

Integration with U-Substitution and Natural Logarithms

We can solve a complex integral that contains a binomial in the denominator of $f(x)$ using u-substitution.

Question 1:

Solve the integral below using u-substitution and natural logarithms.

$$\int \frac{5x}{(x^2+5)} dx$$

Prepare the U-Substitution:

Set μ to $(x^2 + 5)$ then solve for $d\mu$

$$\text{let } \mu = (x^2 + 5)$$

$$\text{let } d\mu = \frac{d}{dx} [\mu] dx$$

$$\therefore d\mu = \frac{d}{dx} [(x^2 + 5)] dx$$

$$\therefore dx = \frac{d\mu}{2x}$$

Solve the equation:

Solve the equation by dividing $5x$ by $2x$ then moving the resulting $\frac{5}{2}$ to the front of the integral. Then, use the natural logarithm function (\ln) to integrate the remaining variables.

$$= \int \frac{5x}{\mu} \frac{d\mu}{2x}$$

$$= \frac{5}{2} \int \frac{1}{\mu} d\mu$$

$$= \frac{5}{2} \ln(\mu) + c$$

$$= \frac{5}{2} \ln(x^2 + 5) + c$$

$$\therefore \int \frac{5x}{(x^2+5)} dx = \frac{5}{2} \ln(x^2 + 5) + c$$

Integrals for Mr.N

Question 1: Easy

Solve with Integration by Parts:

$$\int x^2 \cos(3x) dx$$

Question 2: Medium

Solve by completing the square:

$$\int \frac{(x+4)}{(x^2+2x+5)} dx$$

Question 3: Hard

Solve with Feynman's Technique:

$$\int_0^{\infty} \frac{\sin x}{x} dx$$

Notes

Derivation

The derivation of a function is defined as $f'(x)$

$$f'(x) = \frac{d}{dx}[f(x)]$$

The derivation of the natural logarithm function is as follows:

$$\frac{d}{dx} \ln(\mu) = \frac{\mu'}{\mu}$$

Natural Logarithms and Integrals

The natural logarithm function is denoted by the symbol \ln and is defined as follows:

$$\int \frac{1}{\mu} = \frac{\ln(\mu)}{\mu'} + c$$