Probability Problems

1.) # of ways to ask 8 questions for 15 students: 15^8
# of ways to to select 8 students from 15 in order: $\frac{15!}{7!} = 259459200$
P(no student answers more than one question) $= \frac{259459200}{15^8} = \boxed{0.101237}$

2.) 0 - 100: no even numbers where first two digits are odd.
100 - 1000: first digit ~ (1,3,5,7,9) second digit ~ (1,3,5,7,9) third digit ~ (0,2,4,6,8)
# of even numbers with 2 odd digits first $= 5 \cdot (5-1) \cdot 5 = 100$
1000 - 10000: First and second digit ~ (1,3,5,7,9) third digit ~ (0-9) fourth ~ (0,2,4,6,8)
# of even numbers with 2 odd digits first $= 5 \cdot (5-1) \cdot (9-2) \cdot (5) = 700$
10000 - 99999: first and second ~ (1,3,5,7,9) third and fourth ~ (0-9) fifth ~ (0,2,4,6,8)
# of even numbers with 2 odd digits first $= 5 \cdot (5-1) \cdot (9-2) \cdot (9-3) \cdot (5) = 4200$

Total # of even numbers with 2 odd digits first $= 100 + 700 + 4200 = 5000$
Total #'s between 0 - 99999 $= 10^5$

P(required #) $= \frac{5000}{10^5} = 0.05$ — Let $n = 8$ $p = 0.05$

Let X = # of ints that meet crit. out of 8.

$P(X=5) = 8C_5 (0.05)^5 (1-0.05)^3 = \boxed{1.5004 \times 10^{-5}}$

3.) 4 or above: $\frac{3}{6}$ A ~ P(2 or more show 4 or more) $= P(X=2) \cdot P(X=3) = \frac{1}{2}$
B ~ P(all three show same) $= \frac{6}{6^3} = \frac{1}{36}$
$P(A \cap B)$ = P(all are 4) + P(all are 5) + P(all are 6) $= \frac{1}{72}$

Since $P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{36} = \frac{1}{72} = P(A \cap B)$ $\boxed{\text{A and B are independent}}$

4.) $P(\text{Flush}) = \frac{4(13C_5)}{52C_5} = 0.001980792$ X = # of hands until one flush

$X \sim \text{Geometric}(0.001980792)$ — $E[X] = \frac{1}{p} = \frac{1}{0.001980792} = \boxed{504.8486}$

5) $P(\text{win} \mid \text{super}) = 0.7$

$P(\text{win} \mid \text{no super}) = 0.5$

$P(\text{super}) = 0.75$ for next 5 games

$P(\text{win } \frac{4}{5} \mid \text{super}) = 5C_4 (0.7)^4 (0.3) = 0.36015$

$P(\text{win } \frac{4}{5} \mid \text{no super}) = 5C_4 (0.5)^5 = 0.15625$

$P(\text{win } \frac{4}{5}) = (0.15625)0.25 + (0.36015)0.75 = 0.309175$

$$P(\text{super} \mid \text{win } \tfrac{4}{5}) = \frac{(0.36015)0.75}{0.309175} = \boxed{0.8737}$$