Integration with U-Substitution and Natural Logarithms

The best way to solve complex logarithms that contain a binomial in both the numerator and denomator of f(x) is to use u-substitution.

Question 1:

Solve the integral below using u-substitution and natural logarithms.

> $\int \frac{(x+5)}{(x^2+5)} dx$

U-Substitution:

Set μ to $(x^2 + 5)$ then solve for $d\mu$

> $\text{let } \mu = (x^2 + 5)$
>
> $\text{let } d\mu = \frac{d}{dx}[\mu]\, dx$
>
> $\therefore\ d\mu = \frac{d}{dx}\left[(x^2 + 5)\right] dx$
>
> $\therefore\ dx = \frac{d\mu}{2x}$

Solve the equation:

Solve the equation by moving the numerator constants to the front of the integral and using the natural logarithm function (ln) to integrate the remaining variables.

> $= \int \frac{x+5}{\mu} \frac{d\mu}{2x}$
>
> $= \frac{1}{2} \int \frac{5}{\mu} d\mu$
>
> $= \frac{5}{2} \int \frac{1}{\mu} d\mu$
>
> $= \frac{5}{2} \int \frac{1}{(x^2+5)} dx$
>
> $= \frac{5}{2} \ln(x^2 + 5) + C$
>
> $\therefore\ \int \frac{(x+5)}{(x^2+5)} dx = \frac{5}{2} \ln(x^2 + 5) + C$