



# COUNTEREXAMPLES

**Statements**

An **invalid** argument is one in which it is possible to have true premises and a false conclusion.

An **invalid argument form** is one that has some invalid substitution instances.

We demonstrate that an argument form is invalid by providing a **counterexample**.

A **counterexample** to an argument form is a substitution instance in which the premises are true and the conclusion is false.

# example

1. If Uma Thurman is a ninja, then she is stealthy.
2. It's not true that Uma is a ninja.
3. So, Uma is not stealthy.

## The form

1. If P, then Q.
2. Not P.
3. Therefore, not Q.



# the counterexample recipe

First, extract the logical form. Use capital letters to stand for statements, leaving the logical connectives as they are:

1. If P, then Q.
2. Not P.
3. Therefore, not Q.

Second, come up with statements to plug in uniformly for P and Q so that

- the premises are true
- the conclusion is false

# counterexample

## The logical form

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1. If P, then Q.
2. Not P.
3. Therefore, not Q.

Let P: We live in SF.

Let Q: We live in CA.

## Substitution instance:

1. If we live in SF, then we live in CA. (TRUE)
2. It's false that we live in SF. (TRUE)
3. Therefore, it's false that we live in CA. (FALSE)

# a formal fallacy: Denying the Antecedent

1. If P, then Q.
2. Not P.
3. Therefore, not Q.

This is an invalid argument form.

## another formal fallacy

1. If aliens probed Eric, then Eric has a funny walk.
2. Eric has a funny walk.
3. Therefore aliens probed Eric.

### The logical form

1. If P, then Q.
2. Q.
3. Therefore, P.

It's official name is:

*Affirming the Consequent*



# counterexample

## The logical form

- 1. If P, then Q.
- 2. Q.
- 3. Therefore, P.

Let P: Rocks are conscious.

Let Q: Something is conscious.

## Substitution instance:

- 1. If rocks are conscious, then something is conscious.  
(TRUE)
- 2. Something is conscious.  
(TRUE)
- 3. Therefore, rocks are conscious. (FALSE)

What if you are arguing with somebody who believes firmly that rocks are conscious?

A **good counterexample** to an argument form is a substitution instance in which the premises are well-known truths and the conclusion is a well-known falsehood.

# counterexample

## The logical form

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1. If P, then Q.
2. Q.
3. Therefore, P.

Let P: There is an invisible cat in the room.

Let Q: We see no cat in the room.



## Substitution instance:

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1. If there is an invisible cat in the room, then we see no cat in the room. (TRUE)
2. We see no cat in the room. (TRUE)
3. Therefore, there is an invisible cat in the room. (FALSE)

## *a worry*

Scientific theories make predictions and are confirmed when those predictions are borne out.

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One way this proceeds:

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1. If the general theory of relativity is true, then the chalk falls to the ground after being dropped.
2. The chalk falls to the ground when dropped.
3. Thus, the general theory of relativity is true.

Although the argument is invalid, it is not thereby *bad*. These kind of arguments are not offered as deductive arguments in the first place. They are inductive arguments.

# more counterexamples

consider this argument:

1. Philosophy is important if ideas are important.
2. And assuming that ideas change lives, ideas are important.
3. Hence, if philosophy is important, then ideas change lives.

Let **P**: philosophy is important.

Let **Q**: ideas are important.

Let **R**: ideas change lives.

**Step One:** Extract the form

1. If Q then P.
2. If R then Q.
3. Hence, if P then R.

## The logical form

1. If Q then P.
2. If R then Q.
3. Hence, if P then R.

**Step Two:** Find statements to plug in for the letters so that we get premises that are well-known truths and a conclusion that is a well-known falsehood.

# counterexample

## The logical form

- 1. If Q then P.
- 2. If R then Q.
- 3. Hence, if P then R.

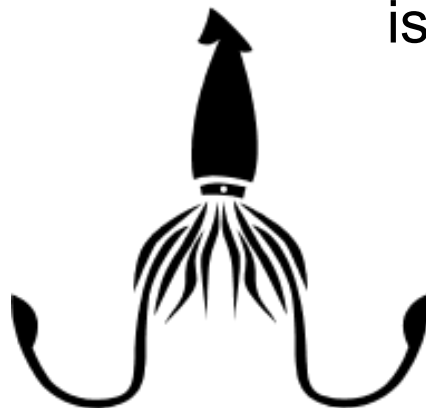
Let P: Some squids are cephalopods.

Let Q: All squids are cephalopods.

Let R: Everything is a cephalopod.

## Substitution instance:

- 1. If all squids are cephalopods then some squids are cephalopods. (TRUE)
- 2. If everything is a cephalopod then all squids are cephalopods. (TRUE)
- 3. Hence if some squids are cephalopods then everything is a cephalopod. (FALSE)



# more counterexamples

consider this argument:

1. Either God exists or we evolved through natural selection.
2. We evolved through natural selection.
3. Therefore God does not exist.

Let **P**: God exists

Let **Q**: We evolved through natural selection.

**Step One:** Extract the form

1. Either P or Q.
2. Q.
3. Therefore, not P.



## The logical form

1. Either P or Q.
2. Q.
3. Therefore, not P.

**Step Two:** Find statements to plug in for the letters so that we get premises that are well-known truths and a conclusion that is a well-known falsehood.

# counterexample

## The logical form

- 1. Either P or Q.
- 2. Q.
- 3. Therefore, not P.

Let P: Heath Ledger played a Joker role.

Let Q: Jack Nicholson played a Joker role.



## Substitution instance:

- 1. Either Heath Ledger played a Joker role or Jack Nicholson played a Joker role. (TRUE)
- 2. Jack Nicholson played a Joker role. (TRUE)
- 3. Therefore, Heath Ledger did not play a Joker role. (FALSE)

