difference is due to that z_2 is not parallel to z_3 (first joint axis of the wrist), requiring that o_3 is placed at o_2 and not the wrist center. To properly decouple the wrist from the rest of the robot, an end effector-like frame could be inserted at link 3. See Example 3.4 for a case where $z_2||z_3$.

Problem 3-13

Given a desired position, $d = [d_x, d_y, d_z]^\top$, for the end effector, we are suppose to find $q = [\theta_1, d_2, d_3]$. By inspecting Figure 4 we see that

$$\theta_1 = \text{Atan2}(d_x, d_y)$$
$$d_2 = d_z - 1$$
$$d_3 = \sqrt{d_x^2 + d_y^2} - 1$$

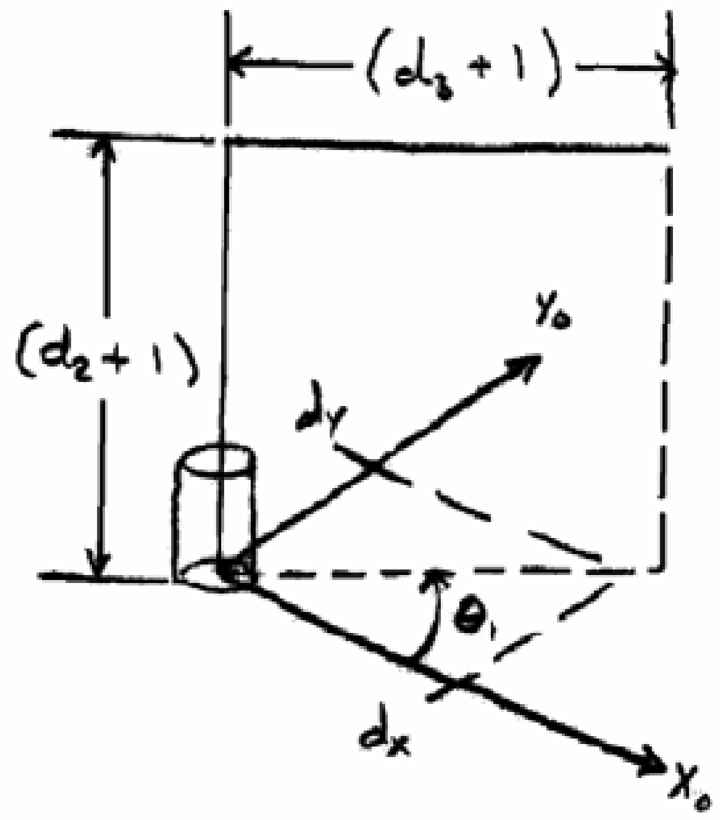

Figure 4: Finding the inverse kinematics in Problem 3-13.