Momentum and Energy Summary

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1 Equations

1.1 Energy without a spring

The variables E_k and $E_k\prime$ are dependant on whether the object has any motion. The variable E_g is dependant on whether the object begins with / ends with a specific height.

$E_{tot} = E_{tot}\prime$

$E_k + E_g = E_k\prime + E_g\prime$

$(\frac{1}{2}mv^2) + (mgh) = (\frac{1}{2}mv^2)\prime + (mgh)\prime$

1.2 Energy with a spring

If a force/object is pushing on the spring then the variable E_e has a value. Else, set it's value to zero. With this being, if an object is being dropped onto a spring, only $E_e\prime$ has a value and $E_k\prime$ is zero.

$E_{tot} = E_{tot}\prime$

$E_k + E_g + E_e = E_k\prime + E_g\prime + E_e\prime$

$(\frac{1}{2}mv^2) + (mgh) + (\frac{1}{2}kx^2) = (\frac{1}{2}mv^2)\prime + (mgh)\prime + (\frac{1}{2}kx^2)\prime$

1.3 Elastic - Momentum

The two objects do ***not*** move together after colliding. Instead, they seperate from eachother, both moving in different paths.

$P_{tot} = P_{tot}\prime$

$P_a + P_b = P_a\prime + P_b\prime$

$(m_a)(v_a) + (m_b)(v_b) = (m_a)(v_a)\prime + (m_b)(v_b)\prime$

1.4 Inelastic - Momentum

The two objects have conjoined and are both moving together after their collision. Problems that require these equations are inelastic.

$P_{tot} = P_{tot'}$

$P_a + P_b = P_{ab}$

$(m_a)(v_a) + (m_b)(v_b) = (m_{a+b})(v_{a\times b})$

2 Units

- $F_s = N$
- $k = \frac{N_{ewtons}}{m_{eter}}$
- $x = m_{eters}$
- $P_{tot} = \frac{kgm}{s}$
- $E_e = J$
- $W = J$
- $\Delta E = J$
- $E_{tot} = J$

3 Collisions

Elastic vs. In-Elastic

In an ELASTIC collision, the kinetic energy of the SYSTEM is equal before and after the collision. In an INELASTIC collision, the kinetic energy of the SYSTEM is NOT equal between the before and after of the collision.

An elastic collision occurs when two objects collide and are physically the same after the collision as they were before the collision. An inelastic collision occurs when the physical shape of the objects involved has been altered and is different after compared to before the collision.

Calculation Steps

If you get stuck on a problem, try to use $P_{tot} = P_{tot'}$ and/or $E_{tot} = E_{tot'}$ to solve for what you need.

1. Diagram
2. Givens
3. What are you looking for?
4. $P_{tot} = P_{tot'}$ or $E_{tot} = E_{tot'}$

4 Energy and Springs

Calculation Steps (No 5 Steps)

Recall Hooke's Law (The extension or compression of a spring). E_e is Elastic Potential Energy - The stored energy in a spring from it's compression or extension.

1. $F_s = kx$ or $k = \frac{F_s}{x}$
2. $E_k + E_g + E_e = E_k\prime + E_g\prime + E_e\prime$

5 Impulses

Equations

Impulses do not require components. Rearrange the equations below to solve for what you need.

- $\Delta P = m\Delta v$
- $\Delta P = ma\Delta t$
- $\Delta P = F_{net}\Delta t$
- $a = \frac{\Delta v}{\Delta t}$
- $\Delta v = a\Delta t$
- $F_{net} = ma$
- $\Delta v = (v_2 - v_1)$

Calculation Steps

Impulses are about the push back of an opposing force.

1. Diagram
2. Givens
3. What are you looking for?
4. Use the equations above to solve for what you need.

6 2D Momentum

Calculation Steps

Similar to collisions except it includes both x and y components.

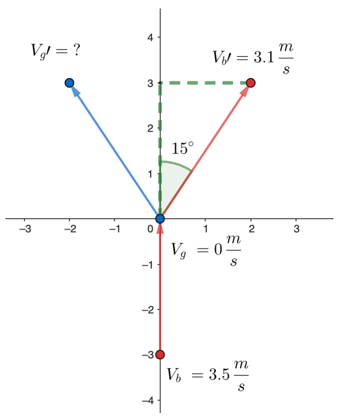
1. Diagram
2. Givens
3. What are you looking for?
4. Components
5. $P_{tot_x} = P_{tot_x}\prime$ and $P_{tot_y} = P_{tot_y}\prime$
6. $c^2 = a^2 + b^2$

7 Example Equations

7.1 2D Momentum - Question

A billiard ball with a mass of 0.155kg is rolling directly away from you at 3.5 $\frac{m}{s}$. It collides with a stationary golf ball with a mass of 0.05kg. The billiard ball rolls off at an angle of 15° clockwise from it's original direction with a velocity of 3.1$\frac{m}{s}$. What is the after velocity of the golf ball?

7.1 2D Momentum - Graph and Givens



- $m_b = 0.155kg$
- $m_g = 0.052kg$
- $V_b = 3.5\frac{m}{s}$
- $V_g = 0\frac{m}{s}$
- $V_b\prime = 3.1\frac{m}{s}$ $[15°clockwise]$
- $V_g\prime = ?$

7.1 2D Momentum - Solve

This solve requires both x and y components. Since v_b only has a y direction, v_{b_x} is zero. Also, since v_g isn't moving both v_{g_x} and v_{g_y} are zero.

1. $P_{tot_x} = P_{tot_x}\prime$

$$(m_g)(\cancel{v}^0_{g_x}) + (m_b)(\cancel{v}^0_{b_x}) = (m_g)(v_{g_x}\prime) + (m_b)(v_{b_x}\prime)$$

$$\therefore v_{g_x}\prime = -\left(\frac{(m_b)(v_{b_x}\prime)}{m_g}\right) = -\left(\frac{(0.155)(3.1\sin 15°)}{(0.052)}\right)$$

2. $P_{tot_y} = P_{tot_y}\prime$

$$(m_g)(\cancel{v}^0_{g_y}) + (m_b)(v_{b_y}) = (m_g)(v_{g_y}\prime) + (m_b)(v_{b_y}\prime)$$

$$\therefore v_{g_y}\prime = \left(\frac{(m_b)(v_{b_y})-(m_b)(v_{b_y}\prime)}{m_g}\right) = \left(\frac{(0.155)(3.5)-(0.155)(3.1\cos 15°)}{0.052}\right)$$

3. Use $c^2 = a^2 + b^2$

$$\therefore v_g\prime = \sqrt{(v_{g_x}\prime)^2 + (v_{g_y}\prime)^2} = \sqrt{(-2.391)^2 + (1.507)^2} \approx 2.766\frac{m}{s}$$

7.2 Inelastic Momentum - Question

A child with a mass of 22 kg runs at a horizontal velocity of 4.2 m/s [forward] and jumps onto a stationary rope swing of mass 2.6 kg. The child ”sticks” on the rope swing and swings forward.

7.2 Inelastic Momentum - Givens

Determine the horizontal velocity of the child plus the swing just after impact then determine the maximum height that the child and swing rise.

- $m_c = 22kg$
- $m_r = 2.6kg$
- $v_c = 4.2\frac{m}{s}$
- $v_r = 0\frac{m}{s}$
- $v_{cr} =?$
- $h_{cr} =?$

7.2 Inelastic Momentum - Solve

In the formula $E_{tot} = E_{tot'}$ the variable $E_{k'}$ is set to zero because when the child reaches his max height $v_c = 0$ and because of this $E_{k'} = 0$

1. $P_{tot} = P_{tot'}$

$$(m_c)(v_c) + (m_r)(\cancelto{0}{v_r}) = (m_{c+r})(v_{cr})$$

$$\therefore v_{cr} = \left(\frac{(m_c)(v_c)}{(m_{c+r})}\right) = \left(\frac{(22)(4.2)}{(22+2.6)}\right) \approx 3.7\frac{m}{s}$$

2. $E_{tot} = E_{tot'}$

$$(m_{c+r}g\cancelto{0}{h}) + (\tfrac{1}{2}m_{c+r}v_{cr}^2) = (m_{c+r}gh)' + \cancelto{0}{(\tfrac{1}{2}m_{c+r}v_{cr}^2)'}$$

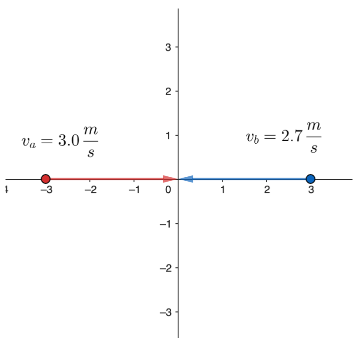
$$\therefore h = \left(\frac{(m_{c+r}v_{cr}^2)}{2(m_{c+r}g)}\right) = \left(\frac{(22+2.6)(3.7)^2}{2(22+2.6)(9.81)}\right) \approx 0.7m$$

7.3 Elastic Momentum - Question

Marcus and Griffin throw two live hand grenades at each other. Hand grenade A is travelling at a velocity of 3.0m/s with a mass of 1.2kg. Hand grenade B is travelling at a velocity of 2.7m/s with a mass of 1.3kg. What is the velocity of the two objects after they collide?

7.3 Elastic Momentum - Graph and Givens

Determine the velocity of Hand Grenade A and Hand Grenade B post-collision.



- $m_a = 1.2g$
- $m_b = 1.3kg$
- $v_a = 3.0\frac{m}{s}$
- $v_b = 2.7\frac{m}{s}$
- $v_a' = ?$
- $v_b' = ?$

7.3 Elastic Momentum - Solve

Because of the law of conservation of momentum this equation is Elastic. The kinetic energy of the objects will be same as they were before the collision.

1. $P_{tot} = P_{tot}'$

$$(\cancel{m_a})(v_a) + (\cancel{m_b})(v_b) = (\cancel{m_a})(v_a)' + (\cancel{m_b})(v_b)'$$

$$(v_a) + (v_b) = (v_a)' + (v_b)'$$

$$\therefore v_a' = v_a = 3.0\frac{m}{s}$$

$$\therefore v_b' = v_b = 2.7\frac{m}{s}$$

2. Proof of Elastic with $E_{tot} = E_{tot}'$

$$\frac{1}{2}mv_a + \frac{1}{2}mv_b = \frac{1}{2}mv_a' + \frac{1}{2}mv_b'$$

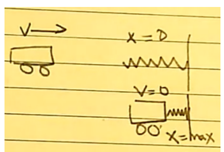
$$\frac{1}{2}(1.2)(3.0) + \frac{1}{2}(1.3)(2.7) = \frac{1}{2}(1.2)(3.0) + \frac{1}{2}(1.3)(2.7)$$

$$\therefore \text{Elastic } 3.055 = 3.055$$

7.4 Energy with Spring - Question

A low friction cart with a mass of 0.25kg travels along a horizontal track and collides head on with a spring that has a spring constant of $155\frac{N}{m}$. If the Spring was compressed by 6.0cm, how fast was the cart initially travelling?

7.4 Energy with Spring - Graph and Givens



- $x = 6.0cm$
- $k = 155\frac{N}{m}$
- $m_c = 0.25kg$
- $v_c = ?$

7.4 Energy with Spring - Solve

Because the cart is not initially colliding with the spring, E_e is zero. Because the cart's velocity post-collision is zero, $E_{k'}$ is also zero. Because the cart and the spring are at the same height both before and after the collision, both E_g and $E_{g'}$ are set to zero.

1. $E_{tot} = E_{tot'}$

$$E_k + \cancelto{0}{E_g} + \cancelto{0}{E_e} = \cancelto{0}{E_{k'}} + \cancelto{0}{E_{g'}} + E_{e'}$$

$$E_k = E_{e'}$$

$$\frac{1}{2}m_c v_c^2 = \frac{1}{2}kx^2$$

$$\therefore v_c = \sqrt{\frac{kx^2}{m_c}} = \sqrt{\frac{(155)(6.0)^2}{(0.25)}} \approx 149\frac{m}{s}$$

2. $h_c = 9.2m$

$$E_k + E_g + \cancelto{0}{E_e} = \cancelto{0}{E_{k'}} + \cancelto{0}{E_{g'}} + E_{e'}$$

$$E_k + E_g = E_{e'}$$

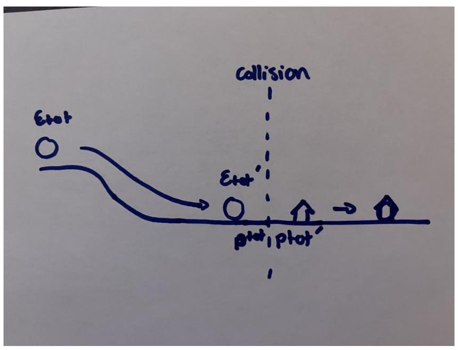
$$\frac{1}{2}m_c v_c^2 + m_c gh = \frac{1}{2}kx^2$$

$$\therefore v_c = \sqrt{\frac{(kx^2) - (m_c gh)}{m_c}} = \sqrt{\frac{(155)(6.0)^2 - (0.25)(9.81)(9.2)}{(0.25)}} \approx 149\frac{m}{s}$$

7.5 Energy + Momentum - Question (Boulder)

A 55.6 kg boulder sat on the side of a mountain beside a lake. The boulder was 14.6 m above the surface of the lake. One winter night, the boulder rolled down the mountain, directly into a 204 kg ice-fishing shack that was sitting on the frozen lake. What was the velocity of the boulder and shack at the instant that they began to slide across the ice?

7.5 Energy + Momentum - Graph and Givens



- $m_b = 55.6kg$
- $m_s = 204kg$
- $h_b = 14.6m$
- $h_s = 0m$
- $v_{bs} = ?$

7.5 Energy + Momentum - Solve

Since we already have the velocity of the shed ($0\frac{m}{s}$), we need to solve for the velocity of the boulder before it collides with the shed. To do this we use the formula $E_{tot} = E_{tot}\prime$. After finding the velocity of the boulder, we use the formula $P_{tot} = P_{tot}\prime$ to solve for the merged velocities.

1. $E_{tot} = E_{tot}\prime$

$$\cancelto{0}{E_k} + E_g = E_k\prime + \cancelto{0}{E_g\prime}$$

$$(m_b)(g)(h_b) = \frac{1}{2}m_b v_b^2$$

$$\therefore v_b = \sqrt{\frac{(2)(m_b)(g)(h_b)}{m_b}} = \sqrt{\frac{(2)(55.6)(9.81)(14.6)}{(55.6)}} \approx 16.9\frac{m}{s}$$

2. $P_{tot} = P_{tot}\prime$

$$m_b v_b + m_s \cancelto{0}{v_s} = m_{b+s} v_{bs}$$

$$\therefore v_{bs} = \frac{m_b v_b}{m_{b+s}} = \frac{(55.6)(16.9)}{(55.6+204)} \approx 3.61\frac{m}{s}$$

7.6 Impulses - Question

Jen has a mass of 50kg and is in a car going 35$\frac{m}{s}$ when she gets in an accident. The airbags deploy and bring her body to a stop in 0.500s. What is the force applied to her body?

7.6 Impulses - Givens

- $v_1 = 35\frac{m}{s}$
- $v_2 = 0\frac{m}{s}$
- $\Delta t = 0.500s$
- $m_J = 50kg$

7.6 Impulses - Solve

We can solve this problem using the equations provided in the Impulses section of this big book.

1. $\Delta P = F_{net}\Delta t$

$$F_{net} = \frac{\Delta P}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{m(v_2 - v_1)}{\Delta t}$$

$$\therefore F_{net} = \frac{50(0-35)}{0.500} \approx -3500N$$

2. $t = 0.100s$

$$F_{net} = \frac{\Delta P}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{m(v_2 - v_1)}{\Delta t}$$

$$\therefore F_{net} = \frac{50(0-35)}{0.100} \approx -17500N$$

7.6 Impulses - Note

Therefore if the time is lower, the less force the collision has on the person. This is why airbags are so important. Airbags reduce the time within the equation.