12/2/20

Tutorial-04

1) Express $f(x)=x$ for half range
(i) sine series $0<x<2$ (ii) cosine series in $0<x<2$

A)

(i) $f(x)=\sum_{n=1}^{\infty} b_n \sin\frac{n x\pi}{l}$

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$$

$l=2$

$$b_n = \int_0^2 x \sin\frac{n\pi x}{2}dx$$

$$= \left(x\left(-\cos\frac{n\pi x}{2}\right)\times\frac{2}{n\pi}\right)_0^2 + \frac{2}{n\pi}\int_0^2 \cos\frac{n\pi x}{2}dx$$

$$= \frac{-4\cos n\pi}{n\pi} + \frac{2}{n\pi}\times\frac{2}{n\pi}\left(\sin\frac{n\pi x}{2}\right)_0^2$$

$$\boxed{b_n = \frac{4(-1)^{n+1}}{n\pi}}$$

$$f(x)=x=\left(\frac{4}{\pi}\right)\left[\sin\frac{\pi x}{2} - \frac{1}{2}\times\sin\left(\frac{2\pi x}{2}\right) - \left(\frac{1}{3}\right)\sin\frac{3\pi x}{2} - - - -\right]$$

(ii) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$

$l = 2$

$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$ $\quad$ $a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

$a_0 = \int_0^2 x\,dx$

$= \frac{1}{2}\times 4 = 2$

$a_n = \int_0^2 x\cos\frac{n\pi x}{2}\,dx$

$= \left(x\sin\frac{n\pi x}{2}\times\frac{2}{n\pi}\right)_0^2 + \left(\frac{2}{n\pi}\right)^2\left(\cos\frac{n\pi x}{2}\right)_0^2$

$\downarrow$

0

$= \frac{4}{n^2\pi^2}\left(\cos n\pi - 1\right)$

$f(x) = x = 1 - \frac{8}{\pi^2}\left[\cos\frac{\pi x}{2} - \frac{1}{3^2}\times\cos\frac{3\pi x}{2} - \cdots\cdots\right].$

2) Express $f(x) = \cos x$, $0 < x < \pi$, in half range sine series.

A) $l = \pi$ $\quad$ $f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{\pi}$

$b_n = \frac{2}{\pi}\int_0^{\pi} \cos x \sin nx\,dx$

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$$= \frac{1}{\pi}\int_0^{\pi} [\sin(n+1)x + \sin(n-1)x] \, dx$$

$$= \frac{-1}{\pi}\left[\frac{+\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right]_0^{\pi}$$

$$= \frac{-1}{\pi}\left[\frac{\cos(n+1)\pi}{n+1} + \frac{\cos(n-1)\pi}{n-1} - \frac{2n}{n^2-1}\right]$$

if 'n' is even,

$$b_n = \frac{4n}{(n^2-1)\pi}$$

$-\cancel{\left(\frac{2n}{n^2-1}\right)}$

if 'n' is odd

$$b_n = 0.$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{4}{\pi}\left[\left(\frac{2}{3}\right)\sin 2x + \left(\frac{4}{15}\right)\sin 4x + \left(\frac{6}{35}\right)\sin 6x + \cdots\right.$$

$$= \frac{8}{\pi}\left[\left(\frac{1}{3}\right)\sin 2x + \left(\frac{2}{15}\right)\sin 4x + \left(\frac{3}{35}\right)\sin 6x + \cdots\right].$$

3) Half range Sine series,

$f(x) = x(\pi - x)$

$(0, \pi)$.

deduce $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} - \cdots = \pi^3/32$.

A) $l = \pi$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx.$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx.$$

$$= \frac{2\cancel{\pi}}{\cancel{\pi}}\left[\int_0^{\pi} x\sin nx\,dx\right] - \frac{2}{\pi}\int_0^{\pi} x^2 \sin nx\,dx$$

$\Downarrow$

$$\left[x\left(\frac{-\cos nx}{n}\right)\right]_0^{\pi} + \frac{1}{n^2}(\sin nx)_0^{\pi}$$

$$= \frac{-\pi\cos n\pi}{n}$$

$$\int_0^{\pi} x^2 \sin nx\,dx = \left[x^2\left(\frac{-\cos nx}{n}\right)\right]_0^{\pi} + \frac{1}{n}\int_0^{\pi} 2x\cos nx\,dx$$

$$= \frac{-\pi^2\cos n\pi}{n} + \frac{2}{n}\left[\left(\frac{x\sin nx}{n}\right)_0^{\pi} + \frac{1}{n^2}(\cos nx)_0^{\pi}\right]$$

$\downarrow$

0

$$= \frac{-\pi^2\cos n\pi}{n} + \frac{2}{n^3}[\cos n\pi - 1].$$

$$b_n = \cancel{\frac{-2\pi\cos n\pi}{n}} + \cancel{\frac{2\pi\cos n\pi}{n}} - \frac{4}{\pi n^3}[\cos n\pi - 1].$$

$$b_n = \cancel{\frac{\pi \cos n\pi}{n}} - \frac{4}{n^3\pi}(\cos n\pi - 1)$$

$$= \cancel{\frac{\pi \cos n\pi}{n}} + \frac{4}{n^3\pi}(1-\cos n\pi)$$

'n' even, $b_n = \frac{\pi}{n} - \frac{\pi}{n} = 0.$

'n' odd, $b_n = \cancel{\frac{-\pi}{n}} + \frac{4}{n^3\pi}(2)$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \cancel{dx}$$

$$x(\pi-x) = \left(\cancel{-\pi}+\frac{8}{\pi}\right)\sin x + \cancel{\frac{\pi}{2}\sin 2x} + \left(\cancel{\frac{-\pi}{3}}+\frac{8}{2\pi}\right)\sin 3$$

$$+ \cancel{\frac{\pi}{4}\sin 4x} + \cdots$$

$$f(x) = \sum_{n=1}^{\infty} \frac{4}{n^3\pi}(1-(-1)^n)\sin nx$$

$$x(\pi-x) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1-(-1)^n}{n^3}\sin nx$$

$x = \frac{\pi}{2}$

$$\frac{\pi^3}{16\times 2} = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$$

4) Find the i) half range sine ii) half range cosine for $f(x) = x(l-x)$ in $0<x<l$.

A) i)

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$b_n = \frac{2}{l}\int_0^l f(x).\sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\times l\left[\int_0^l x\sin\frac{n\pi x}{l}\,dx\right] - \frac{2}{l}\int_0^l x^2 \sin\frac{n\pi x}{l}\,dx.$$

$$b_n = \frac{4l^2}{n^3\pi^3}(1-\cos n\pi).$$

if 'n' is even,

$b_n = 0$

'n' is odd,

$$b_n = \frac{8l^2}{n^3\pi^3}$$

$$f(x) = x(l-x) = \frac{8l^2}{\pi^3}\left[\sin\frac{\pi x}{l} + \frac{1}{3^3}\times\sin\frac{3\pi x}{l} + \dots\right]$$

ii)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}.$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx = \frac{\cancel{2}}{\cancel{l}}\times \cancel{l}\times\frac{l^2}{\cancel{2}} - \frac{2}{3\cancel{l}}\times l^{\cancel{3}2}$$

$$= l^2 - \frac{2l^2}{3} = \frac{l^2}{3}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$$

$$= 2\int_0^l x\cos\frac{n\pi x}{l}\,dx - \frac{2}{l}\int_0^l x^2\cos\frac{n\pi x}{l}\,dx$$

$$= 2\left[\left(x\times\frac{l}{n\pi}\sin\frac{n\pi x}{l}\right)_0^l + \frac{l^2}{n^2\pi^2}\left(\cos\frac{n\pi x}{l}\right)_0^l\right]$$

$$\downarrow 0$$

$$-\frac{2}{l}\left[\left(x^2\times\frac{l}{n\pi}\sin\frac{n\pi x}{l}\right)_0^l - \frac{2}{n\pi}\int_0^l x\sin\frac{n\pi x}{l}\,dx\right]$$

$-l\cos n\pi$

$$= \frac{+2l^2}{n^2\pi^2}(\cos n\pi - 1) - \frac{2}{l}\left[\frac{-2}{n\pi}\left[\left(x\left(-\cos\frac{n\pi x}{l}\right)\times\frac{l}{n\pi}\right)_0^l + \frac{l^2}{n^2\pi^2}\left(\sin\frac{n\pi x}{l}\right)_0^l\right]\right]$$

$$= \frac{+2l^2}{n^2\pi^2}(\cos n\pi - 1) + \frac{4}{ln^2\pi^2}(-l\cos n\pi)$$

$$f(x) = x(l-x) = \frac{l^2}{6} + \left(\frac{4l^2}{\pi^2}\right)\times\left\{\cos\left(\frac{\pi x}{l}\right) + \frac{1}{3^2}\cos\frac{3\pi x}{l} + \cdots\right\}$$

5) Find series of sines multiples of x,

$f(x) = x$ if $0 < x \le 1$

$f(x) = 2 - x$ if $1 < x < 2$.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx.$$

$$b_n = \frac{2}{2}\left[\int_0^1 x\sin\frac{n\pi x}{2}\,dx + \int_1^2 (2-x)\sin\frac{n\pi x}{2}\,dx\right]$$

$$= \left(x\times\frac{2}{n\pi}\left(-\cos\frac{n\pi x}{2}\right)\right)_0^1 + \frac{4}{n^2\pi^2}\left(\sin\frac{n\pi x}{2}\right)_0^1$$

$$- \frac{4}{n\pi}\left(\cos\frac{n\pi x}{2}\right)_1^2$$

$$- \left[\left(x\times\frac{2}{n\pi}\left(-\cos\frac{n\pi x}{2}\right)\right)_1^2 + \frac{4}{n^2\pi^2}\left(\sin\frac{n\pi x}{2}\right)_1^2\right]$$

$$= \frac{2}{n\pi}\left(-\cos\frac{n\pi}{2}\right) - \frac{4}{n\pi}\left(\cancel{\cos n\pi} - \cos\frac{n\pi}{2}\right) + \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$- \left[\cancel{\frac{4}{n\pi}\left(-\cos n\pi\right)} + \frac{2}{n\pi}\cos\frac{n\pi}{2} - \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}\right]$$

$$= \cancel{\frac{-4}{n\pi}\cos\frac{n\pi}{2}} + \cancel{\frac{4}{n\pi}\cos\frac{n\pi}{2}} + \frac{8}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$f(x) = \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{2}$$

$n \to 2n-1$

$$f(x) = 8/\pi^2 \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n-1)^2}\sin\left\{\frac{(2n-1)\pi x}{2}\right\}$$

6) Periodic func with period 2π, $f(x)=x$ if $-\pi<x\le 0$ & $f(x)=2x$ if $0<x\le\pi$.

Deduce, $\pi^2/8 = 1+1/3^2+1/5^2+ \cdots$

A)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0} x\,dx + \int_{0}^{\pi} 2x\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{2}(-\pi^2) + \pi^2\right]$$

$$\boxed{a_0 = \frac{\pi}{2}}$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} x\cos nx\,dx + 2\int_{0}^{\pi} x\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\left(\underbrace{\frac{x}{n}\sin nx}_{0}\right)_{-\pi}^{0} + \frac{1}{n^2}(\cos nx)_{-\pi}^{0}\right.$$

$$\left. + 2\left[\left(\underbrace{\frac{x}{n}\sin nx}_{0}\right)_0^{\pi} + \frac{1}{n^2}(\cos nx)_0^{\pi}\right]\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n^2}(1-\cos n\pi) + \frac{2}{n^2}(\cos n\pi - 1)\right]$$

$$= \frac{+1}{n^2\pi}(\cos n\pi - 1)$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} x\sin nx\,dx + 2\int_{0}^{\pi} x\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{x}{n}(-\cos nx)\right)_{-\pi}^{0} + \frac{1}{n^2}\underbrace{\left(\sin nx\right)_{-\pi}^{0}}_{0}\right.$$

$$\left. + 2\left[\left(\frac{x}{n} - \cos nx\right)_{0}^{\pi} + \frac{1}{n^2}\underbrace{\left(\sin nx\right)_{0}^{\pi}}_{0}\right]\right].$$

$$= \frac{1}{\pi}\left[\frac{\pi\cos n\pi}{n} - \frac{2}{n}\left(\pi\cos n\pi\right)\right]$$

$$= -\frac{3}{n}\cos n\pi$$

$$= \frac{3}{n}(-1)^{n+1}$$

$$\left(x - \frac{\pi}{4}\right) \times \frac{\pi}{\cancel{4}}$$

$$= \frac{\pi^2}{8\,4}$$

$x = \frac{\pi}{2}$

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \cdots\right)$$

$$+ 3\left(\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \cdots\right)$$

Put $x = 0$,

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \cdots$$

7) $f(x) = e^x$

$$e^x = \sum_{n=1}^{\infty} b_n \sin nx \quad \text{in } (0, \pi)$$

Then,

$$\sum_{n=1}^{\infty} (b_n)^2 \text{ converges to,}$$

a) $(e^{\pi}-1)/\pi$ b) $(e^{\pi}+1)/\pi$ c) $(e^{2\pi}-1)/\pi$

d) $(e^{2\pi}+1)/\pi$

A)

$$b_n = \frac{2}{\pi}\int_0^{\pi} e^x \sin nx\, dx$$

$$\rightarrow \left[\frac{e^x}{n}(-\cos nx)\right]_0^{\pi}$$

$$+ \frac{1}{n}\int_0^{\pi} e^x \cos nx\, dx$$

$$\frac{e^{\pi}}{n}(-\cos n\pi) + \frac{1}{n} + \frac{1}{n}\left[\left(\frac{e^x}{n}\sin nx\right)_0^{\pi}\right]$$

$$0$$

$$- \frac{1}{n}\int_0^{\pi} e^x \sin$$

$$\int_0^{\pi} e^x \sin nx\, dx \left(\frac{\cancel{n^2+1}}{n^2}\right) = \frac{n}{\cancel{n}}\left(1 - e^{\pi}\cos n\pi\right)$$

$$\int_0^{\pi} e^x \sin nx\, dx = \frac{n}{n^2+1}\left(1 - e^{\pi}\cos n\pi\right)$$

$$b_n = \frac{2n}{n^2+1}\left(\frac{1-e^{\pi}\cos n\pi}{\pi}\right)$$

$$b_n^2 = \frac{4n^2}{(n^2+1)^2\pi^2}\left(1+e^{2\pi}-2e^{\pi}\cos n\pi\right)$$

$$b_1 = \frac{2}{\pi}\int_0^{\pi} e^x \sin x \, dx$$

8) $x \in [-\pi, \pi]$

$f(x) = (\pi + x)(\pi - x) = \pi^2 - x^2$

$g(x) = \begin{cases} \cos(1/x) & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

P: Fourier series of f converges uniformly to f on $[-\pi, \pi]$.

Q: " " " g " " " g on $[-\pi, \pi]$

A) $f(x) = f(-x)$ [even func.].

$f(x) = \pi^2 - x^2$ $[-\pi, \pi]$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$

$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\, dx$

$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\, dx$

$a_0 = \frac{2}{\pi}\left[\pi^3 - \frac{\pi^3}{3}\right]$

$\boxed{a_0 = \frac{4\pi^2}{3}}$

$a_n = \frac{2}{\pi}\left[\pi^2 \int_0^{\pi} \cos nx\, dx - \int_0^{\pi} x^2 \cos nx\, dx\right]$

$a_n = \frac{-2}{\pi}\left[\left(\frac{x^2 \sin nx}{n}\right)_0^{\pi} - \frac{2}{n}\int_0^{\pi} x \sin nx\, dx\right]$

$= \frac{4}{n\pi}\left[\left(\frac{x(-\cos nx)}{n}\right)_0^{\pi} + \frac{1}{n^2}(\sin$

$a_n = \frac{-4}{n^2} \cos n\pi$

$f(x) = \frac{2\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \cos nx$

We know the func. in $[-\pi, \pi]$

It will converge to 'f' itself.

$$g(x) = \begin{cases} \cos(1/x) & \text{if } x \neq 0 \\ 0 & ; \text{if } x = 0 \end{cases}$$

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9) $[-4,4]$, $f(x) = -x$ if $-4 \le x \le 0$

$f(x) = x$ if $0 < x < 4$.

$l = 4$

A) $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n x \pi}{l} + b_n \sin\frac{n x \pi}{l}\right)$$

$$a_n = \frac{1}{l}\left[-\int_{-4}^{0} x \cos\frac{n\pi x}{l}\,dx + \int_{0}^{4} x \cos\frac{n\pi x}{l}\,dx\right]$$

$$= \frac{1}{l}\left[-\left[\left(\frac{xl}{n\pi}\sin\frac{n x \pi}{l}\right)_{-4}^{0} + \frac{1}{n^2}\left(\cos\frac{n x \pi}{l}\right)_{-4}^{0}\right] + \left(\underbrace{\frac{xl}{n\pi}\sin\frac{n x \pi}{l}}_{0}\right)_{0}^{4} + \frac{1}{n^2}\left(\cos\frac{n x \pi}{l}\right)_{0}^{4}\right]$$

$$= \frac{1}{l}\left[\underbrace{\frac{-16}{n\pi}\sin n\pi}_{0} - \frac{1}{n^2}(1-\cos n\pi) + \frac{1}{n^2}(\cos n\pi \ldots\right.$$

$$= -\frac{1}{ln^2}(1-\cos n\pi)$$

$$b_n = \frac{1}{4}\left[-\int_{-4}^{0} x \sin\frac{n\pi x}{4}\,dx + \int_{0}^{4} x \sin\frac{n\pi x}{4}\,dx\right]$$

$$= \frac{1}{4}\left[+\left(\frac{x\times 4}{n\pi}\left(+\cos\frac{n\pi x}{4}\right)\right)_{-4}^{0} - \frac{4^2}{n^2\pi^2}\left(\sin\frac{n\pi x}{4}\right)_{-4}^{0} + \left(\frac{4x}{n\pi}\left(-\cos\frac{n\pi x}{4}\right)\right)_{0}^{4} + \frac{16}{n^2\pi^2}\left(\sin\frac{n\pi x}{4}\right)_{0}^{4}\right]$$

$\to 0$ $\quad$ $\to 0$

$b_n = 0$

so, no sine term

(B) No sine term

10) $f(x) = \cos x$ for $0<x<\pi$ & $f(x) = -\cos x$ for $-\pi<x<0$. show that fourier series which converges to $f(x)$ is

$$\frac{\pi}{4}\left(\frac{2}{1\times 3}\sin 2x + \frac{4}{3\times 5}\sin 4x + \frac{6}{5\times 7}\sin 6x + \cdots\right)$$

Draw the graph of the sum func. of series for $-2\pi \le x \le 2\pi$.

A)

$$a_0 = \frac{1}{\pi}\left[\int_0^{\pi}\cos x\,dx + (-1)\int_{-\pi}\cos x\,dx\right]$$

$$= \frac{1}{\pi}\left[0\right] = 0 \qquad a_n = 0.$$

Means no cosine term.

$$b_n = \frac{1}{2\pi}\left[2\int_0^{\pi}\cos x \sin nx\,dx - 2\int_{-\pi}^{0}\cos x \sin nx\,dx\right]$$

11}

$$b_n = \frac{1}{2\pi}\left[\int_0^{\pi}(\sin(n+1)x + \sin(n-1)x)\,dx - \cancel{2}\int_{-\pi}^{0}(\sin(n+1)x + \sin(n-1)x)\right]$$

be

A)

$$= \frac{1}{2\pi}\left[\cancel{2}\left(\frac{-\cos(n+1)x}{n+1} - \frac{\cos(n-1)x}{n-1}\right)_0^{\pi} + \cancel{2}\left(\frac{\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right)_{-\pi}^{0}\right]$$

$\frac{1}{n+1} + \frac{1}{n-1}$

$$= \frac{1}{\cancel{2}\pi}\left[\frac{-\cancel{4}\cos(n+1)\pi}{n+1} - \frac{\cancel{4}\cos(n-1)\pi}{n-1} + \frac{\cancel{4}n}{n^2-1} + \cancel{2}\left(\frac{2n}{n^2-1}\right)\right]$$

n is odd, $\qquad f(x) = \sum_{n=1}^{\infty} b_n \sin nx$

Putting b_n into it

Ar $\frac{4}{\pi}\left(\frac{2}{1\times3}\sin 2x + \frac{4}{3\times5}\sin 4x + \cdots\right)$

11) ST,

$-\pi \leq x \leq \pi$

$$\cot Kx = \frac{\sin Kx}{\pi}\left(\frac{1}{K} - \frac{2K\cos x}{K^2-1^2} + \frac{2K\cos 2x}{K^2-2^2} + \cdots\right),$$ K

being non-integral & deduce

1) $\pi \cot K\pi = \frac{1}{K} + 2K \sum \frac{1}{K^2-n^2}$

2) $\frac{\pi}{\sin K\pi} = \sum (-1)^n \left(\frac{1}{n+K} + \frac{1}{n+K-K}\right)$

A)

12) Obtain a sine series which will be equal to x^2 for $0 \le x < \pi$.

Graph of sum func for $-2\pi \le x \le 2\pi$.

A) $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$

$$b_n = \frac{2}{\pi}\int_0^{\pi} x^2 \sin nx \, dx$$

$$= \frac{2}{\pi}\left[\left(x^2\left(\frac{-\cos nx}{n}\right)\right)_0^{\pi} + \frac{1}{n}\int_0^{\pi} 2x\cos nx \, dx\right]$$

$$= \frac{2}{\pi}\left[\frac{-\pi^2\cos n\pi}{n} + \frac{2}{n}\left[\left(\frac{x}{n}\sin nx\right)_0^{\pi} + \frac{1}{n^2}(\cos nx)_0^{\pi}\right]\right]$$

$$\left(\frac{x}{n}\sin nx\right)_0^{\pi} \to 0$$

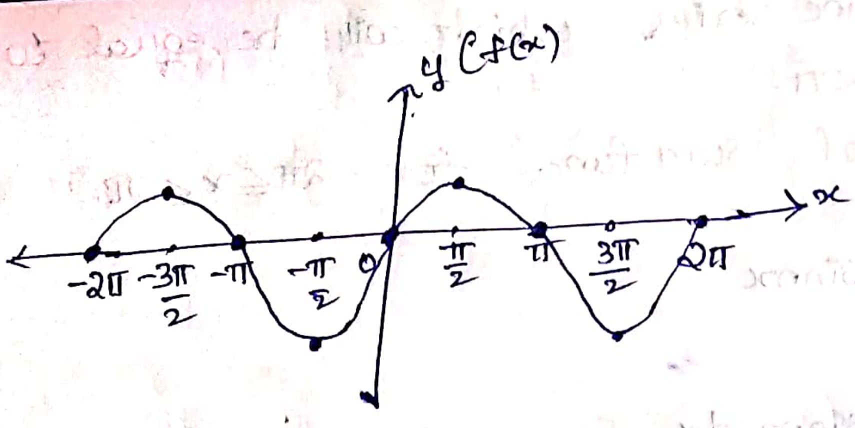
$$b_n = \frac{-2}{\pi}\left[\frac{\pi^2(-1)^n}{n} + \frac{2}{n^3}(1-(-1)^n)\right]$$

$$f(x) = \frac{-2}{\pi}\left[(-\pi^2+4)\sin x + \frac{\pi^2}{2}\sin 2x + \cdots\right]$$

in $-2\pi \le x \le 2\pi$,

at, $-2\pi, -\pi, 0, \pi, 2\pi$, $f(x) = 0$ as it consists of only sine terms.

And some value at, $\frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$, so the graph is,



13) ST in $[0,\pi]$, $f(x)=\frac{\pi^2}{16}-2\sum_{n=1}^{\infty}\frac{\cos(4n-2)x}{(4n-2)^2}$

$f(x)=\frac{\pi x}{4}\quad 0\le x\le\frac{\pi}{2}$

$f(x)=\{\pi(\pi-x)\}/4\quad \pi/2<x\le\pi.$

A)

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\underbrace{\sum_{n=1}^{\infty}b_n\sin nx}_{\text{not there}}$$

Only, cosine series

$$a_0=\frac{2}{\pi}\int_0^{\pi}f(x)\,dx$$

$$=\frac{2}{\pi}\left[\frac{\pi}{4}\int_0^{\pi/2}x\,dx+\frac{\pi}{4}\int_{\pi/2}^{\pi}(\pi-x)\,dx\right]$$

$$=\frac{2}{\pi}\left[\frac{\pi}{8}\times\frac{\pi^2}{4}+\frac{\pi}{4}\left(\pi x-\frac{x^2}{2}\right)_{\pi/2}^{\pi}\right]$$

$\frac{\pi^2}{2}-\frac{\pi^2}{2}$

$$=\frac{2}{\pi}\left[\frac{\pi^3}{32}+\frac{\pi}{4}\left[\pi^2-\frac{\pi^2}{2}-\left(\frac{\pi^2}{2}-\frac{\pi^2}{8}\right)\right]\right]$$

$$=\frac{2}{\pi}\left[\frac{\pi^3}{32}+\frac{\pi^3}{32}\right]=\frac{\pi^2}{16}\times 2=\frac{\pi^2}{8}$$

$$a_n = \frac{2}{\pi}\left[\frac{\pi}{4}\int_0^{\pi/2} x\cos nx\,dx + \frac{\pi}{4}\int_{\pi/2}^{\pi}(\pi - x)\cos nx\,dx\right]$$

$$= \frac{2}{\cancel{\pi}}\times\frac{\cancel{\pi}}{4}\left[\left(\frac{x}{n}\sin nx\right)_0^{\pi/2} + \frac{1}{n^2}\left(\cos nx\right)_0^{\pi/2}\right]$$

$$+ \frac{\pi\times 2}{4n}\left(\sin nx\right)_{\pi/2}^{\pi} - \frac{2}{4}\left[\left(\frac{x}{n}\sin nx\right)_{\pi/2}^{\pi} + \frac{1}{n^2}\left(\cos nx\right)_{\pi/2}^{\pi}\right]$$

$$= \frac{2}{4}\left[\frac{\pi}{2n}\sin\frac{n\pi}{2} + \frac{1}{n^2}\left(\cos\frac{n\pi}{2} - 1\right)\right] - \frac{\pi}{4n}\sin\frac{n\pi}{2}\times 2$$

$$- \frac{2}{4}\left[\left[-\frac{\pi}{2n}\sin\frac{n\pi}{2} + \frac{\cos n\pi - \cos\frac{n\pi}{2}}{n^2}\right]\right]$$

$$= 2\left[\cancel{\frac{\pi}{8n}\sin\frac{n\pi}{2}} + \frac{1}{4n^2}\left(\cos\frac{n\pi}{2} - 1\right) - \cancel{\frac{\pi}{4n}\sin\frac{n\pi}{2}} + \cancel{\frac{\pi}{8n}\sin\frac{n\pi}{2}}\right.$$

$$\left. - \frac{1}{4n^2}\left(\cos n\pi - \cos\frac{n\pi}{2}\right)\right]$$

$$= 2\left[\frac{2}{2n^2\times 2}\cos\frac{n\pi}{2} - \frac{1}{4n^2}\left(1 + \cos n\pi\right) - \cancel{\frac{\pi n}{4n\times n}\sin\frac{n\pi}{2}}\right]$$

$$= \frac{2}{4n^2}\left[2\cos\frac{n\pi}{2} - 1 - (-1)^n + \cancel{\pi n\sin\frac{n\pi}{2}}\right]$$

n is even,

$$a_n = \frac{2\times 2}{4n^2}\left[\cos\frac{n\pi}{2}\right] = \frac{2}{2n^2}\cos\frac{n\pi}{2}$$

If cosine series,

$$f(x) = \frac{\pi^2}{16} - 2\sum_{n=1}^{\infty} \frac{\cos(4n-2)x}{(4n-2)^2}$$

14) ST, $x+x^2 = \frac{1}{3} + \frac{2}{\pi}\sum_{n=1}^{\infty} (-1)^n \left(\frac{2\cos n\pi x}{n^2\pi^2} - \frac{\cos n\pi}{n}\right.$

$-1 < x < 1.$

A) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos \frac{n\pi}{l}x + b_n \sin \frac{n\pi}{l}x\right)$

$l = 1$

$$a_0 = \int_{-1}^{1} (x+x^2)\,dx$$

$$a_0 = \frac{1}{3} + \frac{1}{3} \Rightarrow \boxed{a_0 = \frac{2}{3}}$$

$$a_n = \frac{1}{\cancel{\pi}}\int_{-1}^{1} (x+x^2)\cos n\pi x\,dx$$

$$= \frac{1}{\cancel{\pi}}\left[\int_{-1}^{1} x\cos n\pi x\,dx + \int_{-1}^{1} x^2 \cos n\pi x\,dx\right]$$

$$= \frac{1}{\cancel{\pi}}\left[\left(\frac{x}{n\pi}\sin n\pi x\right)_{-1}^{1} - \frac{1}{n\pi}\int_{-1}^{1}\sin n\pi x\,dx + \left(\frac{x^2}{n\pi}\sin n\pi x\right)_{-1}^{1} - \frac{2}{n\pi}\int_{-1}^{1} x\sin n\pi x\,dx\right]$$

0   0

$$= \frac{1}{\cancel{\pi}}\left[\frac{1}{n^2\pi^2}(\cos n\pi - \cos n\pi) - \frac{2}{n\pi}\left[\left(\frac{-x}{n\pi}\cos n\pi x\right)_{-1}^{1} + \frac{1}{n^2\pi^2}(\sin n\pi x)_{-1}^{1}\right]\right]$$

0

$$= \frac{+4}{n^2\pi^2}(-1)^n$$

$$b_n = \frac{1}{\pi}\left[\int_{-1}^{1} x \sin n\pi x \, dx + \int_{-1}^{1} x^2 \sin n\pi x \, dx\right]$$

$$= \left(\frac{-x}{n\pi}\cos n\pi x\right)_{-1}^{1} + \frac{1}{n^2\pi^2}\left(\sin n\pi x\right)_{-1}^{1}$$

$$+ \left(\frac{-x^2}{n\pi}\cos n\pi x\right)_{-1}^{1} + \frac{2}{n\pi}\int_{-1}^{1} x\cos n\pi x \, dx$$

$$= \frac{-2}{n\pi}\cos n\pi + \frac{2}{n\pi}\left[\left(\frac{x}{n\pi}\sin n\pi x\right)_{-1}^{1} + \frac{1}{n^2\pi^2}\left(\cos n\pi x\right)_{-1}^{1}\right]$$

$$= \frac{-2(-1)^n}{n\pi}$$

$$x + x^2 = \frac{1}{3} + 2\sum_{n=1}^{\infty}(-1)^n\left(\frac{2\cos n\pi x}{n^2\pi^2} - \frac{\sin n\pi x}{n\pi}\right)$$

$0 \le x \le 1$

$$x - \frac{1}{2} = -\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{\sin 2n\pi x}{n}, \quad x^2 - x + \frac{1}{6} = \frac{1}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos 2n\pi x}{n^2}$$

[as here $0 \le x \le 1$]

$$a_0 = 2\left(\int_0^1\left(x - \frac{1}{2}\right)dx\right) = \left(\frac{x^2}{2} - \frac{x}{2}\right)_0^1 \times 2 = 0$$

$$a_n = 2\int_0^1\left(x - \frac{1}{2}\right)\cos n\pi x \, dx = 2\left(\left(\frac{x}{n\pi}\sin n\pi x\right)_0^1 + \frac{1}{n^2\pi^2}\left(\cos n\pi x\right)_0^1 - \frac{1}{2n\pi}\left(\sin n\pi x\right)_0^1\right)$$

But, we need only sine series,

$$\frac{1}{2} b_n = \left(\frac{-x}{n\pi} \cos n\pi x\right)_0^1 + \frac{1}{n^2\pi^2}\left(\sin n\pi x\right)_0^1$$

$$\downarrow$$

$$0 \qquad + \frac{1}{2n\pi}(\cos n\pi x)$$

$$b_n = \frac{-1}{n\pi}\left(1+(-1)^n\right) = \frac{-1}{n\pi}\left(+(-1)^n+1\right)$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x$$

$$x - \frac{1}{2} = \frac{-1}{\pi} \sum_{n=1}^{\infty} \frac{\sin 2n\pi x}{n}$$

(Cos value is only for even).

as, $(0, 2l)$

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x) \sin \frac{n\pi x}{l}\,dx$$

Now, for

$$x^2 - x + \frac{1}{6}$$

$$= \frac{1}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos 2n\pi x}{n^2}$$

$f(x) = x^2 - x + \frac{1}{6}$ [Cosine Series]

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x$$

$$a_0 = 2\int_0^1 \left(x^2 - x + \frac{1}{6}\right) dx$$

$$= 0.$$

$$a_n = 2\left[\int_0^1 x^2 \cos n\pi x\, dx - \int_0^1 x \cos n\pi x\, dx + \underbrace{\frac{1}{6n\pi}\left(\sin n\pi x\right)_0^1}_{0}\right]$$

$$a_n = 2\left[\underbrace{\left(\frac{x^2}{n\pi}\sin n\pi x\right)_0^1}_{0} - \frac{2}{n\pi}\int_0^1 x \sin n\pi x\, dx - \left[\left(\frac{x}{n\pi}\sin n\pi x\right)_0^1 + \frac{1}{n^2\pi^2}\left(\cos n\pi x\right)_0^1\right]\right]$$

$$a_n = 2\left[-\frac{2}{n\pi}\left(\left(\frac{-x}{n\pi}\cos n\pi x\right)_0^1 + \underbrace{\frac{1}{n^2\pi^2}\left(\sin n\pi x\right)_0^1}_{0}\right) - \frac{1}{n^2\pi^2}\left(\cos n\pi - 1\right)\right]$$

$$= 2\left[\frac{-2}{n\pi}\left(-\frac{1}{n\pi}\cos n\pi\right) - \frac{1}{n^2\pi^2}\left(\cos n\pi - 1\right)\right]$$

$$= \frac{+2}{n^2\pi^2}\left[2\cos n\pi + 1\right] = \frac{2}{n^2\pi^2}\left(1 + \cos n\pi\right)$$

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30)

$$x^2 - x + \frac{1}{6} = \frac{1}{\pi^2}\sum_{n=1}^{\infty} \frac{\cos 2n\pi x}{n^2}$$

Cos only for (+)ve values

n only even

$a_n = 0$ (if n is odd)

16) If $f(x) = (a/4) - x \quad 0 \le x \le a/2$

$f(x) = x - (3a/4) \quad a/2 \le x \le a.$

$\forall$ x in $0 \le x \le a.$

A) Cosine series, $l = a.$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{a}$$

$$a_0 = \frac{2}{a}\left[\left(\frac{ax}{4} - \frac{x^2}{2}\right)_0^{a/2} + \left(\frac{x^2}{2} - \frac{3ax}{4}\right)_{a/2}^{a}\right]$$

$$\Rightarrow \boxed{a_0 = 0}$$

$$a_n = \frac{2}{a}\left[\int_0^{a/2}\left(\frac{a}{4} - x\right)\cos\frac{n\pi x}{a}\,dx + \int_{a/2}^{a}\left(x - \frac{3a}{4}\right)\cos\frac{n\pi x}{a}\right.$$

$$a_n = \frac{2}{a}\left[\frac{a}{4} \times \frac{a}{n\pi}\left(\sin\frac{n\pi}{2}\right) - \left[\left(\frac{x \times a}{n\pi}\sin\frac{n\pi x}{a}\right)_0^{a/2} + \frac{a^2}{n^2\pi^2}\left(\cos\frac{n\pi x}{a}\right)_0^{a/2}\right]\right.$$

$$+\left[\left(\frac{ax}{n\pi}\sin\frac{n\pi x}{a}\right)_{a/2}^{a}+\frac{a^2}{n^2\pi^2}\left(\cos\frac{n\pi x}{a}\right)_{a/2}^{a}\right]$$

$$\left.-\frac{a}{2}\times\frac{a}{n\pi}\sin\frac{n\pi}{2}\right]$$

$$a_n=\frac{2a}{n^2\pi^2}\left(\cos n\pi-2\cos\frac{n\pi}{2}+1\right)$$

$$f(x)=\frac{2a}{\pi^2}\sum_{n=1}^{\infty}\frac{\left(\cos n\pi-2\cos\frac{n\pi}{2}+1\right)\cos\frac{n\pi x}{a}}{n^2}$$

replace $n\to(2n-1)$

$$f(x)=\frac{2a}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\frac{(4n-2)\pi x}{a}$$

i) In form of $a_n\sin(n\pi x)$ $f(x)=\sin h\pi x,\ 0\le x<1/2$

$f(x)=0$ for $1/2<x\le 1$.

Deduce,

$$\sum_{n=1}^{\infty}\frac{1}{n^2+(n+1)^2}=\frac{\pi}{2}\tanh\frac{\pi}{2}$$

Sine Series,

$$f(x)=\sum_{n=1}^{\infty}b_n\sin n\pi x$$

$$b_n=\frac{\cancel{2}}{\cancel{1}}\int_0^{1/2}\left(e^{\pi x}-e^{-\pi x}\right)\sin n\pi x\,dx$$

$$b_n = \int_0^{1/2} \left(e^{\pi x} - e^{-\pi x}\right) \sin n\pi x \, dx$$

$$\int_0^{1} e^{\pi x} \sin n\pi x \, dx = \left(\frac{+e^{\pi x}}{n\pi} (\cos n\pi x) \right)_0^{1/2} + \frac{1}{n\pi^2} \int_0^{1/2} e^{\pi x} \cos n\pi x \, dx$$

$$= \frac{-e^{\pi/2}}{n\pi} \cos \frac{n\pi}{2} + \frac{1}{n\pi} + \frac{\pi}{n\pi^2} \left[\left(\frac{e^{\pi x}}{n\pi} \sin n\pi x \right)_0^{1/2} - \frac{\pi}{n\pi^2} \int_0^{1/2} e^{\pi x} \sin n\pi x \, dx \right]$$

18)

$$\left(1 + \frac{\pi^2}{n^2\pi^4} \right) \int_0^{1/2} e^{\pi x} \sin n\pi x \, dx = \frac{1}{n\pi} \left(1 - e^{\pi/2} \cos \frac{n\pi}{2} \right) + \frac{\pi}{n^2\pi^3} e^{\pi/2} \sin \frac{n\pi}{2}$$

As

$$\int_0^{1/2} e^{\pi x} \sin n\pi x \, dx = \frac{n\pi^3}{1+n^2\pi^4} \left(1 - e^{\pi/2} \cos \frac{n\pi}{2} \right) + \frac{\pi^2}{1+n^2\pi^4} e^{\pi/2} \sin \frac{n\pi}{2}$$

$$\int_0^{1/2} e^{-\pi x} \sin n\pi x \, dx = \left(\frac{-e^{-\pi x}}{n\pi} \cos n\pi x \right)_0^{1/2} - \frac{\pi}{n\pi^2} \int_0^{1/2} e^{-\pi x} \cos n\pi x \, dx$$

$$= \frac{-e^{-\pi/2}}{n\pi} \cos \frac{n\pi}{2} + \frac{1}{n\pi} - \frac{\pi}{n\pi^2} \left[\left(\frac{e^{-\pi x}}{n\pi} \sin n\pi x \right)_0^{1/2} + \frac{\pi}{n\pi^2} \int e^{-\pi x} \sin n\pi x \, dx \right]$$

$$= \frac{1}{n\pi}\left(1 - e^{-\pi/2}\cos\frac{n\pi}{2}\right) - \frac{1}{n^2\pi^4}\int_0^{1/2} e^{-\pi x}\sin n\pi x\,dx$$

$$- \frac{\pi}{n^2\pi^3} e^{-\pi/2}\sin\frac{n\pi}{2}$$

$$\int_0^{1} e^{-\pi x}\sin n\pi x\,dx = \frac{n\pi^3}{\pi^2 + n^2\pi^4}\left(1 - e^{-\frac{\pi}{2}}\cos\frac{n\pi}{2}\right) - \frac{\pi^2}{1+n^2\pi^4} e^{-\pi/2}\sin\frac{n\pi}{2}$$

$$b_n = \frac{n\pi^3\cos n\pi}{\pi^2 + n^2\pi^4}\left(-e^{\pi/2} + e^{-\pi/2}\right) + \frac{\pi^2}{\pi^2 + n^2\pi^4}\sin\frac{n\pi}{2}\left(e^{\pi/2} - e^{-\pi/2}\right)$$

$$b_n = \frac{2n}{(1+n^2)\pi}\left[\cos\frac{h\pi}{2}\sin\frac{n\pi}{2} - \sin\frac{h\pi}{2}\cos\frac{n\pi}{2}\right] \quad n = \text{even}, \; f(x) = 0$$

$$f\left(\frac{1}{2}\right) = \sum_{n=1}^{\infty} \frac{2\cosh\pi/2}{(1+4n^2+4n+1)\pi} \sin^2\left(\frac{2n+1}{2}\right)\pi \Rightarrow \frac{\pi}{2}\tanh\frac{\pi}{2} = \sum_{n=1}^{\infty}\frac{1}{n^2+2(n+1)}$$

$$y^2 = \frac{a^2}{3} + \frac{16a^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2x-1)^3}\cos\frac{(2x-1)\pi x}{2a} + \frac{8a^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2}\cos\frac{n\pi x}{a}.$$

$[-2a, 2a]$

$$f(x) = 0, \quad -2a \le x \le -a$$

$$f(x) = a^2 - x^2, \quad -a \le x \le a$$

$$= 0, \quad a \le x \le 2a.$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{a} + b_n\sin\frac{n\pi x}{a}\right)$$

$$a_0 = \frac{1}{2a}\left[\int_{-2a}^{-a} 0\,dx + \int_{-a}^{a} a^2\,dx - \int_{-a}^{a} x^2\,dx\right]$$

$$= \frac{1}{2a}\left[0 + 2a^3 - \frac{2}{3}(a^3)\right] = \frac{2a^2}{3}$$

$$a_n = \frac{1}{2a}\int_{-a}^{a}(a^2-x^2)\cos\frac{n\pi x}{a}\,dx$$

$$= \frac{1}{2a}\left[\left(\frac{(a^2-x^2)\left(\sin\frac{n\pi x}{a}\right)}{n\pi}\right)_{-a}^{a}\right] + \frac{1}{a}\int_{-a}^{a}\frac{x\left(\sin\right.}{\frac{n\pi}{a}}$$

$$+ \frac{1}{2a}\int_{-a}^{a}$$

$$a_n = \frac{a}{n^2\pi^2}\left[-a\cos n\pi - a\cos n\pi\right]$$

$$\boxed{a_n = \frac{-2a^2(-1)^n}{n^2\pi^2}}$$

$$b_n = \frac{1}{2a}\int_{-a}^{a}(a^2-x^2)\sin\frac{\pi x}{a}\,dx$$

$$b_n = \frac{1}{2n\pi}[0] + \frac{1}{n\pi}\int_{-a}^{a}x\cos\frac{n\pi x}{a}\,dx$$

$$\boxed{b_n = 0}$$

$$y^2 = \frac{a}{3} + \sum_{n=1}^{\infty}\frac{(-1)^n(-2a^2)}{n^2\pi^2}\cos\frac{n\pi x}{a}$$

19) $f(x) = \begin{cases} -3\pi, & -\pi < x \leq 0 \\ 3\pi, & 0 < x < \pi \end{cases}$ Period $= 2\pi$

coeff. of $3x$, in FSE of $f(x)$ in $[-\pi, \pi]$ ____

A) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

where $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$, $n = 1, 2, 3, \ldots$

b_3 is req. coeff. of $\sin 3x$,

$$b_3 = \frac{1}{\pi} \left[\int_{-\pi}^{0} f(x) \sin 3x \, dx + \int_{0}^{\pi} f(x) \sin 3x \, dx \right]$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} (-3\pi) \sin 3x \, dx + \frac{1}{\pi} \int_{-\pi}^{0} (3\pi) \sin 3x \, dx$$

using given value of $f(x)$.

$$= \left[\cos 3x\right]_{-\pi}^{0} - \left[\cos 3x\right]_{0}^{\pi}$$

$$= 4$$

20) a) $f(x) = x$, $-l \leq x \leq l$

A) [Odd func.]

$$a_0 = \frac{1}{l} \int_{-l}^{l} x \, dx = 0$$

$a_n = a_0 = 0$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$a_n = 0$.

$$b_n = \frac{2}{l}\int_0^l x \sin\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\left[\left(\frac{-xl}{n\pi}\left(\cos\frac{n\pi x}{l}\right)\right)_0^l + \frac{l^2}{n^2\pi^2}\left(\sin\frac{n\pi x}{l}\right)_0^l\right]$$

$$\downarrow$$
$$0$$

$$= \frac{2}{l}\left[\frac{-l^2}{n\pi}\cos n\pi\right]$$

$$= \frac{-2l}{n\pi}(-1)^n$$

$$f(x) = \frac{2l}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\left(\frac{n\pi x}{l}\right)$$

b) Period $= 2\pi$, $f(x) = x^2$, $-\pi \le x \le \pi$

A) Even func.

$\therefore b_n = 0$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\left(\frac{x^3}{3}\right)_0^{\pi} = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\left[(x^2)\frac{\sin nx}{n} - (2x)\left(\frac{-\cos nx}{n^2}\right)\right.$$
$$\left. + (2)\left(\frac{-\sin nx}{n^3}\right)\right]$$

C)

A)

$$= \frac{2}{\pi}\left[\frac{2\pi(-1)^n}{n^2}\right] = \frac{4(-1)^n}{n^2}$$

$$\therefore x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx$$

c) $[-2, 2]$

$f(x) = 0, \ -2 < x < 0$

$f(x) = 1, \ 0 < x < 2.$

A) $l = 2$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{2} + b_n\sin\frac{n\pi x}{2}\right)$$

$$a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\,dx$$

$$= \frac{1}{2}(x)_0^2 = 1$$

$$a_n = \frac{1}{2}\int_{-2}^{2} f(x)\cos\frac{n\pi x}{2}\,dx$$

$$= \frac{1}{2}\left[\frac{\sin\frac{n\pi x}{2}}{n\pi/2}\right]_0^2 = 0$$

$$b_n = \frac{1}{2}\int_{-2}^{2} f(x)\sin\frac{n\pi x}{2}\,dx = \frac{1}{2}\left[-\frac{\cos\frac{n\pi x}{2}}{\frac{n\pi}{2}}\right]_0^2$$

$$= \frac{1}{n\pi}\left[1-(-1)^n\right]$$

$$\therefore f(x) = \frac{1}{2} + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n}\sin\frac{n\pi x}{2}$$

$n \to 2n-1$

$$f(x) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin \frac{(2n-1)\pi x}{2}$$

d) $f(x) = x, \; -\pi \le x < 0$

$f(x) = -x, \; 0 \le x < \pi$

A)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx).$$

$$a_0 = \frac{1}{\pi} \left[\int_{-\pi}^{0} x\,dx + \int_{0}^{\pi} -x\,dx \right]$$

$$= \frac{1}{\pi} \left[\frac{1}{2}(-\pi^2) - \frac{1}{2} \times \pi^2 \right] = -\pi$$

$$b_n = \frac{1}{\pi} \left[\int_{-\pi}^{0} x \sin nx\,dx + (-1)\int_{0}^{\pi} x \sin nx\,dx \right]$$

$$= \frac{1}{\pi} \left[\left(-\frac{x}{n} \cos nx\right)_{-\pi}^{0} + \frac{1}{n^2} (\sin nx)_{-\pi}^{0} \right.$$

$$\left. - \left[\left(-\frac{x}{n} \cos nx\right)_{0}^{\pi} + \frac{1}{n^2} (\sin nx)_{0}^{\pi} \right] \right]$$

0 0

$$= \frac{1}{\pi} \left[\frac{-\pi}{n} \cos n\pi + \frac{\pi}{n} \cos n\pi \right]$$

$$= 0$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} x\cos nx\,dx + (-1)\int_{0}^{\pi} x\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\left(\underbrace{\frac{x}{n}\sin nx}_{0}\right)_{-\pi}^{0} + \frac{1}{n^2}(\cos nx)_{-\pi}^{0} - \left[\left(\underbrace{\frac{x}{n}\sin nx}_{0}\right)_{0}^{\pi} + \frac{1}{n^2}(\cos nx)_{0}^{\pi}\right]\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n^2}[1-\cos n\pi] - \frac{1}{n^2}(\cos n\pi - 1)\right]$$

$$= \frac{2}{n^2\pi}(1-\cos n\pi)$$

$$f(x) = \frac{-\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2}$$

$n \to 2n-1$

$$f(x) = \frac{-\pi}{2} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2}$$

e) $f(x) = x, \quad 0 < x < 1$

$= 2-x, \quad 1 \le x < 2$ $\qquad (0, 2l)$

$f(2), f(2.5), f(-2.5) = ?$ $\qquad 2l = 2$, $l = 1$

A) $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos n\pi x + b_n\sin n\pi x)$$

$$a_0 = \frac{1}{1}\left[\int_0^1 x\,dx + \int_1^2 (2-x)\,dx\right] = \frac{1}{2} + 2 - \frac{3}{2} = \underline{1}$$

$$b_n = \frac{1}{1}\left[\int_0^1 x \sin n\pi x\,dx + 2\int_1^2 \sin n\pi x\,dx - \int_1^2 x \sin n\pi x\,dx\right.$$

$$b_n = \left[\left(\frac{-x}{n\pi}(\cos n\pi x)\right)_0^1 + \frac{1}{n^2\pi^2}(\sin n\pi x)_0^1\right.$$

$$\downarrow$$

$$0$$

$$-\frac{2}{n\pi}(\cos n\pi x)_1^2$$

$$+\left(\frac{x}{n\pi}(\cos n\pi x)\right)_1^2$$

$$-\frac{1}{n^2\pi^2}(\sin n\pi x)_1^2$$

$$\downarrow$$

$$0$$

$$b_n = \frac{-1}{n\pi}\cos n\pi - \frac{2}{n\pi}(1-\cos n\pi) + \frac{2}{n\pi} - \frac{1}{n\pi}\cos$$

$$b_n = 0$$

$$a_n = \int_0^1 x\cos n\pi x\,dx + 2\int_1^2 \cos n\pi x\,dx - \int_1^2 x\cos n\pi x\,dx$$

$$a_n = \frac{-2}{n^2\pi}(1-(-1)^n)$$

$$n \to (2n-1)$$

$$f(x) = \frac{1}{2} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(2n-1)\pi x}{(2n-1)^2}$$

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A)

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$f(2) = 0$

$2.5 = \frac{5}{2}$ $\quad f(2.5) = f\left(\frac{5}{2}\right) = \frac{1}{2} - \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{\cos(2n-1)\frac{5\pi}{2}}{(2n-1)^2}$ $\;^{0}$

$f(2.5) = \frac{1}{2} = 0.5$

as $\cos(-x) = \cos(x)$

$f(-2.5) = 0.5$

4) $f(x) = x + x^2, \quad -\pi < x < \pi$

ST, $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

4)
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi}\left[\left(\frac{x^2}{2}\right)_{-\pi}^{\pi} + \left(\frac{x^3}{3}\right)_{-\pi}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi^3}{3} + \frac{\pi^3}{3}\right]$$

$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x\cos nx\,dx + \int_{-\pi}^{\pi} x^2 \cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\underbrace{\left(\frac{x}{n}\sin nx\right)_{-\pi}^{\pi}}_{0} - \frac{1}{n}\int_{-\pi}^{\pi} \sin nx\,dx\right.$$

$$\left. + \underbrace{\left(\frac{x^2}{n}\sin nx\right)_{-\pi}^{\pi}}_{0} - \frac{2}{n}\int_{-\pi}^{\pi} x\sin nx\,dx\right]$$

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$$a_n = \frac{1}{\pi}\left[\frac{-2}{n}\int_{-\pi}^{\pi} x\sin nx\,dx\right]$$

$$= \frac{-2}{n\pi}\left[\left(\frac{-x}{n}\cos nx\right)_{-\pi}^{\pi} + \frac{1}{n^2}\underbrace{\left(\sin nx\right)_{-\pi}^{\pi}}_{0}\right]$$

$$= \frac{-2}{n^2\pi}\left[-\pi\cos n\pi - \pi\cos n\pi\right]$$

$$= \frac{4(-1)^n}{n^2}$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} x\sin nx\,dx + \int_{-\pi}^{\pi} x^2\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{-x}{n}\cos nx\right)_{-\pi}^{\pi} + \frac{2}{n}\int_{-\pi}^{\pi} x\cos nx\,dx\right]$$

$$= \frac{-2(-1)^n}{n}$$

$$f(x) = x + x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx - 2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin nx$$