Determining Truth-Values of Complex Sentences
Philosophy 3

Take the following sentence of propositional logic:

- $\neg(p \vee q) \to r$

If we suppose that p is true, and that q and r are false, what is the truth-value of the sentence above? The answer depends on how we calculate the truth-value of more complex sentence from the simpler sentences that compose them.

Each complex sentence is composed of a logical operator and either one or two sentences. Negation sentences are composed of a negation sign and a sentence; all other complex sentences are composed of two sentences and a conjunction, disjunction, conditional, or biconditional sign.

The above sentence is a conditional sentence composed of the following two sentences:

- $\neg(p \vee q)$

- r

As we all know by now, the conditional will be false if the former sentence is true, and the latter sentence is false. We already know that the latter is false, so we just need to figure out if the former sentence is false. The former is a negation, composed of the negation sign and this sentence:

- $p \vee q$

If this sentence, a disjunction, is false, then its negation is true. A disjunction is false just in case neither of its constituents, its disjuncts, is true. We stipulated that p is true, so the disjunction is in fact true.

Since the disjunction is true, its negation is false. Its negation was the antecedent of our original sentence. Since that sentence is a conditional, it is true if its antecedent is false. So, it is true.