

ASIGNMENT 3:

LINEAR ALGEBRA:

Total marks: 50 marks

Q1. It can be shown that the algebraic multiplicity of an Eigen value λ is always greater than or equal to the dimension of the Eigen space corresponding to λ . Find h in the matrix A below such that the Eigen space for $\lambda = 4$ is two dimensional.

$$A = \begin{bmatrix} 4 & 2 & 3 & 3 \\ 0 & 2 & h & 3 \\ 0 & 0 & 4 & 14 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Q2. Find a basis for the subspace S in \mathbb{R}^4 spanned by all solution of

$$x_1 + x_2 + x_3 + x_4 = 0$$

Find orthogonal basis corresponding to above basis.

Q3. Find QR decomposition of the matrix.

$$\begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}$$

Q4. Complete these matrices so that $\det A = 25$. Then check that $\lambda = 5$ is repeated the trace is 10 so the determinant of $(A - I\lambda)$ is $(\lambda - 5)^2$. Find the Eigen vector with $Ax = 5x$. These matrices will not be diagonalizable because there is no second line of Eigen vector.

$$A = \begin{bmatrix} 9 & 4 \\ & 1 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 10 & 5 \\ -5 & \end{bmatrix}$$

Q5. A computer science student, Ali, uses three online coding platforms for practice:

- I. HackerRank (H)
- II. LeetCode (L)
- III. Codeforces (C)

His usage pattern follows a Markov process. The transition behavior is as follows:

- If Ali uses HackerRank on a given day, then
 - there is a 40% chance he will use HackerRank the next day,
 - a 45% chance he will use LeetCode, and
 - a 15% chance he will use Codeforces.
- If Ali uses LeetCode on a given day, then
 - there is a 50% chance he will use HackerRank the next day,
 - a 30% chance he will use LeetCode, and
 - a 20% chance he will use Codeforces.
- If Ali uses Codeforces on a given day, then
 - there is a 60% chance he will use HackerRank the next day,
 - a 20% chance he will use LeetCode, and
 - a 20% chance he will use Codeforces.

(a) Write down the transition matrix for this Markov chain.

(b) Draw the transition diagram.

(c) If Ali uses HackerRank today, what is the probability that he uses HackerRank tomorrow?

(d) If Ali uses Codeforces today, what is the probability that after four days, he is again using Codeforces?

(e) Determine the long-term usage preferences for Ali.