Conditional Proof

Philosophy 3

Suppose you and a friend are trying to decide what to do on Friday night. You want to stay in and study logic, and your friend wants to go to a party. You tell your friend,

> Look, if you don't study logic tonight, you won't pass the final exam. And if you don't pass the final exam, you won't pass the class. And if you don't pass the class, you'll get kicked out of school. And if you get kicked out of school, I won't be your friend anymore.

Your friend responds,

> If I don't study logic tonight, you won't be my friend anymore?!

Your friend's inference is a valid one. They deduced it by considering the consequences of the proposition,

> I [your friend] won't study logic tonight.

In other words, your friend asked, supposing that proposition *were* true, what would be able to be deduced? And, by working through all those conditionals you told your friend, they were able to deduce the proposition

> You will not be my friend anymore.

This is an instance of conditional proof. Conditional proof is a kind of hypothetical reasoning. In non-hypothetical forms of reasoning, every line of a proof is taken to be true. Hypothetical reasoning, on the other hand, allows us to reason from a *hypothesis* that we do not take to be true. Rather, we are only seeing what the consequences of assuming it true would be.

This is exactly how your friend is reasoning. They haven't decided what they are doing tonight yet. So we don't know yet whether the proposition,

> I won't study logic tonight,

is true or not. Yet we can still reason out its consequences. And if we wrote out your friend's chain of reasoning to their conclusion, none of the steps (premises aside) would necessarily be propositions your friend actually believes. Hence we *discharge* all the steps in a hypothetical chain of reasoning once we have reached its end. We do not want to use steps in that chain of reasoning that we don't actually take to be true.

Can you write a formal proof that emulates your friend's chain of reasoning?