Assignment 1

- MCQs:-

1. C	14. A
2. D	15. A
3. B	16. C
4. A	17. D
5. D	18. C
6. B	19. D
7. D	20. B
8. B	21. D
9. B	22. B
10. B	23. D
11. A	24. D
12. B	25. A
13. C	

- Question Answer:-

Q1 a: i) $\frac{2}{x} < \frac{3}{x-2}$

$= \frac{2}{x} - \frac{3}{x-2} < 0$

$= \frac{2(x-2) - 3(x)}{x(x-2)}$

$= \frac{2x-4-3x}{x(x-2)}$

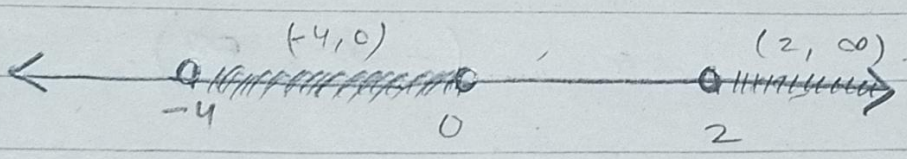
$= \frac{-x-4}{x(x-2)} = 0$

$= -x-4=0, \quad x=0, \quad x-2=0$

$= x = -4^{\times}, \quad x = 0^{\times}, \quad x = 2^{\times}$

$= (-\infty, -4), (-4, 0), (0, 2), (2, \infty)$.

$= S.S = (-4, 0) \cup (2, \infty)$.



R.W

$g \circ f = 2\left(\frac{1}{2}x - 3\right) + 5$

$g \circ f = x - 6 + 5 = x - 1$

$g \circ f(4) = 4 - 1 = 3$

$f \circ g = 4\left(\frac{1}{x}\right) - \left(\frac{1}{x}\right)^2$

$f \circ g = \frac{4}{x^{1/2}} - \frac{1}{x^2}$

$f \circ g\left(\frac{1}{2}\right) = \frac{4}{1/2} - \frac{1}{(1/2)^2}$

$f \circ g\left(\frac{1}{2}\right) = 8 - 4 = 4$

$2(x+5)^2 - 3(x+5) + 1$

$2x^2 + 20x + 50 - 3x - 15 + 1$

$2x^2 + 17x + 36$

$4(a+1)^2 - (a+1) + 1$

$4a^2 + 8a + 4 - a - 1 + 1$

$4a^2 + 7a + 4$

$2(3x-2)^2 + 1$

$2(9x^2 - 12x + 4) + 1$

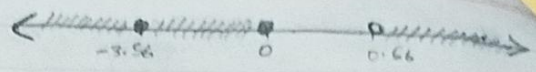
$18x^2 - 24x + 8 + 1$

$18x^2 - 24x + 9$

$18(-2)^2 - 24(-2) + 9$

$72 + 48 + 9 = 129$

ii) $x^3 + 3x^2 - 2x \geq 0$



$x = 0.56$ ✗, $x = -3.56$ ✓, $x = 0$ ✓

$S.S = (-\infty, -3.56], [-3.56, 0], (0.56, \infty)$.

Q1.b) i) $|2x - 3| = 2|3x - 5|$.

① $2x - 3 = 6x - 10$

$2x - 6x - 3 + 10 = 0$

$-4x + 7 = 0$

$x = 7/4$

② $-2x + 3 = -6x + 10$

$-2x + 6x + 3 - 10 = 0$

$4x - 7 = 0$

$x = 7/4$

③ $-2x + 3 = 6x - 10$

$-2x - 6x + 3 + 10 = 0$

$-8x + 13 = 0$

$x = 13/8$

④ $2x - 3 = -6x + 10$

$2x + 6x - 3 - 10 = 0$

$8x - 13 = 0$

$x = 13/8$

ii) $\frac{1}{|2x - 3|} \leq 3$

$= |2x - 3| \geq 1/3$

① $2x - 3 \geq 1/3$

$2x \geq 1/3 + 3$

$2x \geq 10/3$

$x \geq 5/3$

② $2x - 3 \leq -1/3$

$2x \leq -1/3 + 3$

$2x \leq 8/3$

$x \leq 4/3$

Q2.a) $f(x) = \frac{x}{1 + x^2}$, $g(x) = \frac{1}{x}$

- $fog = \frac{1/x}{1 + (1/x)^2} = \frac{1/x}{1 + 1/x^2} = \frac{1/x}{x^2 + 1/x^2} = \frac{1/x \times x^2}{x^2 + 1}$

$fog = \frac{x}{x^2 + 1}$

Domain: $(-\infty, \infty)$.

- $gof = \frac{1}{x/1+x^2} = \frac{1+x^2}{x}$

Domain:: All real numbers except 0.

b) formula for $f^{-1}(x)$, $f(x) = \begin{cases} \frac{7}{2} - x & x < 2 \\ \frac{3}{x} & x \geq 2 \end{cases}$

- $y = \frac{7}{2} - x$

$x = \frac{7}{2} - y$

$f^{-1}(x) = \frac{7}{2} - x$

- $y = \frac{3}{x}$

$x = \frac{3}{y}$

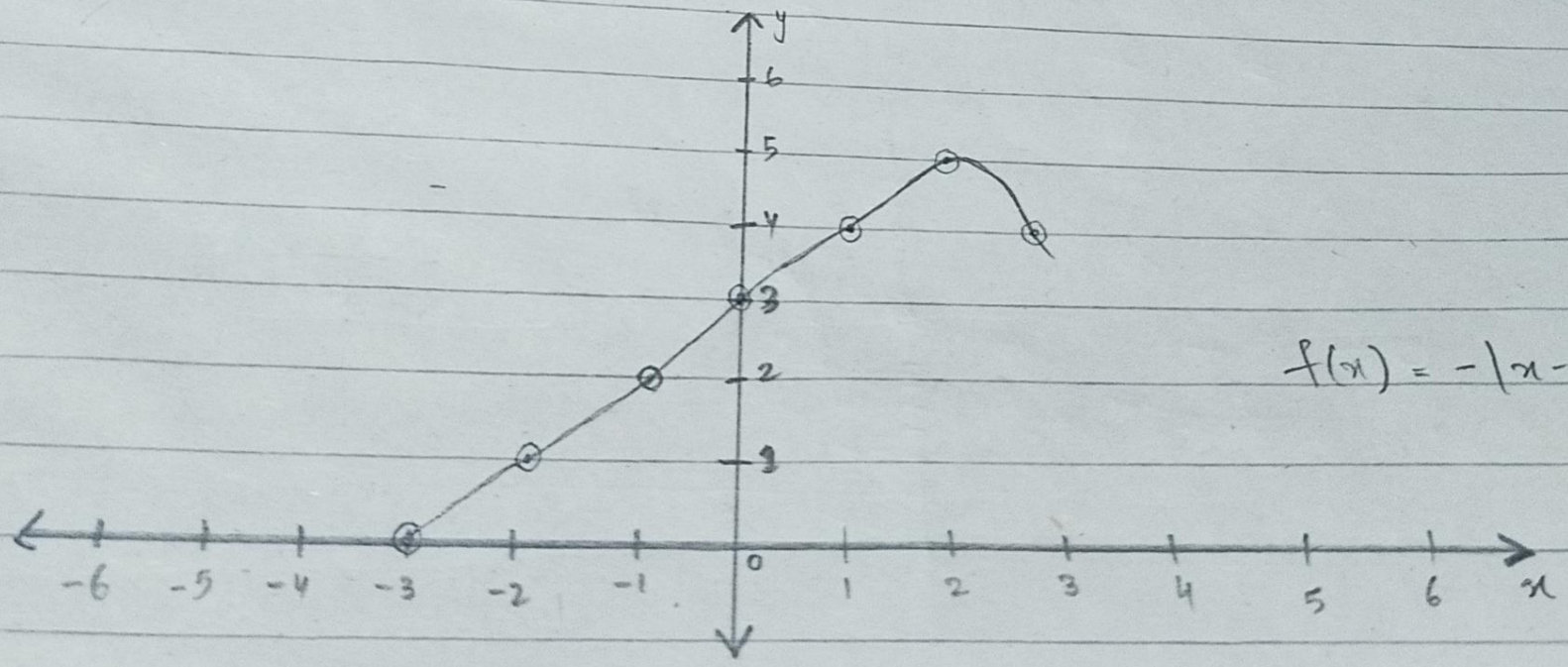
$f^{-1}(x) = \frac{3}{x}$

$= f^{-1}(x) = \begin{cases} \frac{7}{2} - x & x < 2 \\ \frac{3}{x} & x \geq 2 \end{cases}$

83. Sketch the graph of the function:

i) $f(x) = -|x-2| + 5$

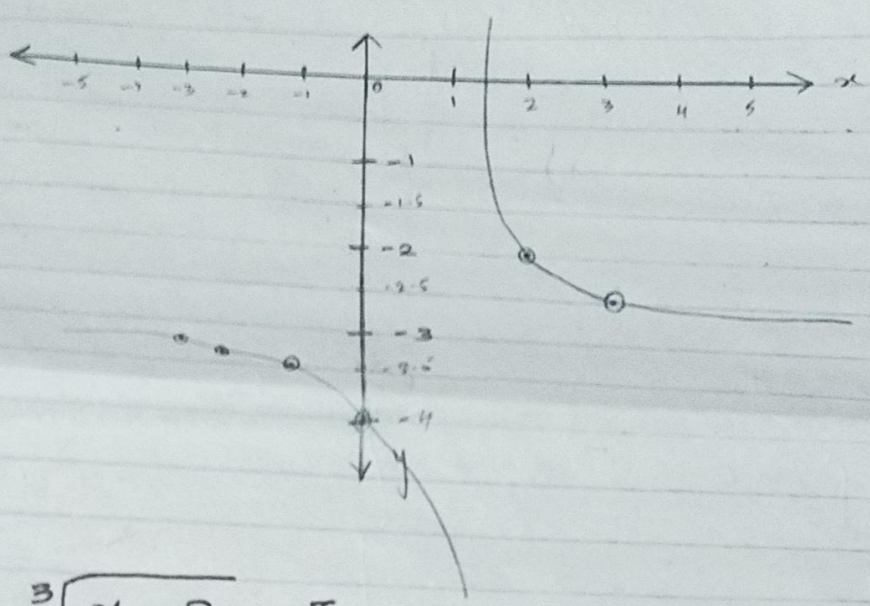
x	-3	-2	-1	0	1	2	3
y	0	1	2	3	4	5	4



$f(x) = -|x-2| + 5$

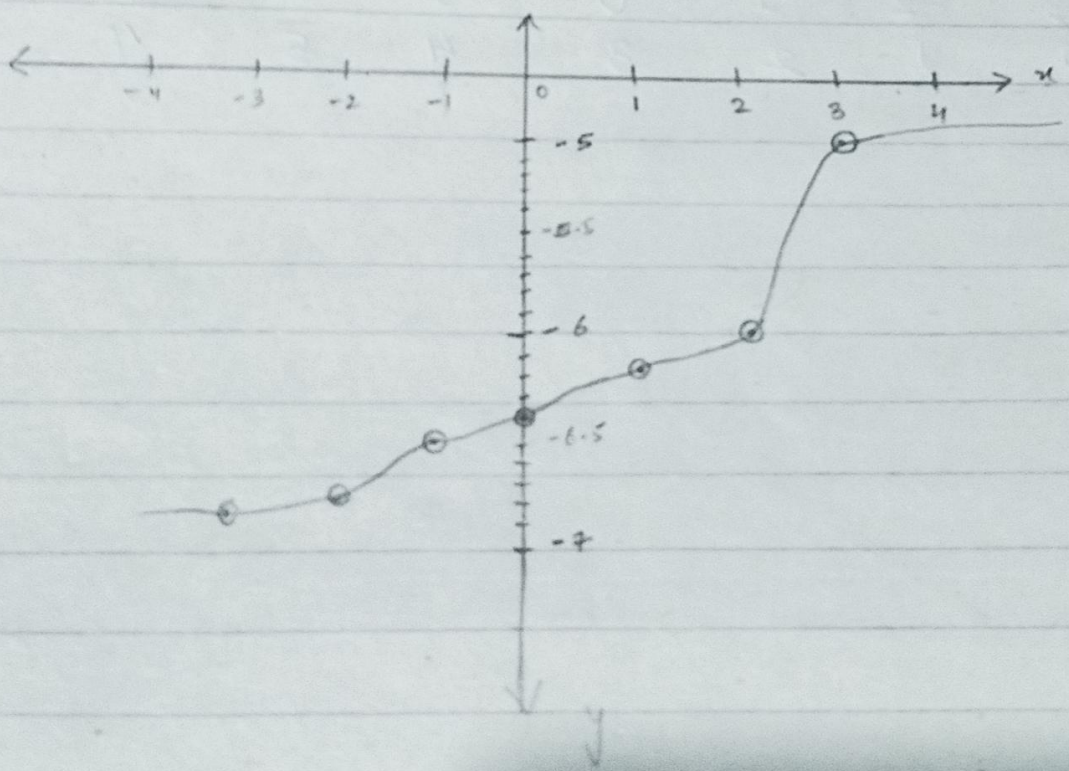
ii) $f(x) = \frac{1}{x-1} - 3$

x	-3	-2	-1	0	1	2	3
y	-3·25	-3·33	-3·5	-4	∞	-2	-2·5

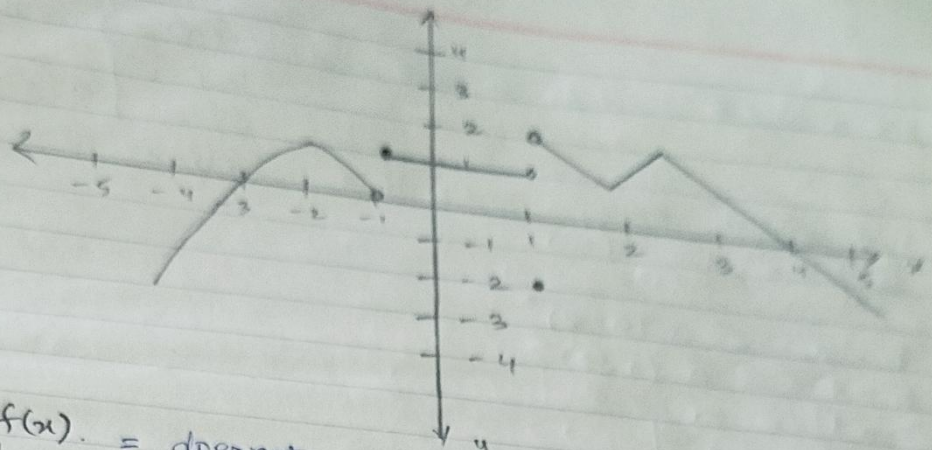


iii) $f(x) = \sqrt[3]{x-3} - 5$

x	-3	-2	-1	0	1	2	3
y	-6·8	-6·7	-6·5	-6·4	-6·2	-6	-5



Q4.



- $\lim_{x \to -1} f(x)$ = doesnot exist

1. $\lim_{x \to -1^-} f(x) = 0$

2. $\lim_{x \to -1^+} f(x) = 1$

- $f(-1) = 1$
- $\lim_{x \to 1} f(x)$ = doesnot exist

1. $\lim_{x \to 1^-} f(x) = 1$

2. $\lim_{x \to 1^+} f(x) = 2$

- $f(1) = -2$
- $\lim_{x \to 3} f(x) = 1$

1. $\lim_{x \to 3^-} f(x) = 1$

2. $\lim_{x \to 3^+} f(x) = 1$

Q5. Let a and b stand for constants and let $f(x) = \begin{cases} b-x, & x < 1 \\ a(x-2)^2, & x \geq 1 \end{cases}$

a) Find equation relating a and b if f is continuous at $x = 1$

- $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$.
- $b - x = a(x-2)^2$
- $b - 1 = a(1-2)^2$
- $b - 1 = a$
- $\boxed{b - a = 1}$

b) Find b if $a = -1$ (Graph and show that function is continuous).

$b - a = 1$

$b = 1 + a$

$b = 1 - 1$

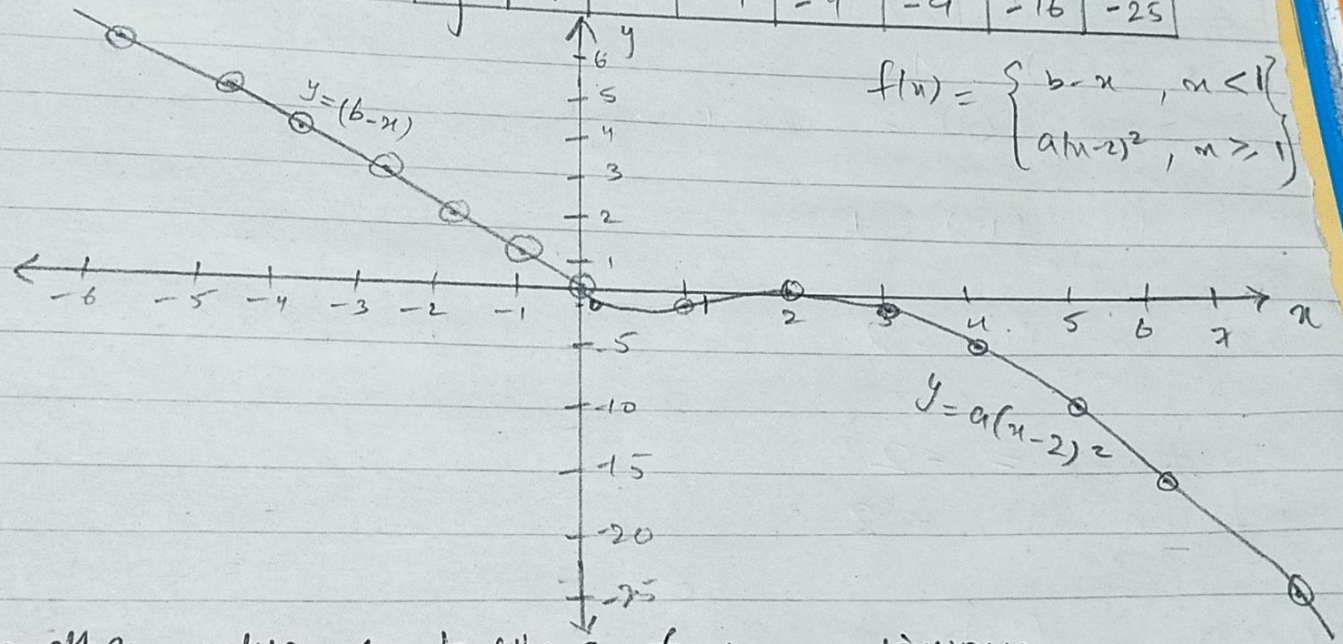
$\boxed{b = 0}$

$(b-x)$

x	-6	-5	-4	-3	-2	-1	0
y	6	5	4	3	2	1	0

$a(x-2)^2$

x	1	2	3	4	5	6	7
y	-1	0	-1	-4	-9	-16	-25



c) Find another value of b where f is continuous.

$b = a + 1$

- $a = 1$, $b = 1 + 1 = 2$.
- $a = 2$, $b = 2 + 1 = 3$
- $a = 3$, $b = 3 + 1 = 4$

Q6. $T(t) = \begin{cases} 2t & t \leq 10 \\ c^2 - 15c - 3t & t > 10 \end{cases}$

i) Find c if T is continuous at $t = 10$ min

- $\lim_{t \to 10^-} T(t) = \lim_{t \to 10^+} T(t)$
- $2t = c^2 - 15c - 3t$
- $20 = c^2 - 15c - 30$
- $c^2 - 15c - 50 = 0$
- $\boxed{c = 17.8, \ -2.8.}$

ii) Explain why T must be continuous at $t = 10$ min

① $T(10) = 20$

② $\lim_{t \to 10^-} T(t) = \lim_{t \to 10^+} T(t)$

$20 = 20$ limit exist

③ ① = ②

$20 = 20$

function is continuous at $t = 10$ min.

Q7. Find value of x where function is discontinuous.

a) $f(x) = x^3 + 3^x$

- Function is continuous
- No point where function is discontinuous.

b) $f(x) = \dfrac{5}{x^2 - 81}$

$x^2 - 81 = 0$

$x = \pm 9$

$f(x)$ is discontinuous at $x = 9$ and $x = -9$.

c) $f(x) = \dfrac{x^2 + 2x - 24}{x^2 - 36}$

$f(x) = \dfrac{(x-4)(x+6)}{(x-6)(x+6)}$

$f(x) = \dfrac{x-4}{x-6}$

$x - 6 = 0$

$x = 6$ (non-removable)

$x^2 - 36 = 0$

$x = \pm 6$

$x = 6$ and $x = -6$ (removable)

- function is discontinuous at $x = 6$ (non-removable) and $x = -6$ (removable).

d) $f(x) = \dfrac{2x+1}{x^2 + 6x + 9}$

- $f(x) = \dfrac{2x+1}{(x+3)^2}$

$(x+3)^2 = 0$

$x + 3 = 0$

$x = -3$

- function is discontinuous at $x = -3$.