General Relativity vs Light

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Contents

1 General Relativity **3**

2 Waves **4**

2.1 Mechanical Waves 4

2.1.1 Different Types of Mechanical Waves 4

2.2 Electromagnetic Waves 4

2.2.1 Different Types of Electromagnetic Waves 5

2.3 Gravitational Waves 5

2.3.1 Gravitational Wave Visualization 6

2.4 Matter Waves 6

3 Light **6**

3.1 Wave Velocity 6

3.2 Components 7

3.3 Films 7

3.4 Polarization 7

3.5 Gravitational Lensing 7

3.5.1 Gravitational Micro-Lensing 7

3.5.2 So how does it work? 8
3.6 Light - A Particle and Wave 8
3.6.1 Light as a Wave 8
3.6.2 Light as a Particle 8

4 Neutron Stars **9**
4.1 Pulsars 9

5 Black Holes **9**
5.1 Light vs Black Holes 9
5.2 Solar Mass Measurement 9
5.3 Schwarzschild Radius 9
5.3.1 What is it? 9
5.3.2 Visualization 10
5.3.3 Schwarzschild Radius Equation 10
5.4 Gravitational Force 10
5.5 Solving for the Gravitational Force 11
5.5.1 Variables 11
5.5.2 Solve for Schwarzschild Radius 11
5.5.3 Solve for Gravitational Force 12
5.5.4 Black Hole Gravitational Force on Light 12
5.6 Gravity in a Black Hole vs Earth 12
5.7 Inside a Black Hole 13
5.7.1 Mass of it's Singularity 13
5.7.2 Gravitational Time Dilation 13
5.7.3 Gravitational Time Dilation of a Singularity 13
5.7.4 Gravitational Time Dilation of a Black Hole 13
5.7.5 Relative Velocity Time Dilation 14

5.8 Quarks in Black Holes 15
5.9 Event Horizon 15
5.10 Hawking Radiation 15
5.11 Black Hole Information Paradox 15

6 White Holes **15**

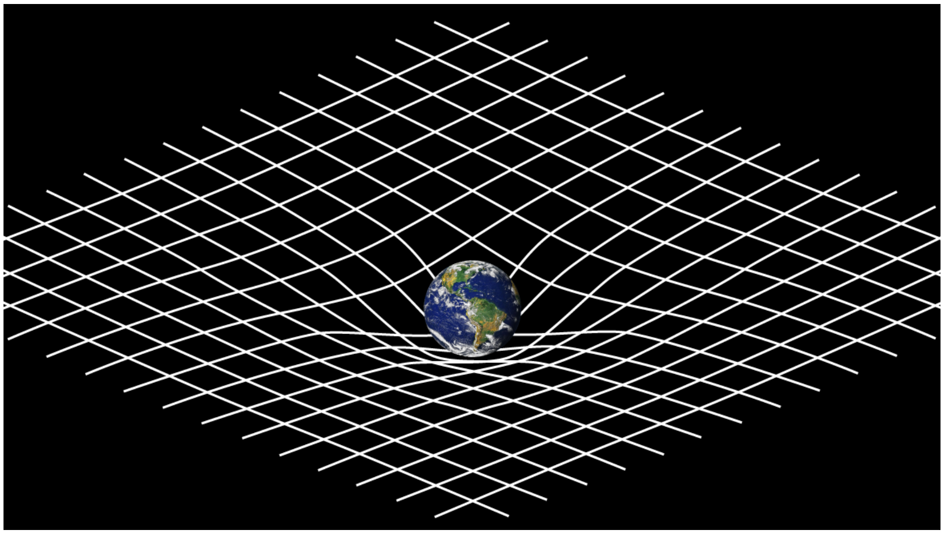
7 Worm Holes **15**

8 Constants **15**

9 Word Bank **16**
9.1 Oscillation 16
9.2 Photons 16

1 General Relativity

Albert Einstein's theory of general relativity is based on the idea that massive objects cause a distortion in space-time. The image below exhibits a clear representation of what that distortion looks like.



2 Waves

2.1 Mechanical Waves

A mechanical wave is an oscillation of matter and is responsible for the transfer of energy through a medium. (Note: Light is not a mechanical wave because it is not matter. The particles that make up light (photons) have no mass.)

2.1.1 Different Types of Mechanical Waves

In total there are ***three*** general forms of mechanical waves. These waves are: Transverse Waves, Longitudinal Waves, and Combined Waves.

2.2 Electromagnetic Waves

Electromagnetic waves are those created by oscillating electric and magnetic fields. A great example of an electromagnetic wave is light. Light has two components: vertical and horizontal (electric and magentic field oscillation). This combination results in an electromagnetic wave.

2.2.1 Different Types of Electromagnetic Waves

In total there are ***seven*** general forms of electromagnetic waves. These waves are: Radio Waves, Microwaves, Infrared, Visible, Ultraviolet, X-Ray, and Gamma Rays.

2.3 Gravitational Waves

First proposed by Albert Einstein in 1916 following his famously recognized Theory of General Relativity, gravitational waves are ripples in space-time (the fabled "fabric" of the Universe) caused by massive objects moving with extreme accelerations.

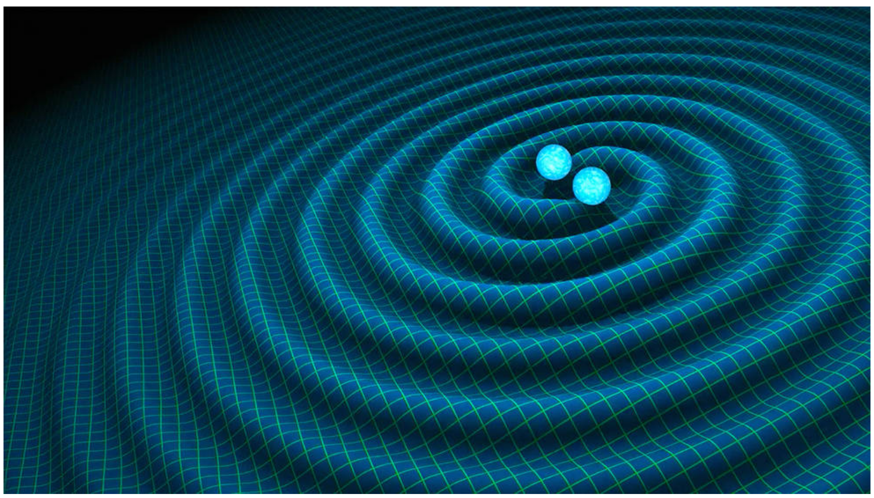
Over 20 years later in 1936, Albert Einstein and Nathan Rosen submitted a manuscript famously contradicting their theory of gravitational waves. This is because gravitational waves were theoretically possible, though they were thought to be physically impossible.

Gravitational waves were later proved to be existent, though it was determined that they're effect by the time they reached us would be so weak, detecting them would be nearly impossible. However, in 2015, the Laser Interferometer Gravitational-Wave Observatory (LIGO) detected gravitational waves from the collision of two black holes.

Gravitational waves are formed by, as an example, also described above, two black holes orbiting eachother, both getting closer and closer together until they collide.

2.3.1 Gravitational Wave Visualization

Gravitational waves are a very difficult concept to visualize. The following image is a basic visualization of gravitational waves. The two light-blue spheres are, for example, black holes. Their circulation around eachother leave ripples in spacetime.



2.4 Matter Waves

Matter waves are a central part of the theory of quantum mechanics, being an example of wave-particle duality. All matter exhibits wave-like behavior. The matter waves describes the relationship between momentum and wavelength.

3 Light

3.1 Wave Velocity

The speed of an electromagnetic wave (therefore light waves) is dependant on the wave length and freuency. The measure of the waves velocity can be calculated by the following equation: $v = \lambda f$ where v is the velocity, λ is the wave length, and f is the frequency. The units of velocity are meters per second (m/s).

3.2 Components

A light wave is made up of two components: electric and magnetic fields. These components can also be represented as vertical and horizontal vectors. These vectors later appear in the polarization of light waves.

3.3 Films

Films allow the light wave to refract and reflect along the innerds of the film. This causing the reflecting light wave (the wave hitting the screen) to either constructively or deconstructively interfere.

3.4 Polarization

The polarization of a light wave allows for either the vertical or horizontal components of a wave to be eliminated. This elimination removes a minimum of 50% of the lights' brightness.

3.5 Gravitational Lensing

Following Einstein's Theory of General Relativity, light can be bent by gravity. This bending is known as gravitational lensing. Through gravitational lensing, lights path around the electromagnetic field of an extremely large mass (eg. a neutron star) is curved.

3.5.1 Gravitational Micro-Lensing

Gravitational micro-lensing allows astronomers to detect objects that would otherwise be hidden in our vast universe (i.e. a black hole). To detect a black hole, taken from a discovery by astronomers in 2019, the light of a star was observed to be distorted by the gravitational lensing of a black hole.

3.5.2 So how does it work?

The best way to describe gravitational lensing is through the visualization of a bowling bowl moving around a circular pit. Because of the momentum of the bowling ball and the curvature of the pit, the ball doesn't just simply fall into the pit, instead, it curves around it. (i.e. light around an object of extremely large mass and distortion in space-time) See Theory of General Relativity

3.6 Light - A Particle and Wave

Not only is light a wave, but it is also a particle. This is known as wave-particle duality which is an essential theory derived from electromagnetics in quantum mechanics.

3.6.1 Light as a Wave

Combatting Isaac Newton's belief that light is a particle was Christiaan Huygens who had instead proposed that light was a wave. At the time Huygens wasn't able to prove his theory. It wasn't until 123 years later (1678 to 1801) that Thomas Young proved light was a wave through his Double Slit experiment.

3.6.2 Light as a Particle

Albert Einstein's quantum theory of light proposes that light is a series of photons, and the flow of photons is a wave. Einstein's essential point is that light's energy is directly related to its oscillation frequency.

4 Neutron Stars

4.1 Pulsars

5 Black Holes

5.1 Light vs Black Holes

It's impossible for anything to escape the event horizon of a black hole. This includes light. Light cannot escape because spacetime is so warped from the sheer mass of the black hole, it's relative region (See Theory of General Relativity) is an infinitely curved space that literally caves in on itself. This means that any direction the light tries to go to, it all just points back to the center of the black hole. However, because of the black holes distortion in spacetime, light bends around it's relative dip. See Gravitational Lensing

5.2 Solar Mass Measurement

The mass of a black hole is measured by it's factor of solar mass. A solar mass is the mass of our sun which is $\approx 1.989 \times 10^{30}$ kilograms. Therefore, a black hole with the mass of two solar masses would be $\approx 2(1.989 \times 10^{30}) \approx 3.978 \times 10^{30}$ kilograms.

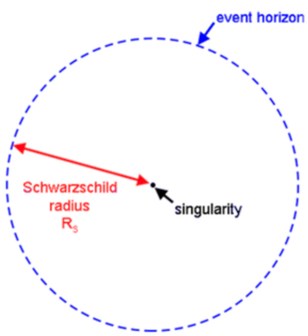
5.3 Schwarzschild Radius

5.3.1 What is it?

The Schwarzschild radius is the radius at which a mass has an escape velocity (the velocity required to escape a planets atmosphere) equal to the speed of light. Any object that is smaller than its Schwarzschild radius is a black hole. This is because any object with an escape velocity greater than the speed of light is a black hole.

5.3.2 Visualization

The following image is a visualization of a black hole and it's Schwarzschild radius.



5.3.3 Schwarzschild Radius Equation

The Schwarzschild radius is calculated using the equation: $R_S = \frac{2GM}{c^2}$ where R_S is the Schwarzschild radius, G is the gravitational constant ($6.67 \times 10^{-11}\ \frac{Nm^2}{kg^2}$), M is the mass of the object, and c is the speed of light ($\approx 299,792,458\ \frac{m}{s}$). The units of the Schwarzschild radius are meters (m).

5.4 Gravitational Force

The gravitational force of a black hole can be calculated using the equation: $F_g = \left(\frac{G \times m_1 m_2}{r^2}\right)$ where F_g is the gravitational force, G is Newtons gravitational constant ($6.67 \times 10^{-11}\ \frac{Nm^2}{kg^2}$), m_1 is the mass of the black hole, m_2 is the mass of the object being pulled in, and r is the radius of the black hole (found using the Schwarzschild Radius Equation). The units for gravitational force in this solution are Newtons per Kilogram ($\frac{N}{kg}$).

5.5 Solving for the Gravitational Force

5.5.1 Variables

- $Given: \ G = 6.67 \times 10^{-11} \ \frac{Nm^2}{kg^2}$
- $Given: \ m_1 = 1.989 \times 10^{30} \ kg$
- $Given: \ m_2 = 1 \ kg$
- $Solved \ for: R_S = 2.95 \times 10^3 \ m$
- $Find: F_g = \left(\frac{Gm_1 m_2}{r^2}\right)$

5.5.2 Solve for Schwarzschild Radius

After solving for the Schwarzschild radius, we need to redact our result by a small increment. This is because if the object is smaller than it's Schwarzschild radius, it is a black hole, which in our case, it is.

$$\therefore \ R_S = \left(\frac{2GM}{c^2}\right) = \left(\frac{2(6.67 \times 10^{-11})(1.989 \times 10^{30})}{(299,792,458)^2}\right) \approx 2.95 \times 10^3 \ m$$

$\therefore \ The \ Schwarzschild \ Radius \ (R_S) \ is \ approximately \ 2.95 \times 10^3 \ m$

$\therefore \ The \ Black \ Hole \ Radius \ (r) \ is \ approximately \ < 2.95 \times 10^3 \ m$

5.5.3 Solve for Gravitational Force

After solving for the Schwarzschild radius, we can now solve for the gravitational force.

$$\therefore \quad F_g = \left(\frac{Gm_1m_2}{r^2}\right) = \left(\frac{(6.67 \times 10^{-11})(1.989 \times 10^{30})(1)}{(2.949 \times 10^3)^2}\right) \approx 4.5 \times 10^{13}\ \frac{N}{kg}$$

$$\therefore \ \textit{The Gravitational Force } (F_g) \textit{ is approximately } 4.5 \times 10^{13}\ \frac{N}{kg}$$

5.5.4 Black Hole Gravitational Force on Light

Discovered by Karl Schwarzschild, the gravitational force on light of a spinning black hole is $F = \left(\frac{hc^3}{GM\lambda}\right)$ and the gravitational force of on light a non-spinning black hole is $F = \left(\frac{hc^3}{4GM\lambda}\right)$ where F is the gravitational force, h is Planck's constant ($6.626 \times 10^{-34}\ \frac{m^2kg}{s}$), c is the speed of light ($\approx 299,792,458\ \frac{m}{s}$), G is Newton's gravitational constant ($6.67 \times 10^{-11}\ \frac{Nm^2}{kg^2}$), M is the mass of the black hole, and λ is the wavelength of the light in meters ($\approx 5.52 \times 10^{-7}\ m$).

5.6 Gravity in a Black Hole vs Earth

The gravitational force (F_g) in a black hole is approximately $4.5 \times 10^{13}\ \frac{N}{kg}$ whereas the gravitational force here on earth is approximately $9.81\ \frac{N}{kg}$. This means that the gravitational force in a black hole is approximately $4.6 \times 10^{12}\ \frac{N}{kg}$ times greater than the gravitational force here on earth.

5.7 Inside a Black Hole

5.7.1 Mass of it's Singularity

A singularity is a point in space where extremely large amounts of matter are crushed into an infinitely small volume and density. Using either it's density or volume we can calulate it's mass using the equation: $m = \rho v$ where m is the mass, ρ is the density, and v is the volume. Since ∞ is not quantifyable, any substitution of it in any equation produces a result of $\pm\infty$ or 0. Therefore, we can assume that the mass of the singularity is ∞.

5.7.2 Gravitational Time Dilation

By the time you've reached the singularity in a black hole, you'd be trapped in time. This is because as you come closer to an object of extremely large mass, because of it's distortion in spacetime (See Theory of General Relativty), time moves slower. Therefore we can assume that because the mass of a singularity is infinite, time in a singularity does not increment.

5.7.3 Gravitational Time Dilation of a Singularity

The equation for gravitational time dilation is $\Delta t\prime = \Delta t \times \sqrt{(1 - \frac{2GM}{rc^2})}$ where $\Delta t\prime$ is the time dilation, Δt is the time in reference (i.e. an hour), G is the gravitational constant, M is the mass of the object, r is the distance from the object, and c is the speed of light. Since we know that the mass of the singularity is infinite, we can substitute it in for M and solve for the time dilation. Like stated above ∞ is not quantifyable, therefore, the time dilation ($\Delta t\prime$) is infinite.

5.7.4 Gravitational Time Dilation of a Black Hole

Following equation for gravitational time dilation: $\Delta t\prime = \Delta t \times \sqrt{(1 - \frac{2GM}{rc^2})}$ we can substitute in the values we've already solved for (R_S, M_{BH}).

$$\begin{aligned}\Delta t\prime &= \Delta t \times \sqrt{(1 - \frac{2GM}{rc^2})} \\ &= 1\ hour \times \sqrt{(1 - \frac{2(6.67 \times 10^{-11})(1.989 \times 10^{30})}{(2.949 \times 10^3)(299,792,458)^2})} \\ &= \sqrt{-1.093 \times 10^{-3}}\end{aligned}$$

Therefore for every one hour on earth, $\sqrt{-1.093 \times 10^{-3}}$ hours pass in a black hole. For perspective, if the square root value was positive (i.e. $\sqrt{1.093 \times 10^{-3}}$), then by mutiplying the result by 8760 (hours in a year), we can determine that approximately 289.6 years would pass on earth for every hour in a black hole.

If we want to calculate the gravitational time dilation of a spacecraft orbiting the black hole, we just add the distance from the spacecraft to our Schwarzschild radius. $R_{SC+S} = R_S + R_{SC}$ where R_{SC} is the spacecraft distance.

5.7.5 Relative Velocity Time Dilation

When something is moving faster than something else, time dilation is experienced due to the discrepancy between velocities. The equation for relative velocity time dilation is $\Delta t\prime = \left(\Delta t \div \sqrt{1 - (v^2 \div c^2)}\right)$ where $\Delta t\prime$ is the time dilation, Δt is the time interval (i.e. an hour), v is the velocity of the object, and c is the speed of light. We can assume that our velocity inside a black hole is equal to c which leaves us with:

$$\begin{aligned}\Delta t\prime &= \left(\Delta t \div \sqrt{1-(c^2 \div c^2)}\right)\\ &= (1 \div \sqrt{1-1})\\ &= \left(1 \div \sqrt{0}\right)\\ &= (1 \div 0)\\ &= undefined\end{aligned}$$

Therefore, calculating the relative velocity time dilation in a black hole is impossible. Our result describes that on earth *undefined* (assuming ∞) hours had passed for the object to travel for one hour inside the black hole. The velocity of an object inside the black hole is equal to the speed of light.

5.8 Quarks in Black Holes

5.9 Event Horizon

5.10 Hawking Radiation

Hawking Radiation is what makes black holes eventually disappear. To accomodate for the law of conservation of energy, the black hole uses it's mass to spew out hawking radiation. Overtime as the hawking radiation builds up, the black hole will eventually disappear. (Could this be the birth of a white hole?). (Clear this section up and clarify and prove some stuff.)

5.11 Black Hole Information Paradox

6 White Holes

7 Worm Holes

8 Constants

- Speed of Light: $c \approx 299,792,458\ \frac{m}{s}$

- Gravitational Constant: $G \approx 6.67 \times 10^{-11}\ \frac{Nm^2}{kg^2}$

- Plancks Constant (Energy and Frequency): $\approx 6.626 \times 10^{-34} \frac{m^2kg}{s}$

- One Solar Mass: $\approx 1.989 \times 10^{30}$ kg

9 Word Bank

9.1 Oscillation

The movement back and forth at a regular speed. Regular variation in magnitude or position around a central point.

9.2 Photons

A packet of electromagnetic energy with no mass nor any charge. A photon is a particle. A photon is the smallest unit of light.