

# Linear Model

## Pro SX and MX

### 1 Setup

Let's connect to the database and get the gate drop times.

```
db_path <- './_data/gate_drop.db'
query_path <- './notebooks/R_Notebooks/main_query.sql'
convenience_routines <- './notebooks/R_Notebooks/convenience_routines.R'
source(convenience_routines, local = knitr::knit_global())

src_db <- DBI::dbConnect(RSQLite::SQLite(), db_path)
main_query <- paste(readLines(query_path), collapse='\n')
main_dat <- DBI::dbGetQuery(conn = src_db, statement = main_query)
DBI::dbDisconnect(conn = src_db)
```

This analysis includes 114 separate observations from 15 Pro SX rounds.

## 2 Histograms

Now, let's do some fundamental analysis to get the margin of error for all gate drop times.

```
t_data <- main_dat |> tibble::as_tibble()

# Drop extra columns we don't need to keep
t_data <- t_data |> dplyr::select (-one_of(c("row_id", "Date", "Venue", "Round", "Comments")))

# make the logistic variables numeric and replace NA with 0
t_data <- t_data |> dplyr::mutate_if(is.logical, as.numeric)
t_data <- t_data |> dplyr::mutate_all(funs(replace_na(., 0)))

t_moe_factor <- qt(0.025, nrow(t_data), lower.tail = FALSE)
t_moe <- t_moe_factor * sd(t_data$`sec to drop`) / sqrt(nrow(t_data))

null_model <- mean(t_data$`sec to drop`)
lower_ci <- null_model - t_moe
upper_ci <- null_model + t_moe

gp1 <- ggplot(data = t_data, aes(x= `sec to drop`))+geom_histogram(binwidth = 0.25,color="light blue",
  geom_vline(aes(xintercept = null_model), color = "dark blue")+
  geom_vline(aes(xintercept = lower_ci), color = "dark blue", linetype = "dashed")+
  geom_vline(aes(xintercept = upper_ci), color = "dark blue", linetype = "dashed")+
  labs(title = "Distribution of Elapsed Seconds",
    subtitle = paste("after 30sec board goes sideways",
      "\nnull model estimate:", round(mean(t_data$`sec to drop`), 2),
      "\nMOE (margin of error) = +-", round(t_moe, 2))) + scale_x_continuous()

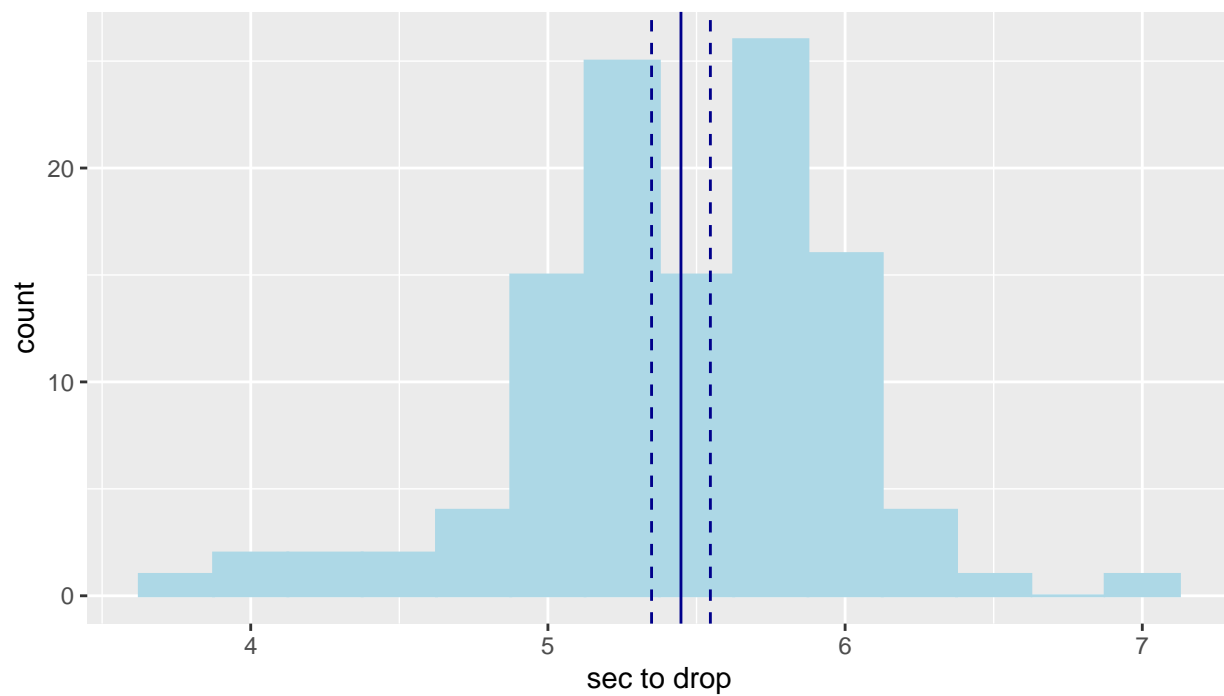
gp1
```

## Distribution of Elapsed Seconds

after 30sec board goes sideways

null model estimate: 5.45

MOE (margin of error) =  $\pm 0.1$



### 3 MLR Model

Let's build a linear model to predict the gate drop times

```
t_data_src_cols <- colnames(t_data)
t_data_src_cols <- t_data_src_cols[!t_data_src_cols == 'sec to drop'] #stage this for later when we mak

t_model <- lm(`sec to drop` ~ . , data = t_data)
print(summary(t_model))
```

```
##
## Call:
## lm(formula = `sec to drop` ~ ., data = t_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.5270 -0.2887  0.0519  0.4210  1.3780
##
## Coefficients: (2 not defined because of singularities)
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   6.0857206   0.5026710   12.107  <2e-16 ***
## `SX Futures`   0.0620000   0.3940379    0.157   0.8753
## `250 SX East` -0.0115817   0.3359756   -0.034   0.9726
## `250 SX West`  0.1168189   0.3331939    0.351   0.7266
## `450 SX`      -0.0007206   0.3272864   -0.002   0.9982
## `450 MX`      NA          NA          NA      NA
## `250 MX`      NA          NA          NA      NA
## Heat          -0.6960409   0.3940913   -1.766   0.0802 .
## LCQ           -0.7179662   0.4064546   -1.766   0.0802 .
## Final         -0.6457206   0.3945085   -1.637   0.1046
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5396 on 106 degrees of freedom
## Multiple R-squared:  0.0392, Adjusted R-squared:  -0.02425
## F-statistic: 0.6178 on 7 and 106 DF,  p-value: 0.7402

t_beta <- round(coefficients(t_model),2)
print(t_beta)
```

```
##      (Intercept)  `SX Futures`  `250 SX East`  `250 SX West`      `450 SX`
##           6.09           0.06          -0.01           0.12           0.00
##      `450 MX`      `250 MX`          Heat          LCQ          Final
##           NA           NA          -0.70          -0.72          -0.65
```

Here's our fitted model:

$$\hat{y} = 6.09 + 0.06x_1 + -0.01x_2 + 0.12x_3 + 0x_4 + NAx_5 + NAx_6 + -0.7x_7 + -0.72x_8 + -0.65x_9$$

So, translating the coefficients, we can determine the following. Looking at the  $y_{\text{intercept}}$ , we can see that, without any other information, we would predict the gate to drop in 6.09 seconds. For all of our features  $\beta_1 - \beta_9$ , they are indicator random variables (aka characteristic random variables) where we have encoded the race classes 'SX Futures', '250 SX East', '250 SX West', '450 SX', '450 MX', and '250 MX.' Further we have applied the convention to the 'Heat', 'LCQ', and 'Final' variables. In summary:

Table 1: Beta Coefficients Discussion

Beta	Value	Discussion
$\beta_1$ SX Futures	0.06	This indicates the SX futures gate drops are slightly longer.
$\beta_2$ 250 SX East	-0.01	This indicates 250 SX East gate drops have no effect on the gate drop time.
$\beta_3$ 250 SX West	0.12	This indicates the 250 SX West gate drops are slightly longer.
$\beta_4$ 450 SX	0	This indicates 450 SX gate drops are subtly quicker.
$\beta_5$ 450 MX	NA	No data for 450 Nationals, so no effects.
$\beta_6$ 250 MX	NA	No data for 250 Nationals, so no effects.
$\beta_7$ Heat	-0.7	This indicates the gates drops for heat races are quicker.
$\beta_8$ LCQ	-0.72	This indicates the gates drops for LCQs are quicker.
$\beta_9$ Final	-0.65	This indicates the gates drops for the finals are quicker.

## 3.1 Model Checking

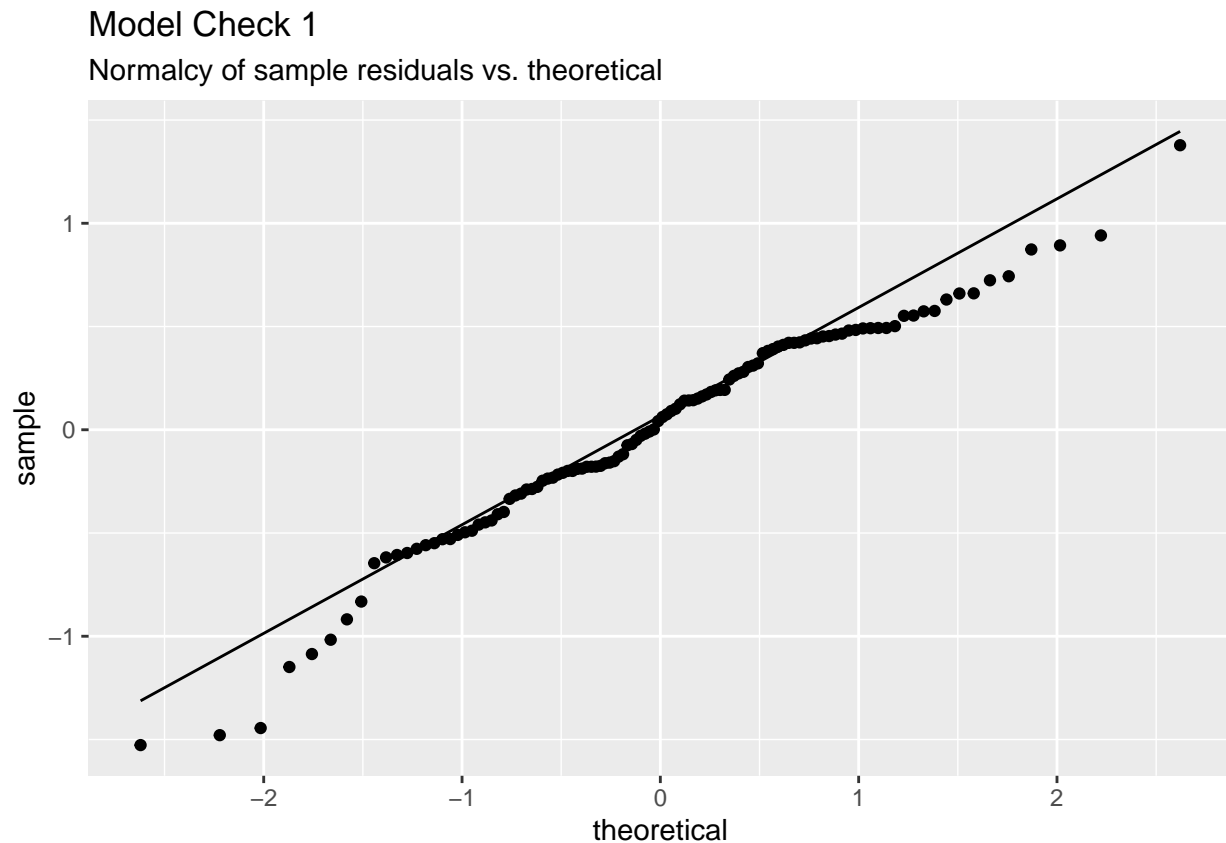
### 3.1.1 Normalcy

Let's do some basic model checking. First, let's check to see if the data follows the normal distribution. We can do this by evaluating the ranked residuals against the theoretical ideal corresponding point in the normal cumulative distribution function. If the ranked residuals generally follow the theoretical line, then we can declare the data follows the normal distribution.

```
unscaled_residual <- tibble(t_model$residuals)

t_data <- t_data |> bind_cols(unscaled_residual)
t_length <- length(t_data)
colnames(t_data)[t_length] <- 'unscaled_residual'

t_qq_plot <- ggplot(data = t_data, aes(sample = unscaled_residual)) + stat_qq() + stat_qq_line() +
  labs(title = "Model Check 1", subtitle = "Normalcy of sample residuals vs. theoretical", y = "sample", x = "theoretical")
t_qq_plot
```

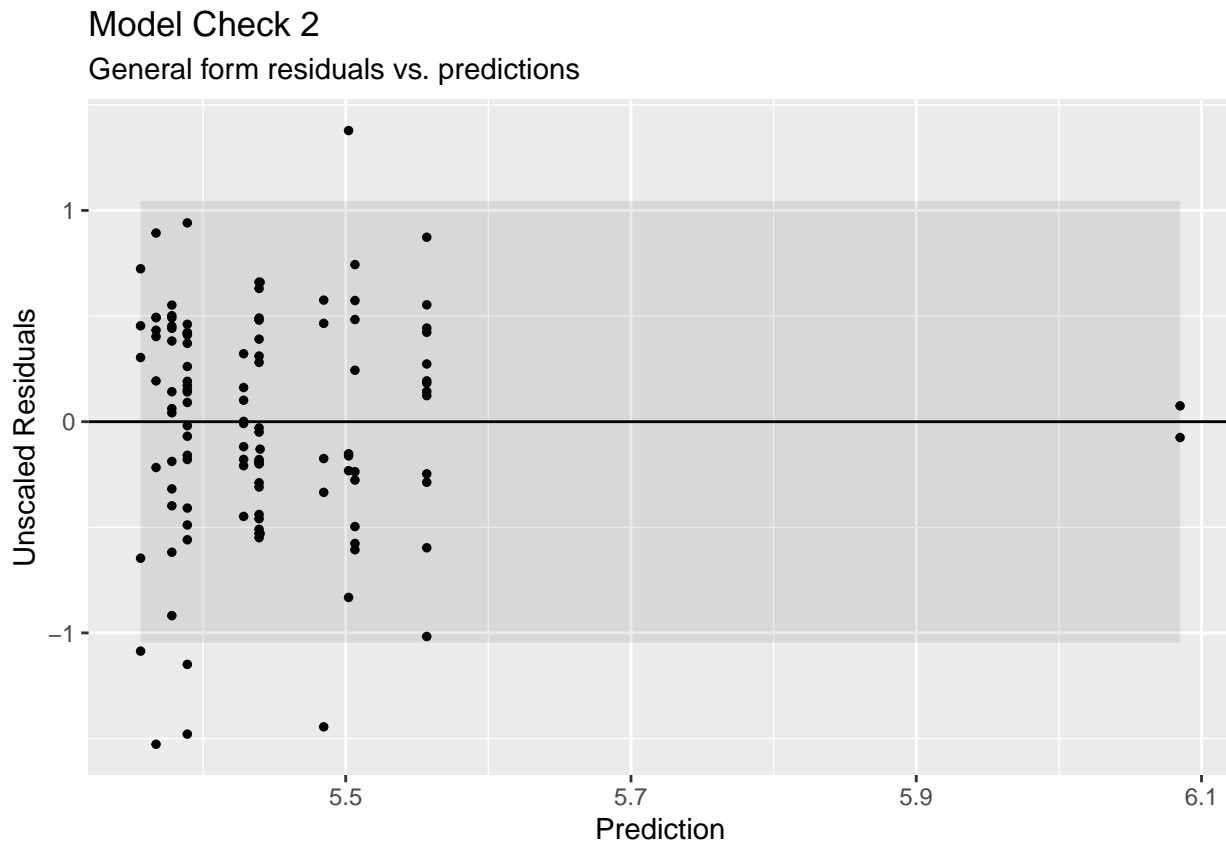


These gate drop times are close to the theoretical normal distribution. However, it is worth noting the data is a little light-tailed on the lower-valued residuals and a little heavy-tailed on the higher-valued residuals. It's unlikely, though this will negatively impact our predictions.

### 3.1.2 Residuals

```
t_predicted_response <- tibble(t_model$fitted.values)
highbound <- 2 * sd(t_data$unscaled_residual)
lowbound <- -2 * sd(t_data$unscaled_residual)

t_data <- t_data |> bind_cols(t_predicted_response)
t_length <- length(t_data)
colnames(t_data)[t_length] <- 't_prediction'
q <- ggplot(data = t_data, aes(y = `unscaled_residual`, x = `t_prediction`)) + geom_point(size=1) +
  labs(title = "Model Check 2", subtitle = "General form residuals vs. predictions", y = "Unscaled Residuals") +
  geom_ribbon(aes(ymin = lowbound, ymax = highbound), alpha = 0.1)
q
```



Above, the shaded region indicates  $\pm 2\sigma$  of the unscaled residuals against each of the predictions. Given that most of the predictions fit within the shaded area, we can say this model passes this check as well.

## 4 Predictions

Let's make some predictions for the 250 and 450 main events!

```
pred_250_main_SX_east <- c(0,1,0,0,0,0,0,0,1)
pred_250_main_SX_west <- c(0,0,1,0,0,0,0,0,1)
pred_450_main_SX <- c(0,0,0,1,0,0,0,0,1)
p_dat <- tibble()
p_dat <- rbind(p_dat, pred_250_main_SX_east, pred_250_main_SX_west, pred_450_main_SX)
names(p_dat) <- t_data_src_cols

a_prediction <- predict(t_model, p_dat, interval = 'confidence')
```

Table 2: Main Event Predictions

Class	Estimate	Lower Bound	Upper Bound
250 SX East	5.43	5.18	5.68
250 SX West	5.56	5.32	5.79
450 SX	5.44	5.24	5.64