

# Integration with U-Substitution and Natural Logarithms

We can solve a complex integral that contains a binomial in the denominator of  $f(x)$  using u-substitution.

## Question 1:

Solve the integral below using u-substitution and natural logarithms.

$$\int \frac{5x}{(x^2+5)} dx$$

## Prepare the U-Substitution:

Set  $\mu$  to  $(x^2 + 5)$  then solve for  $d\mu$

$$\text{let } \mu = (x^2 + 5)$$

$$\text{let } d\mu = \frac{d}{dx} [\mu] dx$$

$$\therefore d\mu = \frac{d}{dx} [(x^2 + 5)] dx$$

$$\therefore dx = \frac{d\mu}{2x}$$

## Solve the equation:

Solve the equation by dividing  $5x$  by  $2x$  then moving the resulting  $\frac{5}{2}$  to the front of the integral. Then, use the natural logarithm function ( $\ln$ ) to integrate the remaining variables.

$$= \int \frac{5x}{\mu} \frac{d\mu}{2x}$$

$$= \frac{5}{2} \int \frac{1}{\mu} d\mu$$

$$= \frac{5}{2} \int \frac{1}{(x^2+5)} dx$$

$$= \frac{5}{2} \frac{\ln(x^2+5)}{2} + c$$

$$= 5 \frac{\ln(x^2+5)}{4} + c$$

$$\therefore \int \frac{5x}{(x^2+5)} dx = 5 \frac{\ln(x^2+5)}{4} + c$$

# Notes

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## Derivation

The derivation of a function is defined as  $f'(x)$

$$f'(x) = \frac{d}{dx}[f(x)]$$

The derivation of the natural logarithm function is as follows:

$$\frac{d}{dx} \ln(\mu) = \frac{\mu'}{\mu}$$

## Natural Logarithms and Integrals

The natural logarithm function is denoted by the symbol  $\ln$  and is defined as follows:

$$\int \frac{1}{\mu} = \frac{\ln(\mu)}{\mu'} + c$$