Integration with U-Substitution and Natural Logarithms

We can solve a complex integral that contains a binomial in the denominator of $f(x)$ using u-substitution.

Question 1:

Solve the integral below using u-substitution and natural logarithms.

> $\int \frac{5x}{(x^2+5)} dx$

Prepare the U-Substitution:

Set μ to $(x^2 + 5)$ then solve for $d\mu$

> $\text{let } \mu = (x^2 + 5)$
>
> $\text{let } d\mu = \frac{d}{dx}[\mu]\, dx$
>
> $\therefore\ d\mu = \frac{d}{dx}\left[(x^2 + 5)\right] dx$
>
> $\therefore\ dx = \frac{d\mu}{2x}$

Solve the equation:

Solve the equation by dividing 5x by 2x then moving the resulting $\frac{5}{2}$ to the front of the integral. Then, use the natural logarithm function (ln) to integrate the remaining variables.

> $= \int \frac{5x}{\mu} \frac{d\mu}{2x}$
>
> $= \frac{5}{2} \int \frac{1}{\mu} d\mu$
>
> $= \frac{5}{2} \int \frac{1}{(x^2+5)} dx$
>
> $= \frac{5}{2} \frac{\ln(x^2+5)}{2} + c$
>
> $= 5 \frac{\ln(x^2+5)}{4} + c$
>
> $= \frac{\ln(x^2+5)^5}{4} + c$
>
> $\therefore \int \frac{5x}{(x^2+5)} dx = \frac{\ln(x^2+5)^5}{4} + c$

Notes:

Derivation:

The derivation of a function is defined as $f(x)\prime$

> $f(x)\prime = \frac{d}{dx}[f(x)]$

The derivation of the natural logarithm function is as follows:

> $\frac{d}{dx}\ln(\mu) = \frac{\mu\prime}{\mu}$

Natural Logarithms and Integrals

The natural logarithm function is denoted by the symbol $\ln$ and is defined as follows:

> $\int \frac{1}{\mu} = \frac{\ln(\mu)}{\mu\prime} + c$