Calculus Assignment 3

1. $Kt = \int \frac{1}{(3-0.4x)(2-0.6x)} dx$

$= \frac{1}{(3-0.4x)(2-0.6x)} = \frac{A}{(3-0.4x)} + \frac{B}{(2-0.6x)}$

$= 1 = 2A - 0.6xA + 3B - 0.4xB$

$= 2A + 3B = 1$

$-0.6xA - 0.4xB = 0$

$A = -2/5 \quad , B = 3/5$

$= \int \frac{-2}{5(3-0.4x)} + \int \frac{3}{5(2-0.6x)} = Kt$

$= -\frac{2}{5} \ln(3-0.4x) + \frac{3}{5} \ln(2-0.6x) = Kt$

$= -\ln(3-0.4x)^{2/5} + \ln(2-0.6x)^{3/5} = Kt$

$= \frac{3/\cancel{5}}{2/\cancel{5}} \ln\left(\frac{2-0.6x}{3-0.4x}\right) = Kt$

$= \frac{3}{2} \ln\left(\frac{2-0.6x}{3-0.4x}\right) = Kt$

2. $\displaystyle\int \frac{(3+6x+4x^2-2x^3)}{x^2(x^2+3)}\,dx$

$x^2(x^2+3) \overline{)\,-2x^3+4x^2+6x+3}$

$-2x^3 \quad -6x$

$4x^2+12x+3$

$\frac{-2x^3}{x^2} = -2x$

$-2x(x^2+3)$

$-2x^3-6x$

$$= \int \frac{4x^2\,dx}{x^2(x^2+3)} + \int \frac{12x\,dx}{x^2(x^2+3)} + \int \frac{3\,dx}{x^2(x^2+3)}$$

$$= \int \frac{4\,dx}{x^2+3} + \int \frac{12\,dx}{x(x^2+3)} + \int \frac{3\,dx}{x^2(x^2+3)}$$

$$= \frac{4}{\sqrt3}\tan^{-1}\left(\frac{x}{\sqrt3}\right) + 6\ln(x^2+3) + \int \frac{3\,dx}{x^2(x^2+3)}$$

$u = x^2+3$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\cancel{12}\,6\int \frac{du}{\cancel{2}u} = 6\ln u$

$= 6\ln(x^2+3)$

$$= \int \frac{3\,dx}{x^2(x^2+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+3}$$

$$= \frac{3}{x^2(x^2+3)} = \frac{A(x)(x^2+3) + B(x^2+3) + (Cx+D)(x^2)}{x^2(x^2+3)}$$

$$= 3 = Ax^3 + 3Ax + Bx^2 + 3B + Cx^3 + Dx^2$$

$= 3B = 3$, $\quad 3Ax = 0$, $\quad Dx^2 + x^2 = 0$, $\quad Cx^3 = 0$

$\boxed{B = 1}$ $\quad \boxed{A = 0}$ $\quad \boxed{D = -1}$ $\quad \boxed{C = 0}$

$$= \int \frac{3}{x^2(x^2+3)} = \int \frac{1}{x^2}\,dx + \int \frac{-1\,dx}{x^2+3}$$

$$= \frac{-1}{x} - \frac{1}{\sqrt3}\tan^{-1}\left(\frac{x}{\sqrt3}\right)$$

$$\frac{4}{\sqrt3}\tan^{-1}\left(\frac{x}{\sqrt3}\right) + 6\ln(x^2+3) - \frac{1}{x} - \frac{1}{\sqrt3}\tan^{-1}\left(\frac{x}{\sqrt3}\right)$$

3. $$\int \frac{dx}{7-3\sin x+6\cos x}$$

$\sin x = \frac{2u}{1+u^2}$

$\cos x = \frac{1-u^2}{1+u^2}$

$dx = \frac{2\,du}{1+u^2}$

$$= \int \frac{\frac{2\,du}{1+u^2}}{7-3\left(\frac{2u}{1+u^2}\right)+6\left(\frac{1-u^2}{1+u^2}\right)}$$

$$= \int \frac{\frac{2\,du}{\cancel{1+u^2}}}{\frac{7+7u^2-6u+6-6u^2}{\cancel{1+u^2}}}$$

$$= \int \frac{2\quad du}{u^2-6u+13}$$

$$= \int \frac{2\quad du}{(u-3)^2+4}$$

$$= 2.\frac{1}{2}\left(\tan^{-1}\frac{u-3}{2}\right)+c$$

$$\tan^{-1}\left(\frac{u-3}{2}\right)+c.$$

$U = \tan x/2$

$$\tan^{-1}\left(\frac{\tan x/2-3}{2}\right)+c.$$

4. Determine if $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$ is convergent or divergent.

$u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$= \int_{-\infty}^{\infty} xe^{-x^2}\,dx$

$= \int_{-\infty}^{\infty} \frac{e^{-u}}{2}\,du$

$= \frac{-e^{-u}}{2}\Big|_{-\infty}^{\infty} = \frac{-e^{-x^2}}{2}\Big|_{-\infty}^{\infty}$

= Substituting limit :-

$= \frac{-e^{-(\infty)^2}}{2} - \left(\frac{-e^{-(-\infty)^2}}{2}\right) = 0$ (Convergent).

5. Determine if $\int_{0}^{4} \frac{x\,dx}{x^2-9}$ is converges or diverges. If converges, determine its value.

$u = x^2 - 9$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$= \int_{0}^{4} \frac{du}{2u}$

$= \frac{1}{2}\ln(u)\Big|_{0}^{4} = \frac{1}{2}\ln(x^2-9)\Big|_{0}^{4}$

= Substituting limit :-

$= \frac{1}{2}\ln(4^2-9) - \frac{1}{2}\ln(0^2-9)$

$= \frac{1}{2}\ln 7 - \frac{1}{2}\ln(9)$

$= -0.125$ (Convergent).

6. Determine area :-

$y = x^2+2$, $y = \sin x$, $x = -1$ and $x = 2$

$= A = \int_{-1}^{2} \left((x^2+2) - \sin x \right) dx$

$A = \int_{-1}^{2} (x^2+2)\, dx - \int_{-1}^{2} \sin x \, dx$

$A = \frac{x^3}{3} + 2x \Big|_{-1}^{2} + \cos x \Big|_{-1}^{2}$

$A = \left[\frac{(2)^3}{3} + 2(2) + \cos(2) \right] - \left[\frac{(-1)^3}{3} + 2(-1) + \cos(-1) \right]$

$A = 8.97$ units^2.

7. Determine the area :-

$x = e^{1+2y}$, $x = e^{1-y}$, $y = -2$ and $y = 1$

$= A = \int_{-2}^{1} \left(e^{1-y} - e^{1+2y} \right) dy$

$A = \int_{-2}^{1} e^{1-y}\, dy - \int_{-2}^{1} e^{1+2y}\, dy$

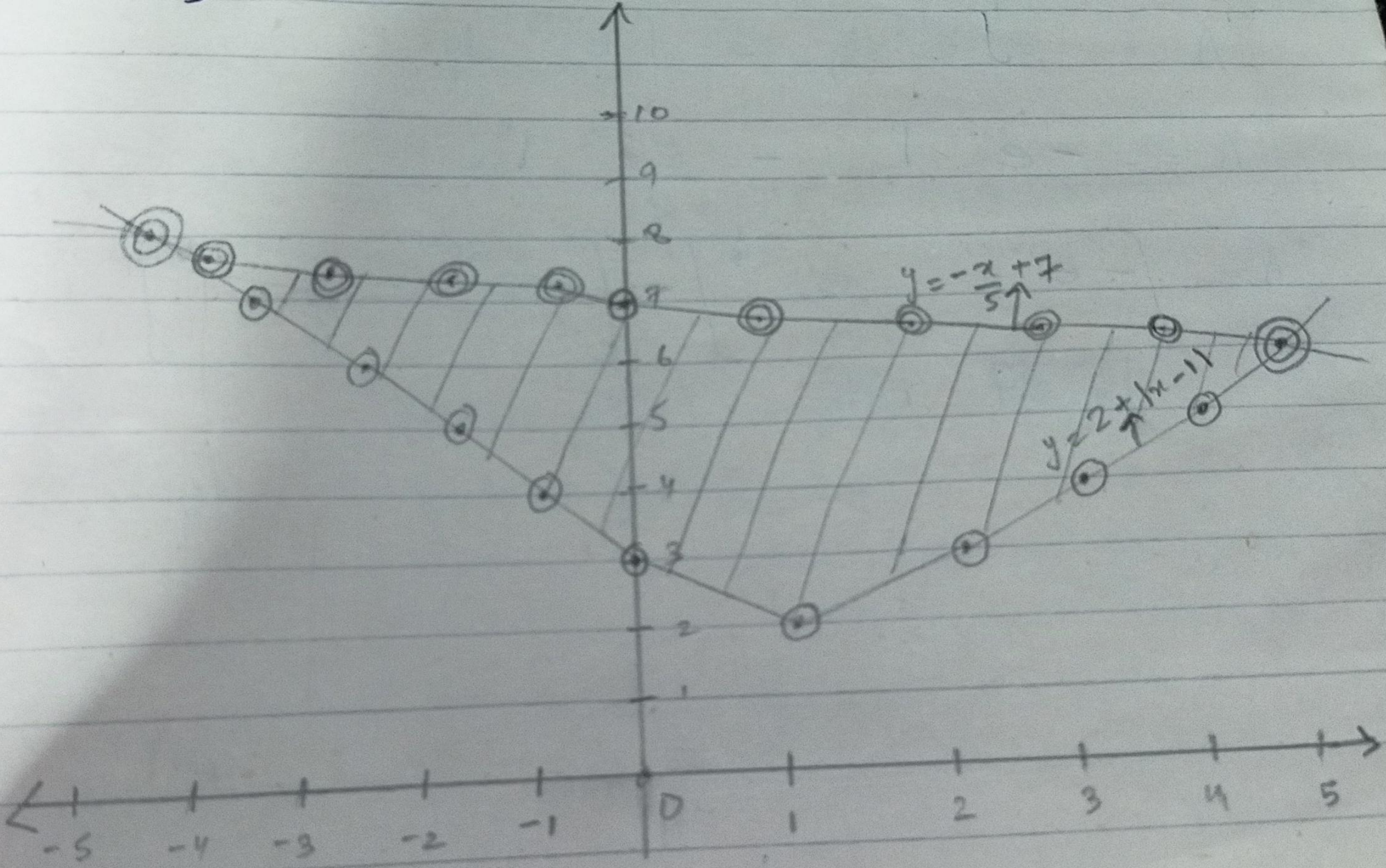
$e^{1-y} \cdot -1 = -e^{1-y}$

$e^{1+2y} \cdot 2 = 2e^{1+2y}$

$A = -e^{1-y} \Big|_{-2}^{1} - \frac{e^{1+2y}}{2} \Big|_{-2}^{1}$

$A = \left[-e^{1-(1)} - \frac{e^{1+2(1)}}{2} \right] - \left[-e^{1-(-2)} - \frac{e^{1+2(-2)}}{2} \right]$

$A =$ 9.06 units^2.



10
9
8
7
6
5
4
3
2
1
$y = -\frac{x}{5} + 7$
$y = 2 + |x - 1|$
-5
-4
-3
-2
-1
0
1
2
3
4
5

8. Sketch the region and find its area:

$y = 2 + |x-1|$ and $y = -\frac{x}{5} + 7$

$$A = \int_{-5}^{1} -\frac{x}{5} + 7 - (-x+3)\, dx + \int_{1}^{5} -\frac{x}{5} + 7 - (x+1)\, dx$$

$$A = \int_{-5}^{1} \frac{4x}{5} + 4 \quad dx + \int_{1}^{5} -\frac{6x}{5} + 6 \quad dx$$

$$A = \frac{\cancel{4}^{2}x^2}{\cancel{10}_{5}} + 4x \Big|_{-5}^{1} + \left(-\frac{\cancel{6}^{3}x^2}{\cancel{10}_{5}} + 6x \Big|_{1}^{5} \right)$$

$$A = \left[\frac{2(1)^2}{5} + 4(1) - \left(\frac{2(-5)^2}{5} + 4(-5) \right) + \left(\frac{-3(5)^2}{5} + 6(5) - \left(\frac{-3(1)^2}{5} + 6(1) \right) \right) \right]$$

$A = 24$ units^2.

9. Determine volume about x-axis:
$y = 2x^2$ and $y = x^3$.

$$x^3 - 2x^2 = 0$$
$$x = 0, 2$$

$$= \quad V = \pi \int_0^2 (x^3)^2 - (2x^2)^2 \, dx$$

$$V = \pi \left[\int_0^2 x^6 \, dx - \int_0^2 4x^4 \, dx \right]$$

$$V = \pi \left(\frac{x^7}{7} - \frac{4x^5}{5} \Big|_0^2 \right)$$

$$V = \pi \left[\left(\frac{(2)^7}{7} - \frac{4(2)^5}{5} \right) - \left(\frac{(0)^7}{7} - \frac{4(0)^5}{5} \right) \right]$$

$$V = \pi \, (7.31)$$

$$V = 7.31 \pi \text{ units}^3.$$

10. Determine volume about y-axis
$y = \sqrt[3]{x}$, $y = \frac{x}{4}$

$$x^{1/3 \times 3} = \left(\frac{x}{4}\right)^{\times 3}$$

$x = y^3$, $x = 4y$.

$$64x = x^3$$
$$x^3 - 64x = 0$$
$$x = 2, -2, 0$$

$$= \quad V = \pi \int_0^2 (y^3)^2 - (4y)^2 \, dy$$

$$V = \pi \left[\int_0^2 y^6 \, dy - \int_0^2 16y^2 \, dy \right]$$

$$V = \pi \left(\frac{y^7}{7} - \frac{16y^3}{3} \right) \Big|_0^2$$

$$V = \pi \left[\left(\frac{(2)^7}{7} - \frac{16(2)^3}{3} \right) - \left(\frac{(0)^7}{7} - \frac{16(0)^3}{3} \right) \right]$$

$$V = \pi \, (24.3)$$

$$V = \quad 24.3 \pi \text{ units}^3.$$