Direct and Formal Counterexamples

We learned two methods to demonstrate an argument's invalidity: *direct* and *formal* counterexamples.

1 Direct Counterexamples

Recall that an argument is valid only if the premises imply the conclusion, that is, if it is impossible for the premises to be true and the conclusion false. Then we can demonstrate that an argument is *invalid* simply by showing that it is *possible* for the premises to be true and the conclusion false.

One way to show that it's possible for an argument's premises to be true and its conclusion to be false is just to describe a situation in which the premises and true and the conclusion false. Consider the following argument:

- Premise 1: If you study hard for your exam, you will receive an A.
- Premise 2: You received an A.
- Conclusion: You studied hard for your exam.

This argument is not valid, and we can give a simple direct counterexample to prove it. The counterexample is: *You could have received an A without studying because you cheated on your exam and didn't get caught.*

The counterexample describes a situation in which the premises of the argument are true and the conclusion false. Premise 1 is only false if its antecedent (You studied hard) is true and its consequent (You received an A) false. But in our counterexample the antecedent is false, so Premise 1 is true. We also specified that you received an A on the exam, so Premise 2 is true. But the conclusion is false - you didn't receive an A because you studied hard, you received an A because you cheated.

2 Formal Counterexamples

2.1 Logical Form

Propositions and arguments have *logical form.* We can represent logical form by abstracting from the content of an argument, leaving only propositional variables and logical connectives, like *if, then, and, or, etc.*. In the above example, we can represent the logical form of the first premise as follows:

- If p, q.

Here p stands for "You study hard for your exam" and q stands for "You receive an A." If we represent the logical form of the first premise in this way, it follows that we can represent the form of the entire argument in the following way:

- Premise 1: If p, q.
- Premise 2: q
- Conclusion: p

2.2 Form and Invalidity

A formal counterexample makes use of the following principle: *every argument with the same logical form as an invalid argument is also invalid.* If we can show that there is an invalid argument with the same logical form as the argument from the previous section, we have provided a *formal counterexample* to it.

Here is a formal counterexample to the argument from the previous section:

- Premise 1: If you are eaten by a Tyrannosaurus Rex, you will die.
- Premise 2: You are dead.
- Conclusion: You were eaten by a Tyrannosaurus Rex.

This argument has the same logical form as the argument from the previous section, and it is clearly invalid. Just because a person is dead does not mean they were eaten by a T-Rex. There are a million other ways that a person could die aside from death by dinosaur, so the conclusion does not follow from the premises. So this argument is a formal counterexample to the original argument.