Robotic Navigation and Exploration Lab 1

Week 2: Kinematic Model & Path Tracking Control

Min-Chun Hu anitahu@cs.nthu.edu.tw
CS, NTHU

Requirement

- Python 3.X (Suggest to install miniconda/anaconda)
- Numpy
- Opencv-Python

Issues in MAC

- Miniconda environment path setting for Mac
  - vim ~/.zshrc
  - export PATH=$PATH:"~/[installation path]/miniconda3/bin“
  - source ~/.zshrc

```
CommandNotFoundError: Your shell has not been properly configured to use 'conda activate'.
```

- source ~/[installation path]/miniconda3/etc/profile.d/conda.sh

```
This application failed to start because no Qt platform plugin could be initialized.
Reinstalling the application may fix this problem.
```

- pip install opencv-python==4.1.2.30

Kinematic Model

- There are two files for motion models: **wmr_model.py** / **bicycle_model.py**. In wheeled mobile robot (WMR) model, we control the car by angular velocity **ω**. In bicycle model, we control the car by steer **δ**.

- Basic Program Flow (main function)

```
car = KinematicModel(<parameters>) # Create the model
car.init_state(<start pose>) # Initialize the pose
while(True):
    car.control(<control parameter>) # Control the model
    car.update() # Update the state
```

TODO

Kinematic Model for WMR

State:

$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

Rotation Matrix:

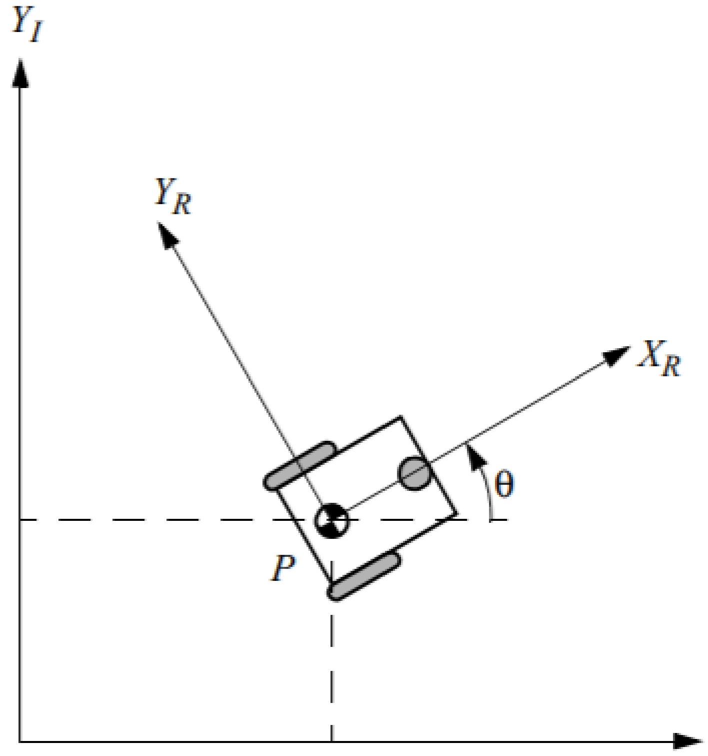
$$R(\theta) = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Kinematic Model:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = R(\theta)^{-1} \begin{bmatrix} \dot{x_R} \\ \dot{y_R} \\ \dot{\theta} \end{bmatrix}$$

$$= \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ 0 \\ \omega \end{bmatrix}$$

$$= \begin{bmatrix} v\cos(\theta) \\ v\sin(\theta) \\ \omega \end{bmatrix}$$

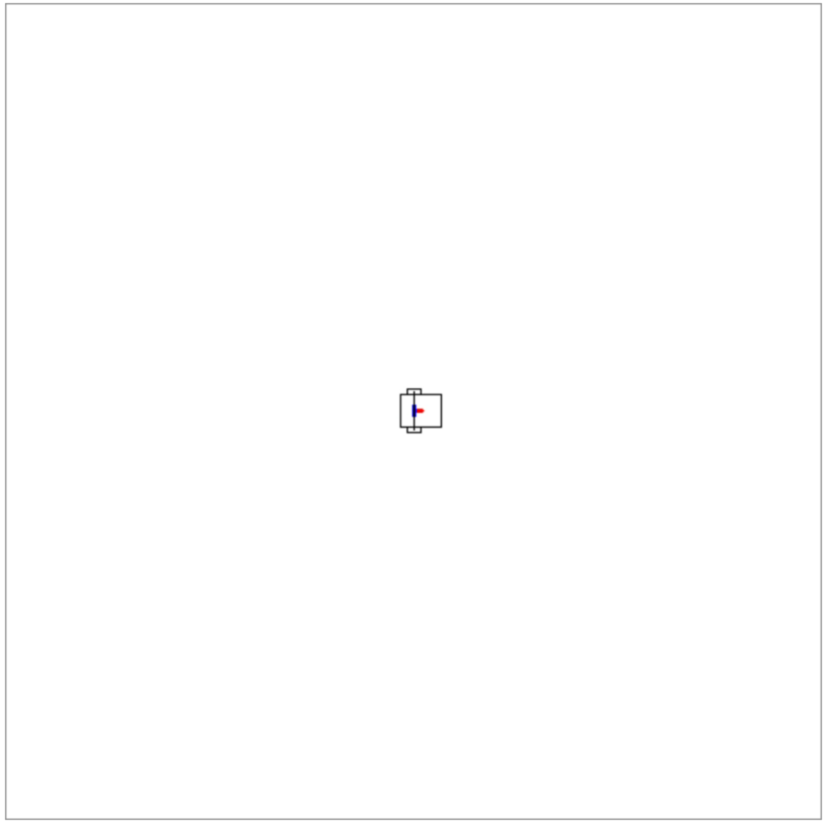


Kinematic Model for WMR

```
def update(self):
    self.v += self.a * self.dt

    # Speed Constrain
    if self.v > self.v_range:
    self.v = self.v_range
    elif self.v < -self.v_range:
    self.v = -self.v_range

    # Motion
    self.x += self.v * np.cos(np.deg2rad(self.yaw)) * self.dt
    self.y += self.v * np.sin(np.deg2rad(self.yaw)) * self.dt
    self.yaw += self.w * self.dt
    self.yaw = self.yaw % 360
    self.record.append((self.x, self.y, self.yaw))
    self._compute_car_box()
```

Kinematic Model for WMR

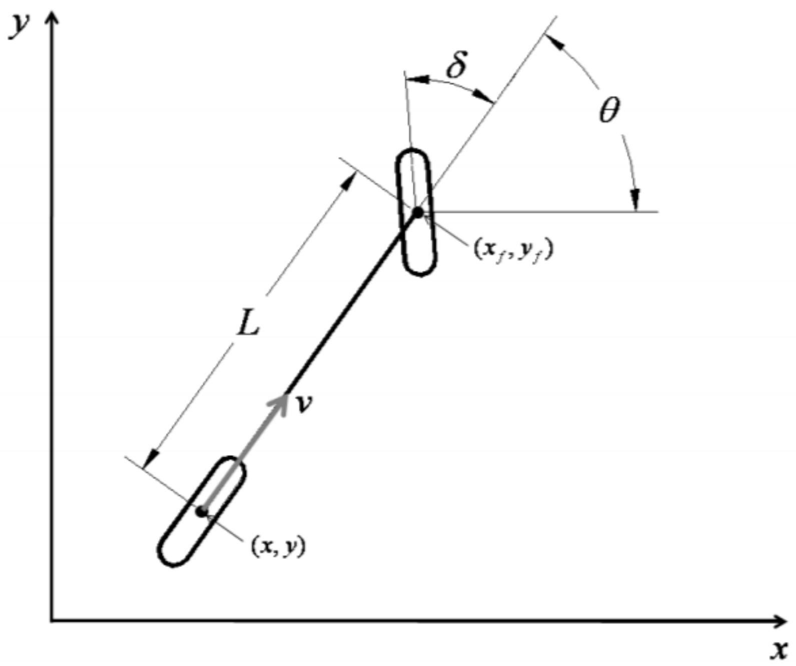
- Keyboard Control



Kinematic Bicycle Model

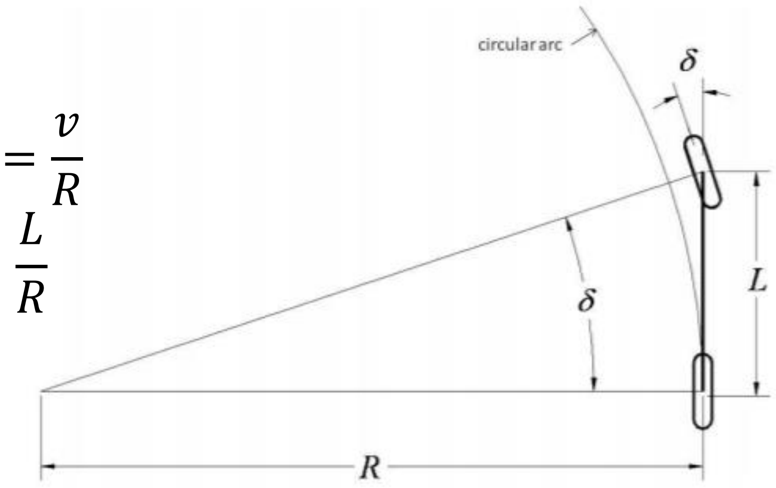
- Kinematic Model

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos(\theta)\\ \sin(\theta)\\ \dfrac{\tan(\delta)}{L}\end{bmatrix} v$$



- Some Property

$$R\dot{\theta} = v$$
$$\frac{v\tan(\delta)}{L} = \frac{v}{R}$$
$$\tan(\delta) = \frac{L}{R}$$



Path Tracking Control

- All path tracking control class (**wmr_pid.py** / **wmr_pure_pursuit.py** / **bicycle_pure_pursuit.py** / **bicycle_stanley.py**) share the same abstract function.

__init__(<parameters>): Initialize the control algorithm.

set_path(path): Set the path of the control algorithm.

_search_nearest(pos): Search the nearest points on the path.

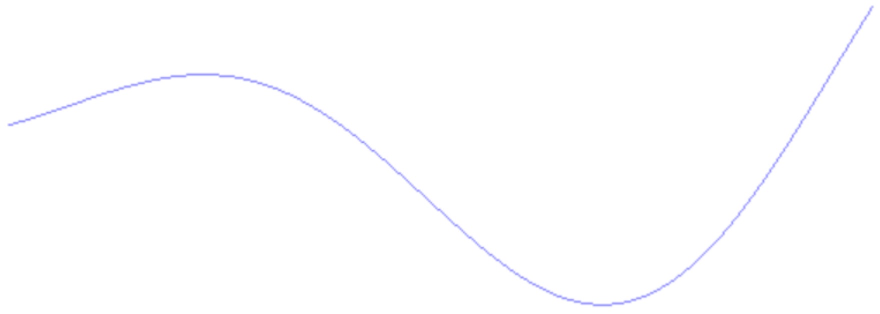
feedback(state): Compute the feedback control signal (TODO).

Path Tracking Control

- Path generator provides two paths, **path1** is a line and **path2** is a curve.

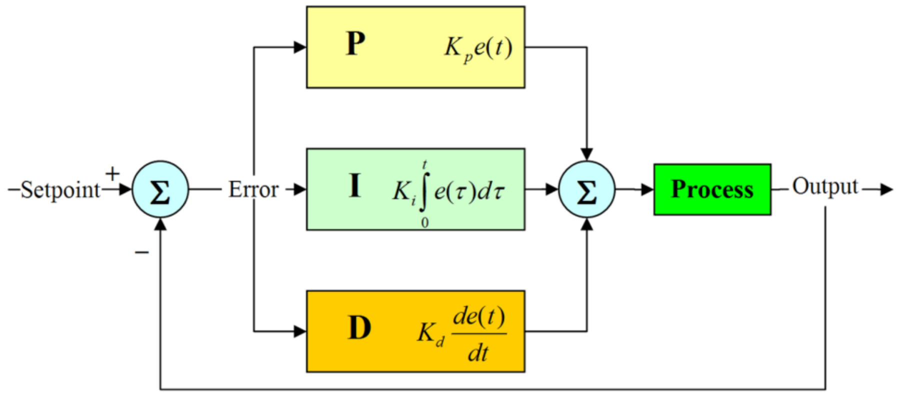
Path 1

Path 2



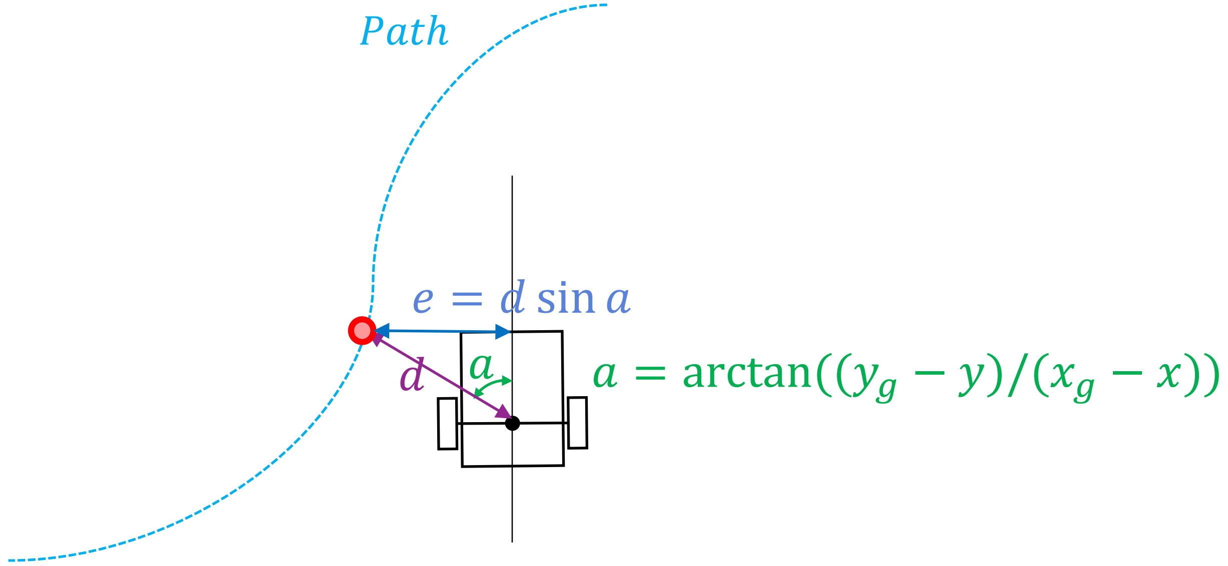
PID Control

- **P**roportional / **I**ntegral / **D**ifferential Control



Discrete Form :

$$Output = K_p e(t) + K_i \sum_{0}^{t} e_t + K_d(e_t - e_{t-1})$$

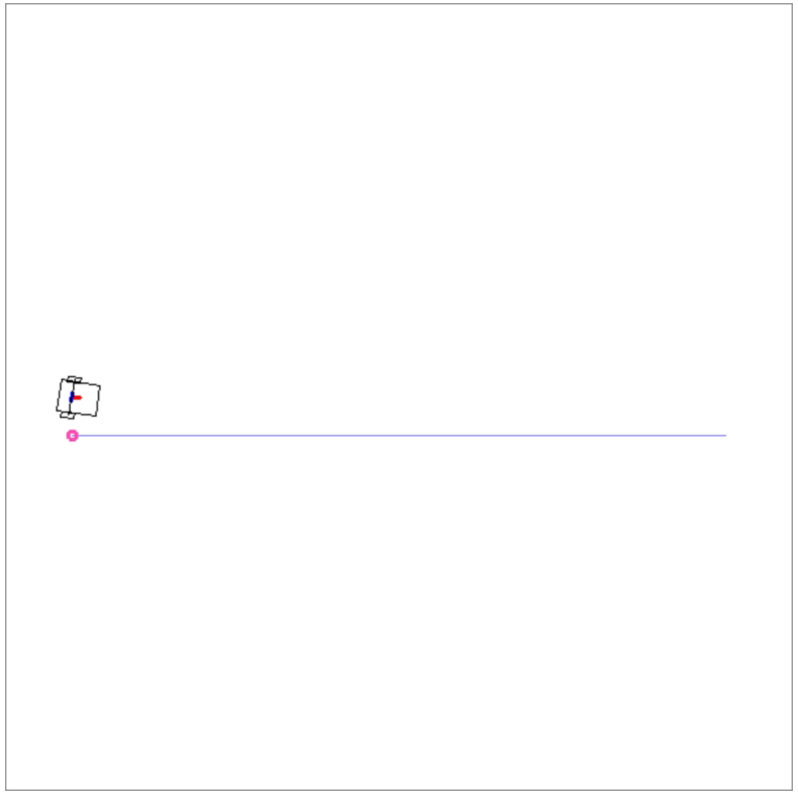


PID Control

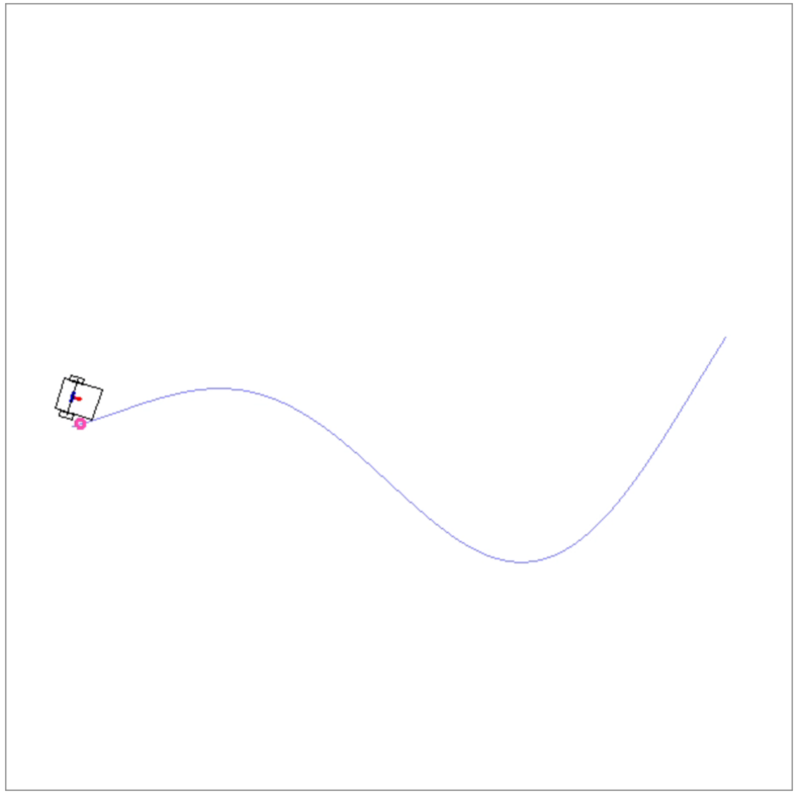
```
def feedback(self, state):
    # Check Path
    if self.path is None:
        print("No path !!")
        return None, None

    # Extract State
    x, y, dt = state["x"], state["y"], state["dt"]

    # Search Nesrest Target
    min_idx, min_dist = self._search_nearest((x,y))

    # PID Control
    ang = np.arctan2(self.path[min_idx,1]-y,
    self.path[min_idx,0]-x)
    ep = min_dist * np.sin(ang)
    self.acc_ep += dt*ep
    diff_ep = (ep - self.last_ep) / dt
    next_w = self.kp*ep + self.ki*self.acc_ep + self.kd*diff_ep
    self.last_ep = ep

    return next_w, self.path[min_idx]
```

PID Control

Path 1



Path 2



Pure Pursuit Control for WMR

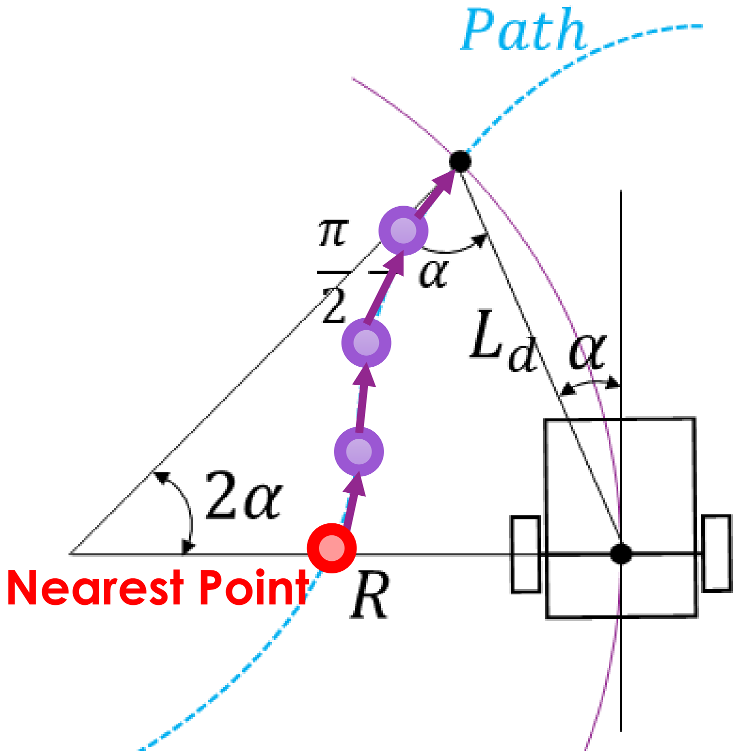
- Concept:
  - Modify the angular velocity to let the center achieve a point on path

$$\alpha = \arctan\left(\frac{y - y_g}{x - x_g}\right) - \theta$$

$$\omega = \frac{2\sin(\alpha)}{L_d}$$

$L_d = (kv + L_{fc})$, where k, L_{fc} are parameters.

1. Set a distance **Ld.**
2. Find the nearest point on the path.
3. Search the following point until the distance of the point larger than or equal to **Ld**.



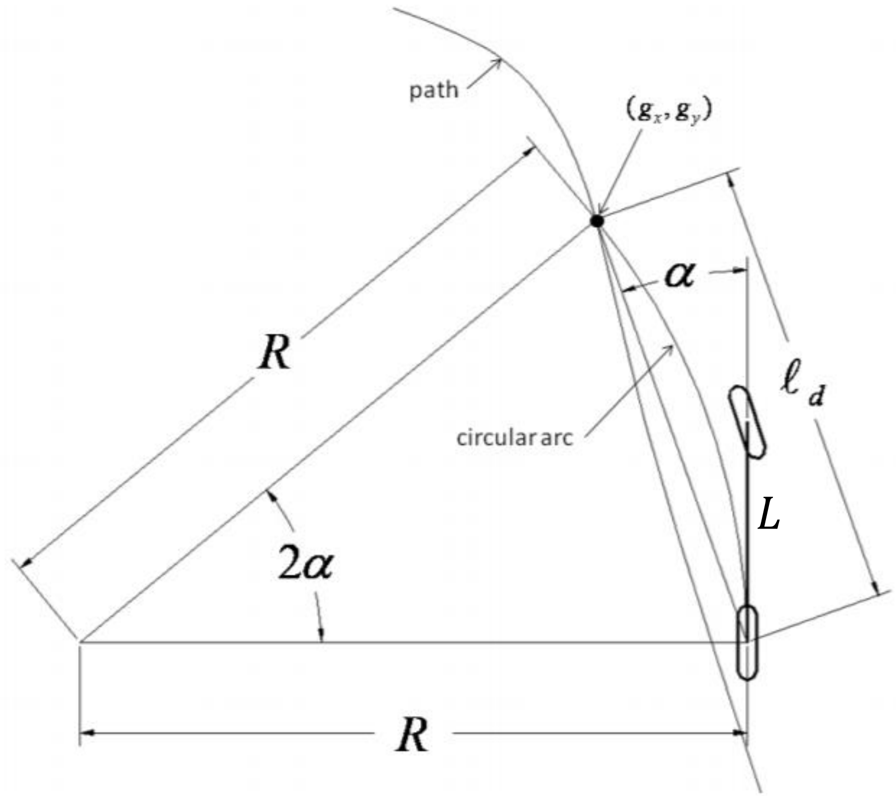
Pure Pursuit Control for Bicycle Model

- Concept:
  - Control the steer to let the rear wheel achieve a point on the path.

$$\alpha = \arctan\left(\frac{y - y_g}{x - x_g}\right) - \theta$$

$$\delta = \arctan\left(\frac{2Lsin(\alpha)}{L_d}\right)$$

$L_d = (kv + L_{fc})$, where k, L_{fc} are parameters.



Stanley Control

- Concept:
  - Exponential stability for front wheel feedback
- Some Implementation Details

$$\theta_e = \theta_p - \theta$$

$$\dot{e} = v_f sin(\delta - \theta_e)$$

$$\delta = \arctan\left(-\frac{ke}{v_f}\right) + \theta_e$$

$$e = \begin{bmatrix} x - x_g \\ y - y_g \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta_p) \\ \sin(\theta_p) \end{bmatrix}$$