Deriving the Mass of Binary Stars

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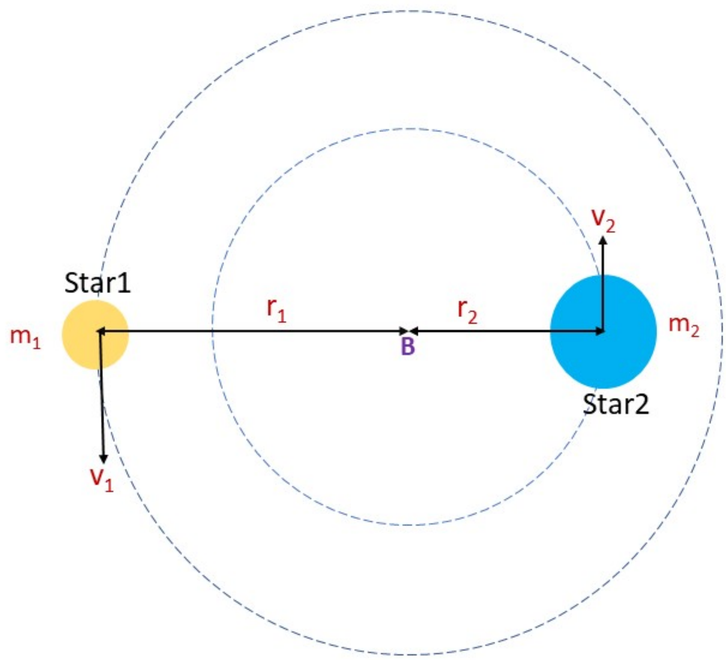

Fig1 : Binary Star System

The above diagram shows two stars in a binary system each:

1. of mass m_1 and m_2
2. having concentric orbits with radii r_1 and r_2
3. having periods T_1 and T_2
4. having orbital velocity v_1 and v_2

Point B in the diagram is called the Barycenter of the system. It is the center of mass of the two bodies that orbit one another and is the common point about which the bodies orbit.

The formula for center of mass is:-

$$x_c m = \frac{\sum_{i=1}^{n} m_i x_i}{M_{total}}$$

The position of Barycenter from Star 1:-

$$r_1 = \frac{m_1(0) + m_2(r_1 + r_2)}{m_1 + m_2}$$
$$r_1 = \frac{m_2(r_1 + r_2)}{m_1 + m_2}$$
$$(m_1 + m_2)r_1 = m_1 r_1 + m_1 r_2$$
$$m_1 r_1 + m_2 r_1 = m_1 r_1 + m_1 r_2$$
$$m_2 r_1 = m_1 r_2$$
$$\frac{m_1}{m_2} = \frac{r_2}{r_1} \tag{1}$$

Further, for the system to be stable the line joining the stars should pass through the center of mass. This means the stars must orbit with the same period else, they would not remain on the opposite sides of the center of mass.

$$T_1 = \frac{2\pi r_1}{v_1} \tag{2}$$
$$T_2 = \frac{2\pi r_2}{v_2} \tag{3}$$
$$\because T_1 = T_2$$
$$\frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2}$$
$$\frac{r_1}{r_2} = \frac{v_1}{v_2} \tag{4}$$

from (1) and (4) we have:

$$\frac{m_2}{m_1} = \frac{r_1}{r_2} = \frac{v_1}{v_2} \tag{5}$$

Now, the Gravitational Force between the two stars provides them their centripetal force:

$$F_G = F_C$$

For Star 1 :-

$$\frac{Gm_1 m_2}{(r_1 + r_2)^2} = \frac{m_1 v_1^2}{r_1}$$
$$m_2 = \frac{v_1^2 (r_1 + r_2)^2}{r_1 G} \times \frac{2\pi}{2\pi}$$
$$m_2 = \frac{v_1 2\pi (r_1 + r_2)^2}{T_1 G} \tag{6}$$

Adding (2) and (3) we have:

$$(r_1 + r_2) = \frac{T_1 v_1 + T_2 v_2}{2\pi} \tag{7}$$

Let $T = T_1 = T_2$

$$(r_1 + r_2) = \frac{T(v_1 + v_2)}{2\pi}$$

Substituting (7) in (6) we have:

$$m_2 = \frac{v_1 T (v_1 + v_2)^2}{2\pi G} \tag{8}$$

using (7)

$$m_1 = \frac{v_2 T (v_1 + v_2)^2}{2\pi G} \tag{9}$$

Equations (8) and (9) , can be used to calculate the masses in our assignment.
Also, from equation (7) we can say that the star with lower mass will have a higher orbital velocity and a bigger radius than the one with higher mass. This can be one of the ways to verify whether the final answer is in the right direction.