

Momentum and Energy Summary

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1 Equations (1)

Energy without a spring

If the object is moving at start, then the variable E_k has a value. Else, if the object is not moving at start, set E_k to zero.

$$\begin{aligned}E_{tot} &= E_{tot}' \\E_k + E_g &= E_k' + E_g' \\(\frac{1}{2}mv^2) + (mgh) &= (\frac{1}{2}mv^2)' + (mgh)'\end{aligned}$$

Energy with a spring

If there is a force/object pushing on the spring, then the variable E_e has a value. Else, it's value is zero. Therefore, if an object is being dropped onto a spring, only E_e' has a value.

$$\begin{aligned}E_{tot} &= E_{tot}' \\E_k + E_g + E_e &= E_k' + E_g' + E_e' \\(\frac{1}{2}mv^2) + (mgh) + (\frac{1}{2}kx^2) &= (\frac{1}{2}mv^2)' + (mgh)' + (\frac{1}{2}kx^2)'\end{aligned}$$

Elastic Momentum

The two objects do **not** move together after colliding. Instead, they separate from each other, both going in different paths.

$$\begin{aligned}P_{tot} &= P_{tot}' \\P_a + P_b &= P_a' + P_b' \\(m_a)(v_a) + (m_b)(v_b) &= (m_a)(v_a)' + (m_b)(v_b)'\end{aligned}$$

Inelastic Momentum

The two objects move together after colliding. Instead of separating from each other, the two objects have conjoined, both moving in the same path.

$$\begin{aligned}P_{tot} &= P_{tot}' \\P_a + P_b &= P_{ab} \\(m_a)(v_a) + (m_b)(v_b) &= (m_{a+b})(v_{a \times b})\end{aligned}$$

2 Units

- $F_s = N$
- $E_e = J$
- $k = \frac{Newtons}{meters}$
- $W = J$
- $x = meters$
- $\Delta E = J$
- $P_{tot} = \frac{kgm}{s}$
- $E_{tot} = J$

3 Collisions

Elastic vs. In-Elastic

When two objects collide and they both stay the same, then the collision is ***Elastic***. Otherwise if they both ***do not*** stay the same, then the collision is ***In-Elastic***.

For Elastic collisions, the useable energy is maintained and the momentum is conserved. For Inelastic collisions, the useable energy is not conserved, though the momentum ***is*** conserved.

Calculation Steps

If you get stuck on a problem, try to use $P_{tot} = P_{tot}'$ and/or $E_{tot} = E_{tot}'$ to solve for what you need.

1. Diagram
2. Givens
3. What are you looking for?
4. $P_{tot} = P_{tot}'$ or $E_{tot} = E_{tot}'$

4 Energy and Springs

Calculation Steps (No 5 Steps)

Recall Hooke's Law (The extension or compression of a spring). E_e is Elastic Potential Energy - The stored energy in a spring from it's compression or extension.

1. $F_s = kx$ or $k = \frac{F_s}{x}$
2. $E_k + E_g + E_e = E_k' + E_g' + E_e'$

5 Impulses

Equations

Impulses do not require components. Rearrange the equations below to solve for what you need.

- $\Delta P = m\Delta v$
- $a = \frac{\Delta v}{\Delta t}$
- $\Delta P = ma\Delta t$
- $\Delta v = a\Delta t$
- $\Delta P = F_{net}\Delta t$
- $F_{net} = ma$

Calculation Steps

Impulses are about the push back of an opposing force.

1. Diagram
2. Givens
3. What are you looking for?
4. Use the equations above to solve for what you need.

6 2D Momentum

Calculation Steps

Similar to collisions except it includes both x and y components.

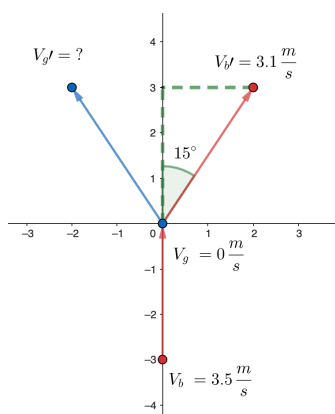
1. Diagram
2. Givens
3. What are you looking for?
4. Components
5. $P_{tot_x} = P_{tot_x}'$ and $P_{tot_y} = P_{tot_y}'$

7 Example Equations

2D Momentum - Question

A billiard ball with a mass of 0.155kg is rolling directly away from you at $3.5 \frac{\text{m}}{\text{s}}$. It collides with a stationary golf ball with a mass of 0.05kg . The billiard ball rolls off at an angle of 15° clockwise from its original direction with a velocity of $3.1 \frac{\text{m}}{\text{s}}$. What is the after velocity of the golf ball?

2D Momentum - Graph and Givens



- $m_b = 0.155\text{kg}$
- $m_g = 0.052\text{kg}$
- $V_b = 3.5 \frac{\text{m}}{\text{s}}$
- $V_g = 0 \frac{\text{m}}{\text{s}}$
- $V_b' = 3.1 \frac{\text{m}}{\text{s}}$ [15° clockwise]
- $V_g' = ?$

2D Momentum - Solve

This solve requires both x and y components. Since v_b only has a y direction, v_{b_x} is zero. Also, since v_g isn't moving, both v_{g_x} and v_{g_y} are zero.

$$1. P_{tot_x} = P_{tot_x}'$$

$$(m_g)(v_{g_x}^0) + (m_b)(v_{b_x}^0) = (m_g)(v_{g_x}') + (m_b)(v_{b_x}')$$

$$\therefore v_{g_x}' = - \left(\frac{(m_b)(v_{b_x}')}{m_g} \right) = - \left(\frac{(0.155)(3.1 \sin 15^\circ)}{(0.052)} \right)$$

$$2. P_{tot_y} = P_{tot_y}'$$

$$(m_g)(v_{g_y}^0) + (m_b)(v_{b_y}) = (m_g)(v_{g_y}') + (m_b)(v_{b_y}')$$

$$\therefore v_{g_y}' = \left(\frac{(m_b)(v_{b_y}) - (m_b)(v_{b_y}')}{m_g} \right) = \left(\frac{(0.155)(3.5) - (0.155)(3.1 \cos 15^\circ)}{0.052} \right)$$

$$3. \text{ Use } c^2 = a^2 + b^2$$

$$\therefore v_{g'} = \sqrt{(v_{g_y}')^2 + (v_{g_x}')^2} = \sqrt{(-2.391)^2 + (1.507)^2} \approx 2.766 \frac{\text{m}}{\text{s}}$$