ML Assignment 5

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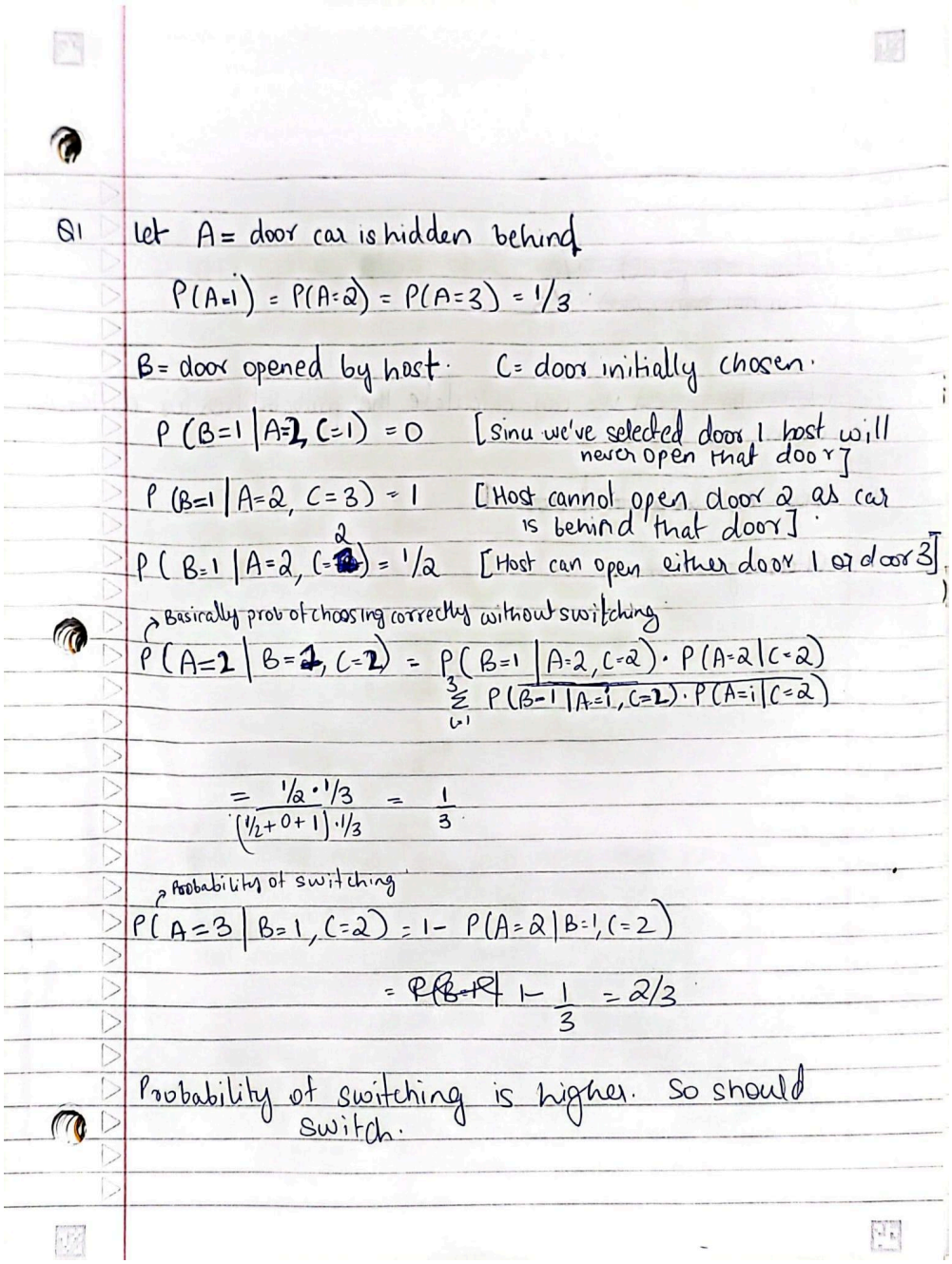
Problem 1



Q1 Let A = door car is hidden behind

$P(A=1) = P(A=2) = P(A=3) = 1/3$

B = door opened by host. C = door initially chosen.

$P(B=1 \mid A=1, C=1) = 0$ [since we've selected door 1 host will never open that door]

$P(B=1 \mid A=2, C=3) = 1$ [Host cannot open door 2 as car is behind that door].

$P(B=1 \mid A=2, C=2) = 1/2$ [Host can open either door 1 or door 3]

Basically prob of choosing correctly without switching

$$P(A=2 \mid B=1, C=2) = \frac{P(B=1 \mid A=2, C=2) \cdot P(A=2 \mid C=2)}{\sum_{i=1}^{3} P(B=1 \mid A=i, C=2) \cdot P(A=i \mid C=2)}$$

$$= \frac{1/2 \cdot 1/3}{(1/2 + 0 + 1) \cdot 1/3} = \frac{1}{3}$$

Probability of switching

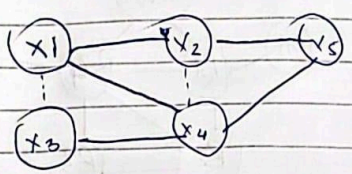
$$P(A=3 \mid B=1, C=2) = 1 - P(A=2 \mid B=1, C=2)$$

$$= 1 - \frac{1}{3} = 2/3$$

Probability of switching is higher. So should switch.

Problem 2

Q2. Probability distribution of a bayesian network can be given by



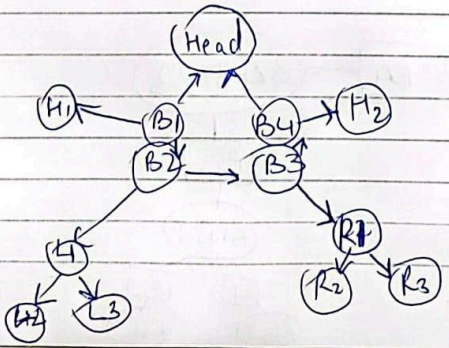
$$p(x_1, x_2, x_3, x_4, x_5) = p(x_1) \cdot p(x_2|x_1) \cdot p(x_4|x_1) \cdot p(x_3|x_1) \cdot p(x_5|x_3, x_1) \cdot p(x_5|x_2, x_1)$$

$$= p(x_1) \cdot p(x_2|x_1) \cdot p(x_4|x_1, x_3) \cdot p(x_3) \cdot p(x_5|x_2, x_4)$$

Using bayes ball

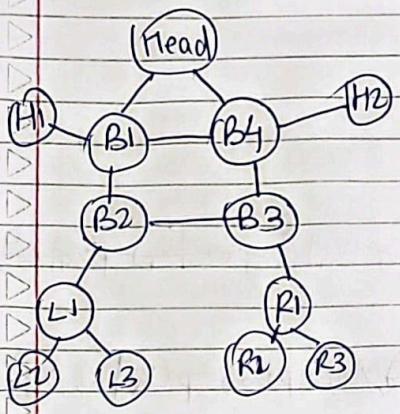
1) False (dependent on x_1)
2) False (depends on x_5)
3) True
4) False
5) True
6) False
7) True
8) True
9) False
10) False

Q3.

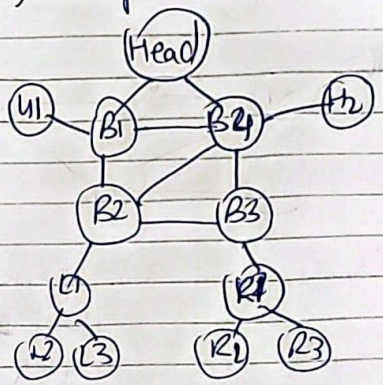


Problem 3

a) Moralization



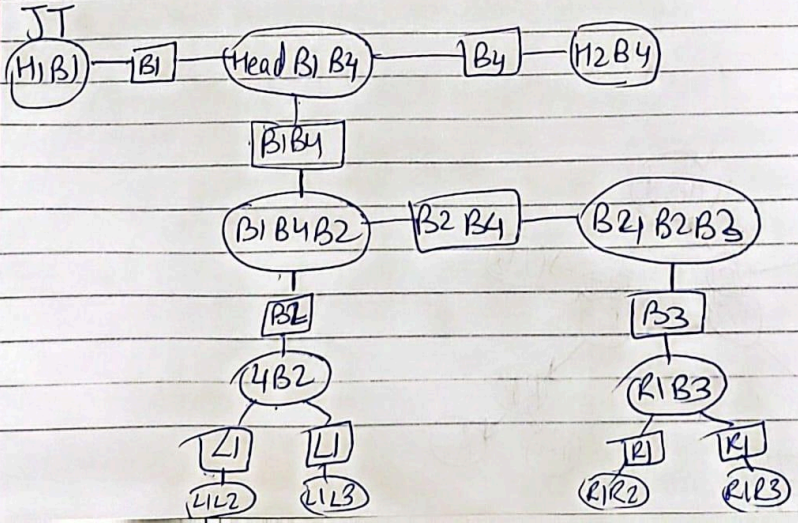
b) Triangulation



c) Cliques

(Head B1 B4), (H1 B1), (B4 H2), (B1 B2 B4), (B4 B2 B3), (B2 L1), (B3 R1), (L1 L2), (L1 L3), (R1 R2), (R1 R3)

d) JT



Problem 4

Implemented the forward and backward message passing using loops on the initialised potentials (either random initialization or using the values provided in the test)

Code included in problem4.py

After passing all the messages left to right and right to left, we normalise the results.

We get the following marginal probabilities§

```
simran@Simrans-MBP Q4 % /usr/bin/python3 /Users/simran/Documents/ML/Assignments/MLAssignment5/Q4/problem4.py
[[0.04046243 0.44508671]
 [0.32369942 0.19075145]
 [0.26011561 0.10404624]
 [0.05780347 0.57803468]
 [0.11921965 0.19869942]
 [0.63945087 0.04263006]
 [0.56900289 0.18966763]
 [0.06033237 0.18099711]]
```