Assignment 3

Q1. $A = \begin{bmatrix} 4 & 2 & 3 & 3 \\ 0 & 2 & h & 3 \\ 0 & 0 & 4 & 14 \\ 0 & 0 & 0 & 2 \end{bmatrix}$, $\lambda = 4$

$A - \lambda I \Rightarrow A - 4I$:

$$\begin{bmatrix} \lambda-4 & -2 & -3 & -3 \\ 0 & \lambda-2 & -h & -3 \\ 0 & 0 & \lambda-4 & -14 \\ 0 & 0 & 0 & \lambda-2 \end{bmatrix}$$

Substituiting $\lambda = 4$:

$$\begin{bmatrix} 4-4 & -2 & -3 & -3 \\ 0 & 4-2 & -h & -3 \\ 0 & 0 & 4-4 & -14 \\ 0 & 0 & 0 & 4-2 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -3 & -3 \\ 0 & 2 & -h & -3 \\ 0 & 0 & 0 & -14 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Row Reduction:-

1) $R_2 + R_1$

$$\begin{bmatrix} 0 & -2 & -3 & -3 \\ 0 & 0 & -h-3 & -6 \\ 0 & 0 & 0 & -14 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

2) $R_4 + R_2/7$

$$\begin{bmatrix} 0 & -2 & -3 & -3 \\ 0 & 0 & -h-3 & -6 \\ 0 & 0 & 0 & -14 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- rank = dimension - nullity

  rank = $4 - 2 = 2$
- h should be -3 for rank to be 2.

Q2. $x_1 + x_2 + x_3 + x_4 = 0$

- Basis for S:

$$x_1 = -x_2 - x_3 - x_4$$

let $x_4 = t$, $x_3 = s$, $x_2 = r$

$$x_1 = -r - s - t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = r \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

- Orthogonal Basis for S:

$U_1 = (-1,1,0,0)$, $U_2 = (-1,0,1,0)$, $U_3 = (-1,0,0,1)$

- $V_1 = U_1 = (-1,1,0,0)$

- $V_2 = U_2 - \dfrac{\langle U_2, V_1 \rangle V_1}{\|V_1\|^2}$

$$V_2 = (-1,0,1,0) - \frac{(1+0+0+0)(-1,1,0,0)}{2}$$

$$V_2 = (-1,0,1,0) - \frac{1}{2}(-1,1,0,0)$$

$$V_2 = (-1,0,1,0) - (-1/2, 1/2, 0, 0)$$

$$V_2 = (-1/2, -1/2, 1, 0)$$

- $V_3 = U_3 - \dfrac{\langle U_3, V_1 \rangle V_1}{\|V_1\|^2} - \dfrac{\langle U_3, V_2 \rangle V_2}{\|V_2\|^2}$

$$V_3 = (-1,0,0,1) - \frac{(1+0+0+0)(-1,1,0,0)}{2} - \frac{(1/2+0+0+0)(-1/2,-1/2,1,0)}{3/2}$$

$$V_3 = (-1,0,0,1) - \frac{1}{2}(-1,1,0,0) - \frac{1}{3}(-1/2,-1/2,1,0)$$

$v_3 = (-1,0,0,1) - (-1/2, 1/2, 0, 0) - (-1/6, -1/6, 1/3, 0)$

$v_3 = (-1/3, -1/3, -1/3, 1)$

Orthogonal Basis = v_1, v_2, v_3

Q3. $\begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}$

$U_1 = (1,-1,-1,1,1)$, $U_2 = (2,1,4,-4,2)$,
$U_3 = (5,-4,-3,7,1)$.

- $V_1 = U_1 = (1,-1,-1,1,1)$
- $V_2 = U_2 - \dfrac{\langle U_2, V_1 \rangle V_1}{\|V_1\|^2}$

$V_2 = (2,1,4,-4,2) - \dfrac{(2-1-4-4+2)(1,-1,-1,1,1)}{5}$

$V_2 = (2,1,4,-4,2) + 1(1,-1,-1,1,1)$

$V_2 = (2,1,4,-4,2) + (1,-1,-1,1,1)$

$V_2 = (3,0,3,-3,3)$

- $V_3 = U_3 - \dfrac{\langle U_3, V_1 \rangle V_1}{\|V_1\|^2} - \dfrac{\langle U_3, V_2 \rangle V_2}{\|V_2\|^2}$

- $V_3 = (5,-4,-3,7,1) - \dfrac{(5+4+3+7+0)(1,-1,-1,1,1)}{5} -$

$\dfrac{(15+0-9-21+3)(3,0,3,-3,3)}{36}$

$-1/6, 1/3, 0)$

$$V_3 = (5,-4,-3,7,1) - 4(1,-1,-1,1,1) + \frac{1}{3}(3,0,3,-3,3)$$

$$V_3 = (5,-4,-3,7,1) - (4,-4,-4,4,4) + (1,0,1,-1,1)$$

$$V_3 = (2,0,2,2,-2)$$

- $q_1 = \dfrac{v_1}{\|v_1\|} = \dfrac{(1,-1,-1,1,1)}{\sqrt{5}} = (1/\sqrt{5}, -1/\sqrt{5}, -1/\sqrt{5}, 1/\sqrt{5}, 1/\sqrt{5})$

- $q_2 = \dfrac{v_2}{\|v_2\|} = \dfrac{(3,0,3,-3,3)}{6} = (1/2, 0, 1/2, -1/2, 1/2)$

- $q_3 = \dfrac{V_3}{\|V_3\|} = \dfrac{(2,0,2,2,-2)}{4} = (1/2, 0, 1/2, 1/2, -1/2)$

$$Q = \begin{bmatrix} 1/\sqrt{5} & 1/2 & 1/2 \\ -1/\sqrt{5} & 0 & 0 \\ -1/\sqrt{5} & 1/2 & 1/2 \\ 1/\sqrt{5} & -1/2 & 1/2 \\ 1/\sqrt{5} & 1/2 & -1/2 \end{bmatrix}$$

- $R = \begin{bmatrix} \langle u_1, q_1\rangle & \langle u_2, q_1\rangle & \langle u_3, q_1\rangle \\ 0 & \langle u_2, q_2\rangle & \langle u_3, q_2\rangle \\ 0 & 0 & \langle u_3, q_3\rangle \end{bmatrix}$

- $R = \begin{bmatrix} \frac{1}{\sqrt{5}}+\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{5}}+\frac{1}{\sqrt{5}} & 1+0+2+2+1 & \frac{5}{\sqrt{5}}+\frac{4}{\sqrt{5}}+\frac{3}{\sqrt{5}}+\frac{7}{\sqrt{5}}+\frac{1}{\sqrt{5}} \\ 0 & \frac{2}{\sqrt{5}}-\frac{1}{\sqrt{5}}-\frac{4}{\sqrt{5}}-\frac{4}{\sqrt{5}}+\frac{2}{\sqrt{5}} & \frac{5}{2}+0-\frac{3}{2}-\frac{7}{2}+\frac{1}{2} \\ 0 & 0 & \frac{5}{2}+0-\frac{3}{2}+\frac{7}{2}-\frac{1}{2} \end{bmatrix}$

- $R = \begin{bmatrix} \sqrt{5} & -\sqrt{5} & 4\sqrt{5} \\ 0 & 6 & -2 \\ 0 & 0 & 4 \end{bmatrix}$

Q4

1) First matrix: $A = \begin{bmatrix} 9 & 4 \\ a & 1 \end{bmatrix}$

- $\det(A) = 9 - 4a = 25$

$= \boxed{a = -4}$

- Trace $= 9 + 1 = 10$

- $A - \lambda I = \begin{bmatrix} \lambda - 9 & -4 \\ 4 & \lambda - 1 \end{bmatrix}$

$= (\lambda - 9)(\lambda - 1) - (-4)(4)$

$= \lambda^2 - \lambda - 9\lambda + 9 + 16$

$= \lambda^2 - 10\lambda + 25$

$= (\lambda - 5)^2 \Rightarrow \lambda = 5$

- $A - 5I = \begin{bmatrix} -4 & -4 \\ 4 & 4 \end{bmatrix}$

1) $R_2 + R_1$

$\begin{bmatrix} -4 & -4 \\ 0 & 0 \end{bmatrix}$

2) $R_1 / -4$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$x_2 = t$, $x_1 = -t$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Only one eigen vector $\Rightarrow$ not diagonalizable.

2) Second matrix:- $A = \begin{bmatrix} 10 & 5 \\ -5 & d \end{bmatrix}$

- $\det(A) = 10d + 25 = 25$

$10d = 0$

$\boxed{d = 0}$

- Trace $= 10 + 0 = 0$

- $A - \lambda I = \begin{bmatrix} \lambda - 10 & -5 \\ 5 & \lambda \end{bmatrix}$

$= (\lambda - 10)(\lambda) - (-5)(5)$

$= \lambda^2 - 10\lambda + 25$

$= (\lambda - 5)^2 \Rightarrow \lambda = 5$

- $A - 5I = \begin{bmatrix} -5 & -5 \\ 5 & 5 \end{bmatrix}$

1) $R_2 + R_1$

$\begin{bmatrix} -5 & -5 \\ 0 & 0 \end{bmatrix}$

2) $R_1 / -5$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

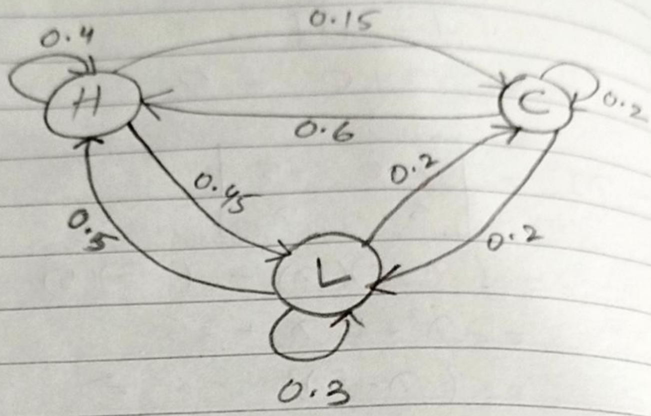
$x_2 = t$, $x_1 = -t$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Only one eigen vector $\Rightarrow$ not diagonalizable.

Q5. a) $P = \begin{bmatrix} 0.4 & 0.45 & 0.15 \\ 0.5 & 0.3 & 0.2 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}$

b)



c) To H : 0.40

To L : 0.45

To C : 0.15

$P(H \text{ tomorrow} \mid H \text{ today}) = 0.4$

d) $P^2 = \begin{bmatrix} 0.475 & 0.345 & 0.180 \\ 0.47 & 0.355 & 0.175 \\ 0.46 & 0.370 & 0.170 \end{bmatrix}$

$P^4 = \begin{bmatrix} 0.470575 & 0.35295 & 0.176475 \\ 0.4706 & 0.352925 & 0.176475 \\ 0.4706 & 0.35295 & 0.17645 \end{bmatrix}$

Probability =

e) $\pi P =$

1) $\pi_H = 0.$

2) $\pi_L = 0$

3) $\pi_C =$

From

0.

From

Fro

Probability = 0.17645

e) $\pi P = \pi$, $\pi_H + \pi_L + \pi_C = 1$

1) $\pi_H = 0.4\pi_H + 0.5\pi_L + 0.6\pi_C$ — ①

2) $\pi_L = 0.45\pi_H + 0.3\pi_L + 0.2\pi_C$ — ②

3) $\pi_C = 0.15\pi_H + 0.2\pi_L + 0.2\pi_C$ — ③.

From ①:-

$0.6\pi_H = 0.5\pi_L + 0.6\pi_C$

$6\pi_H = 5\pi_L + 6\pi_C$ — (A)

From ②:-

$0.7\pi_L = 0.45\pi_H + 0.2\pi_C$

$7\pi_L = 4.5\pi_H + 2\pi_C$ — (B)

From ③:-

$0.8\pi_C = 0.15\pi_H + 0.2\pi_L$

$8\pi_C = 1.5\pi_H + 2\pi_L$ — (C)

Solved through substitution:-

$$\pi = \left(\frac{8}{17}, \frac{6}{17}, \frac{3}{17}\right) = (0.4706, 0.3529, 0.1765)$$

HackerRank = 47.06 %

Leet Code = 35.29 %

CodeForces = 17.65 %