JPMC Quant Mentorship Case Study

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Year: 3rd year

Derivatives, Futures and Options

Question 1 :- Power of Compounding

1) $V(0)$:- Initial Investment, rate of Interest = r% per annum

Interest Rate per Compounding Period:

The annual interest rate r% is divided among the n Compounding periods in a year. Therefore the interest rate per period is:

$$\text{Periodic Interest Rate} = \frac{r}{n}$$

Number of Compounding Periods:

Over T years, the total number of Compounding periods is:

$$\text{Total Periods} = n \times T$$

Growth of Investment per Period:

At each compounding period, the investment grows by the periodic interest rate. After one period, the investment becomes:

$$V(1) = V(0) * \left(1 + \frac{r}{n}\right)$$

after two periods:

$$V(2) = V(1) * \left(1 + \frac{r}{n}\right) = V(0) * \left(1 + \frac{r}{n}\right)^2$$

Continuing this process, after K periods:

$$V(K) = V(0) \times \left(1 + \frac{r}{n}\right)^K$$

Final amount after T years: After T years, which corresponds to $n \times T$ Compounding periods, the investment grows to

$$V(T) = V(0) * \left(1 + \frac{r}{n}\right)^{nT}$$

Since r is given as a percentage, it should be converted to a decimal by dividing by 100:

$$\text{Periodic Interest Rate} = \frac{r}{100 \times n}$$

Combining these adjustments, the Compound interest formula is:

$$\boxed{V(T) = V(0) \times \left(1 + \frac{r}{100 \times n}\right)^{n \times T}}$$

To find the expression for Continuous Compounding, we take the limit as n approaches infinity:

$$V(T) = \lim_{n\to\infty} V(0) \times \left(1 + \frac{r}{100n}\right)^{nT}$$

$$V(T) = V(0) \times \left[\left(1+\frac{r}{100n}\right)^{n}\right]^{T}$$

as $n\to\infty$, the term $\left(1+\frac{r}{100n}\right)^{n} \to e^{r/100}$, where e is the base of the natural logarithm.

Therefore:

$$\lim_{n\to\infty}\left(1+\frac{r}{100n}\right)^{n} = e^{r/100}$$

Substituting this result back into our expression:

$$V(T) = V(0) \times \left(e^{r/100}\right)^{T} = V(0) \times e^{rT/100}$$

$$\boxed{V(T) = V(0) \times e^{rT/100}}$$ → for Continuous Compounding, where interest is Compounded Continuously over time.

<u>Calculations</u> :- at $T=10$

(i) Semi - annual ($n=2$):

$$V(10) = 100\left(1+\frac{5}{200}\right)^{20} = 100 \times (1.025)^{20} \simeq 163.86$$

(ii) Monthly ($n=12$):

$$V(10) = 100\left(1+\frac{10}{1200}\right)^{120} \simeq 270.70$$

(iii) Weekly ($n=52$):

$$V(10) = 100\left(1+\frac{5}{5200}\right)^{520} \simeq 164.87$$

(iv) Continuous ($n\to\infty$):

$$V(10) = 100 \cdot e^{0.10\times 10} = 100 \cdot e^{1} \simeq 271.83$$

2) To determine the value $V(T)$ of an investment at time T, given: an initial investment $V(0)$, a continuous income stream $P(t)$, a time-varying interest rate $r(t)\%$ per annum.
With continuous compounding, we can draw the expression from first principles as follows:

→ Present Value of the Income Stream:

The present value PV of a future cash flow $P(t)$ received at time t is given by discounting it back to the present time $t=0$ using the continuous compounding interest rate $r(t)$:

$$\boxed{PV = P(t) \times e^{-\int_0^t \frac{r(\tau)}{100} d\tau}}$$

Here, $\int_0^t \frac{r(\tau)}{100} d\tau$ represents the accumulated effect of the time-varying interest rate from time 0 to t.

→ Accumulated Value of the Income Stream at time T:

To find the value of the income stream at a future time T, we need to accumulate each infinitesimal payment $P(t)\,dt$ made at time t forward to time T. The future value FV of an infinitesimal payment $P(t)\,dt$ at time T is:

$$\boxed{FV = P(t)\,dt \times e^{\int_t^T \frac{r(\tau)}{100} d\tau}}$$

This accounts for the growth of each payment $P(t)dt$ from time t to T under the time-varying interest rate $r(t)$.

→ Total Value at time T:-

The total value $V(T)$ at time T is the sum of the future value of the initial investment $V(0)$ and the accumulated value of the income stream from time 0 to T:

$$\boxed{V(T) = V(0) \times e^{\int_0^T \frac{r(\tau)}{100} d\tau} + \int_0^T P(t) \times e^{\int_t^T \frac{r(\tau)}{100} d\tau} dt}$$

- The term $V(0) \times e^{\int_0^T \frac{r(\tau)}{100} d\tau}$ represents the growth of the initial investment $V(0)$ over the period from 0 to T under the continuously compounding, time-varying interest rate $r(t)$.

- The integral $\int_0^T P(t) \times e^{\int_t^T \frac{r(r)}{100} dr} dt$ sums up the future values of all infinitesimal payments $P(t)\,dt$ made at each time t, grown forward to time T considering the interest rate $r(t)$.

This formulation allows for the calculation of the investment's value at any time T, accounting for both the initial investment and a continuous income stream under a time-varying interest rate with continuous compounding.

Calculations :-

i) $r(t) = 5$, $V(0) = 100$, $P(t) = 5$, $T=1$

$$V(1) = 100 \cdot e^{0.05} + 5 \int_0^1 e^{0.05(1-s)} ds$$

$$= 105.13 + 5.13$$

$$= 110.26$$

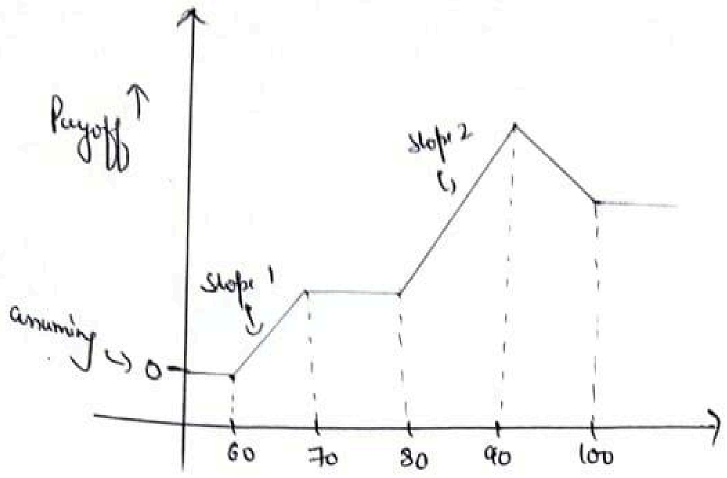
ii) $r(t) = 5$, $V(0) = 100$, $P(t) = 5t$, $T=1$

$$V(1) = 100 \cdot e^{0.05} + 5 \int_0^1 t \cdot e^{0.05(1-s)} ds$$

$$= 105.13 + 2.59$$

$$= 107.72 .$$

Question 2:



Key insights in this payoff graph :-

i) before 60 Constant payoff, likely a long call option with strike 60.

ii) in 60 to 70 it'll be sold, likely a short call option with strike 70.

iii) The payoff chart resembles a bull spread from the left side i.e before \$80

iv) And from the right side it resembles a butterfly spread i.e btw \$80 and \$100.

v) two long call options at strike \$80, a short call at \$90 and a long call at strike price \$100.

Now,

$C(K) = \max(S-K, 0)$: Call option with strike price K.

$P(K) = \max(K-S, 0)$: Put option with strike K

$\therefore$ Portfolio V is Constructed as,

$$V = C(60) - C(70) + 2C(80) - 3C(90) + C(100)$$

Can be written in terms of put options as

$$V = P(60) - P(70) + 2P(80) - 3P(90) + P(100)$$

$\therefore$ V1 & V4 have the same payoff profile as portfolio V.

Question 3 : Keep your friends close, Hedgers Closer!

1) Let amount lent $= x$

$$K = x \cdot e^{rT} \Rightarrow x = \frac{K}{e^{rT}}$$ where r is interest rate. T is time and K is amount to be returned.

2) Keeping Ritwik's watch doesn't eliminate all the risk for Ashutosh and it could have possible issues :-

a) The value of the watch may be insufficient to cover the loan, in case Ritwik doesn't pay back.

b) Since the price is a function of time, its price might reduce later, even if it is sufficient now.

3) • If the price of the watch is sufficient to cover a possible clean default now, to secure his position Ashutosh can buy a put option on the wristwatch. This allow him to sell the watch at a predetermined price and time (say P, T). If the price of the watch drops below P, the put option will enable Ashutosh to sell it for price P, ensuring a minimum return irrespective of market value hence reducing market risk.

• If Ritwik agrees to pay Ashutosh some amount periodically but then defaults, to insure himself from this situation, Ashutosh can choose to buy a credit default swap (CDS). In this arrangement, Ashutosh has to pay a little amount of interest to the CDS provider for their service. In case Ritwik defaults, the CDS provider would compensate Ashutosh for his loss.

Here, Ashutosh might earn a lesser interest but it works in his favour as his risk is lower too.

4) Ritwik can face the following risks :-

• Since the price of the watch is a function of time, its value can also appreciate over time. If he fails to pay the loan, Ashutosh will get the ownership of the watch. In this situation, Ritwik will not be able to benefit from the rise in price of the watch.

- while giving out his watch as a collateral, he might mis-evaluate the worth of his watch. If the original price is significantly higher that what is required to cover defaulting of the loan. Then Rituik will be incurring a loss.
- loss of ownership might have negative financial or emotional effects on Rituik.

To Counter these risks:

- To protect their interests, both parties can opt to get collateral insurance. Rituik could use a custom derivative that prevents mis-calculation of the fair price of the wristwatch so that at the time of sale it protects Rituik from losing more than fair amount for his watch.
- Rituik can choose to buy a call option for his wristwatch at an agreed price. Using this he gets a right to purchase the watch back at that price upon maturity of the loan. Even if the market value of the watch has increased, Rituik gets to purchase the watch at a pre-determined price. This keeps him retain an asset of greater value.

5) If Rituik's portfolio is denoted as V_1 and Ashutosh's portfolio as V_2 then $V_1 = V_2$. This equality arises from the Put-call Parity relationship, which connects the prices of European call call and put options with the same strike price and expiration date. The put-call parity formula is:

$$C + K \cdot e^{-rT} = P + S$$

where:

C = Price of a European call option
P = Price of a European Put option
S = Current price of the underlying asset
K = Strike price
r = risk-free interest rate
T = time to expiration

Explanation:

- V_1 and V_2 are constructed using combinations of calls and puts with the same strikes and expirations.
- By rearranging the put-call parity formula, we can express the value of a call option in terms of a put option and vice versa. This implies that the payoffs of the portfolios are equivalent when the underlying assumptions (eg. no arbitrage, same strikes, same expirations) hold.

Economic Interpretation:

- The equality $V_1 = V_2$ reflects the no-arbitrage principle. If two portfolios have the same payoffs under all possible scenarios, their values must be equal. Otherwise, arbitrage opportunities would exist.

*Bonus Question

Implementing Put-Call Parity to Establish Lower Bounds on D1 & D2

- Lower Bound for a Call option (D1):

The lower bound for a European call option is given by:

$$C \geq S - K \cdot e^{-rT}$$

Interpretation:- The call option price cannot be less than the difference between the current asset price and the present value of the strike price. This ensures that the call option is not undervalued.

- Lower Bound for a Put option (D2):

The lower bound for a European put option is given by:

$$P \geq K \cdot e^{-rT} - S$$

Interpretation:- The put option price cannot be less than the difference between the present value of the strike price and the current asset price. This ensures that the put option is not undervalued.

<u>Economic Interpretation</u> :-

→ These bounds ensure that the options are priced fairly relative to the underlying asset and the time value of money.

→ If the option price falls below the lower bound, arbitrageurs can buy the option and sell the underlying asset (or vice versa) to lock in a risk-free profit, driving the price back to the bound.

Let D1 be a call option and D2 be a put option

From Put-call parity:

$$C + K \cdot e^{-rT} = P + S$$

rearranging for C(D1) and P(D2):

$$C = P + S - K \cdot e^{-rT}$$

$$P = C + K \cdot e^{-rT} - S$$

for D1 (call options):

$$C \geqslant S - K \cdot e^{-rT}$$

This ensures the call option is not undervalued.

for D2 (put option):

$$P \geqslant K \cdot e^{-rT} - S$$

This ensures the put option is not undervalued.

<u>Economic Interpretation</u> :-

- The lower bounds reflect the intrinsic value of the options:
  - → for a call option, the intrinsic value is S-K (if true)
  - → for a put option, the intrinsic value is K-S (if true)

- The bounds ensure that the option price accounts for the time value of money (e^{-rT}) and the current asset price (S).

Question 4 : option - on - option

1)
Underlying Call option : Strike price $(K_1) = \$50$, expiration = 3 months
Compound Call option : Strike price $(K_2) = \$5$, expiration = 1 month
Premium paid = \$2

Value of the underlying call option = $\max(60 - 50, 0)$
$= 10$

In this problem, it is assumed the compound option is exercised $\Rightarrow$ Value of the underlying call option $> K_2$

$\therefore$ Net payoff of the compound call option = payoff of the underlying call option $- K_2 -$ premium paid.

$\Rightarrow 10 - 5 - 2 = 3$

$\Rightarrow$ $\boxed{\text{Net payoff} = \$3}$

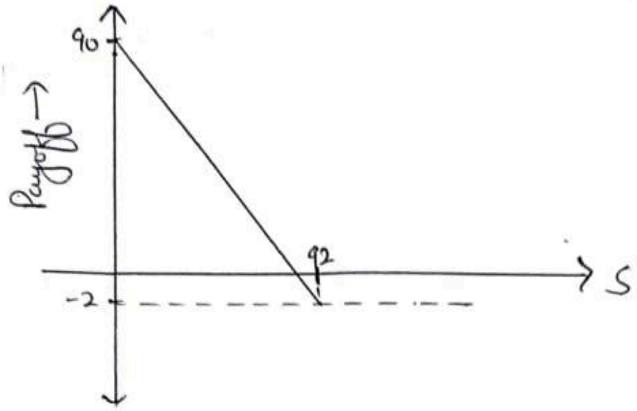
2) underlying Put option : Strike price $= \$100$
Compound Put option : Strike price $= \$8$, premium paid $= \$2$

If exercised , Net payoff $= \text{Max}(100 - S, 0) - 8 - 2$
if not exercised , net payoff $= -2$

we exercise the option only when
$$\max(100 - S, 0) > 8$$
$$\Rightarrow S < \$92$$

for $S \leq 92 \Rightarrow$ exercised $\Rightarrow$ payoff $= 90 - S$
$S \geq 92 \Rightarrow$ not exercised $\Rightarrow$ payoff $= -2$

Plot :-



3) underlying put option : Strike = \$40 ; S = stock price at the expiration of the underlying put option.
Compound option : strike = \$3

At break-even, the net payoff of the call-on-put compound option is zero. This implies

$$\text{Net payoff} = \text{Payoff of put option} - K_2 - P_{\text{Compound}} = 0$$

If exercised, net payoff = $\max(40-S, 0) - 3$
if not exercised, net payoff = 0.

For breakeven on exercising

$$\max(40-S, 0) = 3$$

$$\Rightarrow \boxed{S = \$37}$$

4) underlying call option : strike (K_1) = \$120
Compound option : Strike (K_2) = \$10

$$\text{Net Payoff} = \begin{cases} \max(S-120, 0) - 10, & \text{if exercised} \\ 0, & \text{if not exercised} \end{cases}$$

we exercise only when $\max(S-120, 0) > 10 \Rightarrow S > 130$

Case 1 : $S = 90$:
at \$90 $\Rightarrow$ Payoff = $\max(90-120, 0) = 0 \Rightarrow$ not exercised

$\Rightarrow$ Net payoff = 0

Case 2 : $S = \$110$

$\Rightarrow$ Payoff = max(110-120, 0) = 0 $\Rightarrow$ not exercised

$\Rightarrow$ Net payoff = 0

Case 3 : $S = \$130$

$\Rightarrow$ Payoff = max(130-120, 0) = 10 $\Rightarrow$ not exercised ($\because$ only exercise when $S > 130$)

$\Rightarrow$ net payoff = 0

Case 4 : $S = \$150$

$\Rightarrow$ payoff = max(150-120, 0) = 30 $\Rightarrow$ exercised

$\Rightarrow$ net payoff = 30 - 10 = 20