Integration with U-Substitution and Natural Logarithms

We can solve a complex integral that contains a binomial in both the numerator and denominator of $f(x)$ using u-substitution.

Question 1:

Solve the integral below using u-substitution and natural logarithms.

$$\int \frac{(x+5)}{(x^2+5)} dx$$

Prepare the U-Substitution:

Set μ to $(x^2 + 5)$ then solve for $d\mu$

$$\text{let } \mu = (x^2 + 5)$$

$$\text{let } d\mu = \frac{d}{dx}[\mu] \, dx$$

$$\therefore \; d\mu = \frac{d}{dx}\left[(x^2 + 5)\right] dx$$

$$\therefore \; dx = \frac{d\mu}{2x}$$

Solve the equation:

Solve the equation by moving the numerator constants to the front of the integral and using the natural logarithm function (ln) to integrate the remaining variables.

$$= \int \frac{x+5}{\mu} \frac{d\mu}{2x}$$

$$= \frac{1}{2} \int \frac{5}{2x\mu} d\mu$$

$$= \frac{5}{4} \int \frac{1}{x\mu} d\mu$$

$$= \frac{5}{4} \int \frac{1}{x(x^2+5)} dx$$

$$= \frac{5}{4} \ln(x^3 + 5x) + C$$

$$\therefore \int \frac{(x+5)}{(x^2+5)} dx = \frac{5}{4} \ln(x^3 + 5x) + C$$