Take-Home Quiz 1: Gaussian Elimination, Gauss-Jordan, and Matrix Inversion

Due: 17-09-2025

Please show all your work for full credit.

Instructions:

- This quiz covers Row Reduction to Echelon Form, the Gauss-Jordan Elimination Method, and the Matrix Inversion Algorithm.
- Your answers must be **unique** and based on your **enrollment year (YY)** and the **last four digits of your student ID (ABCD)**.
- For example, if your ID is 6789 and you enrolled in 2023, then YY = 23 and ABCD = 6789, so A=6, B=7, C=8, D=9.
- **Present your work in a clear, step-by-step manner.** Box your final answers.
- Academic integrity is expected. This is an individual assignment.

1. **Row Echelon Form and Rank** [4 Points]
   Let:

$$A = \begin{bmatrix} 1 & 2 & -1 & YY \\ A & 3 & B & 17 \\ 2 & C & 4 & D \end{bmatrix}$$

   where —YY— is your two-digit enrollment year.

   (a) Augment the matrix A with the right-hand side vector $\mathbf{b} = \begin{bmatrix} YY \\ 17 \\ D \end{bmatrix}$.

   (b) Perform Gaussian elimination to reduce the augmented matrix to **Row Echelon Form (REF)**. Clearly indicate each row operation (e.g., $R_2 \to R_2 - 3R_1$).

   (c) Identify the **rank** of the coefficient matrix A and the rank of the augmented matrix $[A|\mathbf{b}]$.

   (d) Based on the ranks, state whether the system $A\mathbf{x} = \mathbf{b}$ is consistent or inconsistent.

2. **Gauss-Jordan Elimination** [3 Points]
   Consider the following system of equations:

$$\begin{aligned} x + 2y - z &= YY \\ Ax + 3y + Bz &= 17 \\ 2x + Cy + 4z &= D \end{aligned}$$

   (a) Write the augmented matrix for this system.

   (b) Using the Gauss-Jordan elimination method, reduce the augmented matrix to **Reduced Row Echelon Form (RREF)**.

   (c) Clearly state the solution (if it exists) as an ordered triple (x, y, z) or describe the solution set if there are infinitely many solutions.

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3. **Matrix Inversion Algorithm** [3 Points]
   Let M be the matrix defined by:

$$M = \begin{bmatrix} 1 & A & B \\ 0 & 1 & C \\ D & 0 & 1 \end{bmatrix}$$

   (a) Form the augmented matrix $[M|I]$, where I is the 3×3 identity matrix.

   (b) Apply the Matrix Inversion Algorithm (using Gauss-Jordan elimination on $[M|I]$) to find M^{-1}, if it exists.

   (c) Show all row operations until you obtain $[I|M^{-1}]$.

   (d) Explicitly state the inverse matrix M^{-1}.

   (e) Verify your result by showing that $M \cdot M^{-1} = I$.

Good Luck!