Consider a space ship which, according to the factory specs, is 120 meters long, and a cylindrical, hollow space station which, again according to factory specs, is 100 meters long. Suppose that both are moving inertially, i.e., that neither is experiencing any acceleration, at a relative velocity of 6/10 the velocity of light. Question: is there any time at which the entire space ship is enclosed within the space station? Look at the diagram below.

Fig. 1

Fig. 2

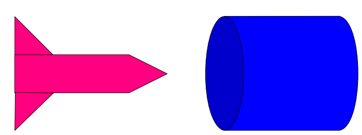


Fig. 3a

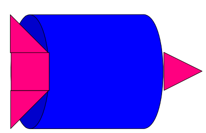
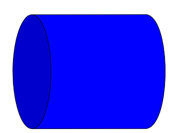


Fig. 3b



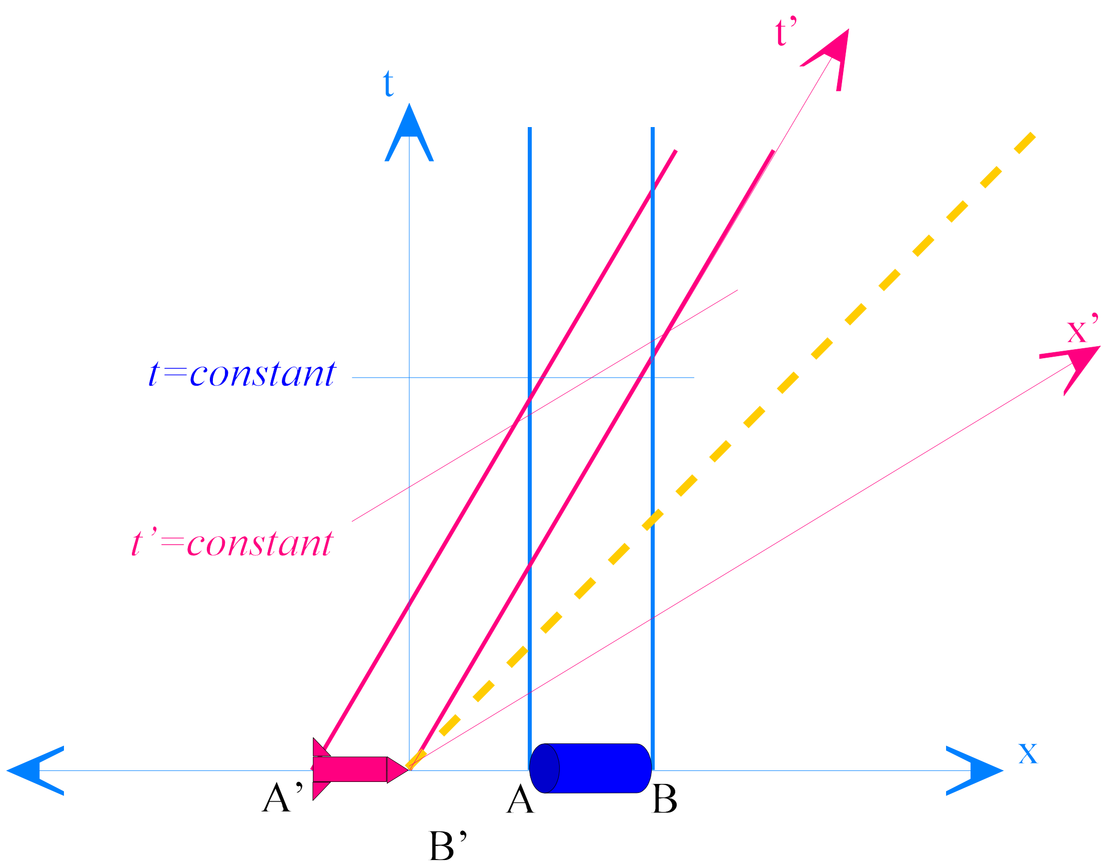
Which of Figure 3a or 3b correctly depicts the case when the space ship is in the space station? It turns out that both of them are correct. If we look at the situation from the space station's point of view, we analyze it as follows. Let the space station be the "rest" (unprimed) frame and the space ship be the "moving" (primed) frame. The Lorentz equations state

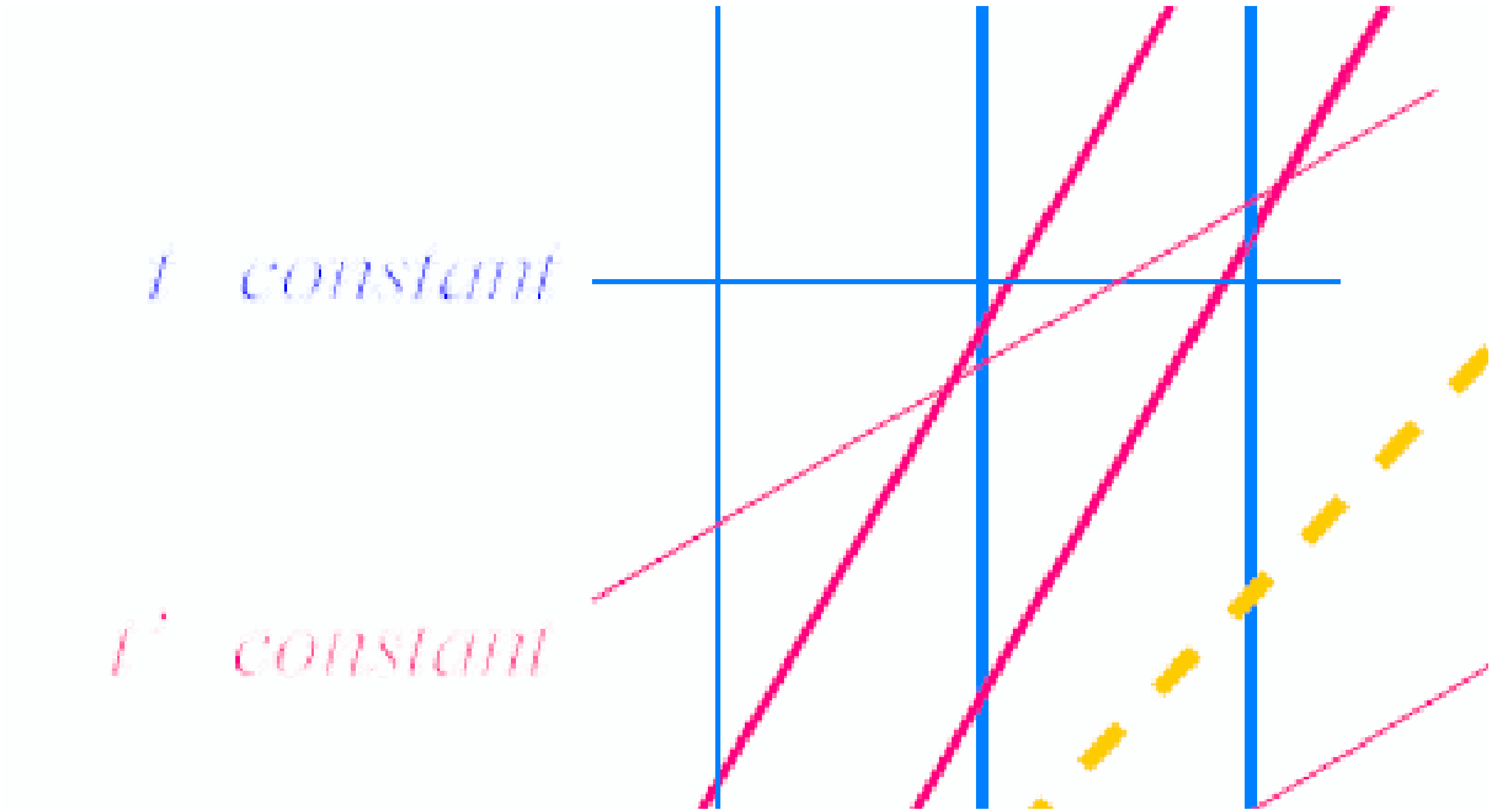
$$\Delta x = \Delta x'\sqrt{1-{v^2}/{c^2}} = 120\sqrt{1-.6^2} = 120\sqrt{1-.36} = 120\sqrt{.64} = 120(.8) = 96m$$

So, if the space ship is 120 meters long in the moving frame, it will be 96 meters long in the rest frame, so it will fit inside the 100 meter-long space station. Now, let's reverse the situation, and let the space ship be the "rest" frame, and the space station the "moving" frame. In that case, $\Delta x' = 100$ meters, so $\Delta x = 80$ meters. That is, if the space station is 100 meters long in the moving frame, it will be 80 meters long in the rest frame, far to short to contain a 120-meter-long space ship. But how can this be? What would happen if someone on the space station closed the end doors just as the front end of the space ship (B') reached the back end of the space station (B)? What would happen at the other end of the space station? Would the door at A come down on the tail of the rocket, or would the tail be inside the space station?

The key to understanding how this is to be answered is to recognize the role that simultaneity is playing here. We have two events: the closing of the door at B and the closing of the door at A, and we ask what would happen if these events were simultaneous. But remember, simultaneity is relative to one's state of motion. In general, people on the space ship and people on the space station will disagree as to which events are simultaneous. Look at the following spacetime diagram.

Notice the two lines, t = constant and t' = constant. These represent "planes of simultaneity" in each frame. Notice that at t = constant, the entire space ship is inside the space station, while at t' = constant, it is not.





This is an enlarged view of the point at which the rocket ship passes through the space station. Notice that along the line t = constant, the ends of the rocket ship (the red lines) are inside the ends of the space station (the blue lines), while along the t' = constant, the ends of the rocket ship are outside the space station. These lines represent the sets of events that are simultaneous in each reference frame. So, in the blue frame, there is a time at which the rocket ship is entirely inside the space station, while in the red frame, there is no such time.