Momentum and Energy Summary

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1 Equations

1.1 Energy without a spring

The value of E_k is dependant on whether the object begins with any motion. The variable E_g is dependant on whether the object begins with / ends with a specific height.

$E_{tot} = E_{tot}\prime$

$E_k + E_g = E_k\prime + E_g\prime$

$(\frac{1}{2}mv^2) + (mgh) = (\frac{1}{2}mv^2)\prime + (mgh)\prime$

1.2 Energy with a spring

If a force/object pushing on the spring, then the variable E_e has a value. Else, set it's value to zero. With this being, if an object is being dropped onto a spring, only $E_e\prime$ has a value.

$E_{tot} = E_{tot}\prime$

$E_k + E_g + E_e = E_k\prime + E_g\prime + E_e\prime$

$(\frac{1}{2}mv^2) + (mgh) + (\frac{1}{2}kx^2) = (\frac{1}{2}mv^2)\prime + (mgh)\prime + (\frac{1}{2}kx^2)\prime$

1.3 Momentum (1)

The two objects do ***not*** move together after colliding. Instead, they seperate from eachother, both going in different paths.

$P_{tot} = P_{tot}\prime$

$P_a + P_b = P_a\prime + P_b\prime$

$(m_a)(v_a) + (m_b)(v_b) = (m_a)(v_a)\prime + (m_b)(v_b)\prime$

1.4 Momentum (2)

The two objects move together after colliding. Instead of seperating from eachother, the two objects have conjoined, both moving in the same path.

$P_{tot} = P_{tot}\prime$

$P_a + P_b = P_{ab}$

$(m_a)(v_a) + (m_b)(v_b) = (m_{a+b})(v_{a \times b})$

2 Units

- $F_s = N$
- $k = \frac{N_{ewtons}}{m_{eter}}$
- $x = m_{eters}$
- $P_{tot} = \frac{kgm}{s}$
- $E_e = J$
- $W = J$
- $\Delta E = J$
- $E_{tot} = J$

3 Collisions

Elastic vs. In-Elastic

In an ELASTIC collision, the kinetic energy of the SYSTEM is equal before and after the collision. In an INELASTIC collision, the kinetic energy of the SYSTEM is NOT equal between the before and after of the collision.

An elastic collision occurs when two objects collide and are physically the same after the collision as they were before the collision. An inelastic collision occurs when the physical shape of the objects involved has been altered and is different after compared to before the collision.

Calculation Steps

If you get stuck on a problem, try to use $P_{tot} = P_{tot}\prime$ and/or $E_{tot} = E_{tot}\prime$ to solve for what you need.

1. Diagram
2. Givens
3. What are you looking for?
4. $P_{tot} = P_{tot}\prime$ or $E_{tot} = E_{tot}\prime$

4 Energy and Springs

Calculation Steps (No 5 Steps)

Recall Hooke's Law (The extension or compression of a spring). E_e is Elastic Potential Energy - The stored energy in a spring from it's compression or extension.

1. $F_s = kx$ or $k = \frac{F_s}{x}$
2. $E_k + E_g + E_e = E_k\prime + E_g\prime + E_e\prime$

5 Impulses

Equations

Impulses do not require components. Rearrange the equations below to solve for what you need.

- $\Delta P = m\Delta v$
- $\Delta P = ma\Delta t$
- $\Delta P = F_{net}\Delta t$
- $a = \frac{\Delta v}{\Delta t}$
- $\Delta v = a\Delta t$
- $F_{net} = ma$

Calculation Steps

Impulses are about the push back of an opposing force.

1. Diagram
2. Givens
3. What are you looking for?
4. Use the equations above to solve for what you need.

6 2D Momentum

Calculation Steps

Similar to collisions except it includes both x and y components.

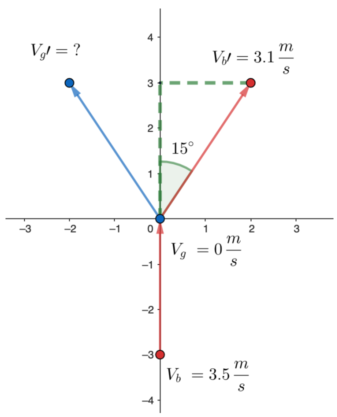
1. Diagram
2. Givens
3. What are you looking for?
4. Components
5. $P_{tot_x} = P_{tot_x}\prime$ and $P_{tot_y} = P_{tot_y}\prime$
6. $c^2 = a^2 + b^2$

7 Example Equations

7.1 2D Momentum - Question

A billiard ball with a mass of 0.155kg is rolling directly away from you at 3.5 $\frac{m}{s}$. It collides with a stationary golf ball with a mass of 0.05kg. The billiard ball rolls off at an angle of 15° clockwise from it's original direction with a velocity of 3.1$\frac{m}{s}$. What is the after velocity of the golf ball?

7.1 2D Momentum - Graph and Givens



- $m_b = 0.155kg$
- $m_g = 0.052kg$
- $V_b = 3.5\frac{m}{s}$
- $V_g = 0\frac{m}{s}$
- $V_b' = 3.1\frac{m}{s}$ $[15° clockwise]$
- $V_g' = ?$

7.1 2D Momentum - Solve

This solve requires both x and y components. Since v_b only has a y direction, v_{b_x} is zero. Also, since v_g isn't moving, both v_{g_x} and v_{g_y} are zero.

1. $P_{tot_x} = P_{tot_x}'$

$$(m_g)(\cancelto{0}{v_{g_x}}) + (m_b)(\cancelto{0}{v_{b_x}}) = (m_g)(v_{g_x}') + (m_b)(v_{b_x}')$$

$$\therefore v_{g_x}' = -\left(\frac{(m_b)(v_{b_x}')}{m_g}\right) = -\left(\frac{(0.155)(3.1\sin 15°)}{(0.052)}\right)$$

2. $P_{tot_y} = P_{tot_y}'$

$$(m_g)(\cancelto{0}{v_{g_y}}) + (m_b)(v_{b_y}) = (m_g)(v_{g_y}') + (m_b)(v_{b_y}')$$

$$\therefore v_{g_y}' = \left(\frac{(m_b)(v_{b_y})-(m_b)(v_{b_y}')}{m_g}\right) = \left(\frac{(0.155)(3.5)-(0.155)(3.1\cos 15°)}{0.052}\right)$$

3. Use $c^2 = a^2 + b^2$

$$\therefore v_g' = \sqrt{(v_{g_x}')^2 + (v_{g_y}')^2} = \sqrt{(-2.391)^2 + (1.507)^2} \approx 2.766\frac{m}{s}$$

7.2 Inelastic Momentum - Question

A child with a mass of 22 kg runs at a horizontal velocity of 4.2 m/s [forward] and jumps onto a stationary rope swing of mass 2.6 kg. The child ”sticks” on the rope swing and swings forward.

7.2 Inelastic Momentum - Givens

Determine the horizontal velocity of the child plus the swing just after impact then determine the height that the child and swing rise.

- $m_c = 22kg$
- $m_r = 2.6kg$
- $v_c = 4.2\frac{m}{s}$
- $v_r = 0\frac{m}{s}$
- $v_{cr} =?$
- $h_{cr} =?$

7.2 Inelastic Momentum - Solve

To solve for the velocity of the child + the rope, we must use the formula $P_{tot} = P_{tot}\prime$ and for the height of the swing after impact, we must use the formula $E_{tot} = E_{tot}\prime$

1. $P_{tot_x} = P_{tot_x}\prime$

$$(m_c)(v_c) + (m_r)(\cancel{v_r}^{0}) = (m_{c+r})(v_{cr})$$

$$\therefore v_{cr} = \left(\frac{(m_c)(v_c)}{(m_{c+r})}\right) = \left(\frac{(22)(4.2)}{(22+2.6)}\right) \approx 3.7\frac{m}{s}$$

2. $E_{tot_y} = E_{tot_y}\prime$

$$(m_c g \cancel{h}^{0}) + (\tfrac{1}{2} m_c v_c^2) = (m_{c+r} g h)\prime + \cancel{(\tfrac{1}{2} m v^2)\prime}^{0}$$

$$\therefore h = \left(\frac{(m_c v_c^2)}{2(m_{c+r} g)}\right) = \left(\frac{(22)(4.2)^2}{2(22+2.6)(9.81)}\right) \approx 1.6m$$

7.3 Elastic Momentum - Question

7.4 Energy + Momentum - Question (Boulder)

7.5 Energy with Spring - Question

7.6 Impulse - Question