Godel's Incompleteness Theorem

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The incompleteness theorems of Godel are two mathematical logic theorems that deal with the provability limits of formal axiomatic system. There will always be claims about natural numbers that are true but unprovable within the system for any such consistent formal system. Godel's incomplete theorems show that pretty much any logical system either has contradictions or statements that cannot be proven!

According to godel's first incompleteness theorem, if you have a consistent logical system (i.e. a set of axioms with no contradictions) in which you can do a given amount of arithmetic, there are proposition in that system that are unprovable using only the axioms of that system. To put it another way, your logical system is incomplete as long as it is complicated enough to incorporate addition and multiplication. There are some things that are impossible to prove true or incomplete!

Godel's second incompleteness theorem gives a specific example of such an unprovable statement. And the example is quite a doozy. The theorem says that inside of a similar consistent logical system (one without contradictions) , the consistency of the system itself is unprovable! This second theorem is stronger than the first incompleteness theorem because the statement constructed in the first incomplete theorem does not directly express the consistency of the system.

The incompleteness results affect the philosophy of mathematics, particularly versions of formalism, which use a single system of formal logic to define their principles.

The existence of incomplete theories is hardly surprising. Take ant theory, even a complete one and drop some axiom; unless the axiom is redundant, the resulting system is incomplete. The incompleteness theorems, however, deal with a much more radical kind of incompleteness phenomenon. Unlike the above sort of trivially incomplete theories, which can be easily completed, there is no way of completing the relevant theories; all their extensions, inasmuch as they are still formal systems and hence axiomatizable are also incomplete. They remain, so to speak, externally incomplete and can never be completed. They are "essentially incomplete".