LA Quiz 1

Date:

1. a) $$A|b = \left[\begin{array}{cccc|c} 1 & 2 & -1 & 24 & 24 \\ 0 & 3 & 7 & 17 & 17 \\ 2 & 4 & 4 & 6 & 6 \end{array}\right]$$

b) i) $R_3 \longrightarrow R_3 - 2R_1$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 24 & 24 \\ 0 & 3 & 7 & 17 & 17 \\ 0 & 0 & 6 & -42 & -42 \end{array}\right]$$

ii) $R_3 \longrightarrow R_3/6$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 24 & 24 \\ 0 & 3 & 7 & 17 & 17 \\ 0 & 0 & 1 & -7 & -7 \end{array}\right]$$

c) Rank of $A = 3$
Rank of $A|b = 3$

d) Since rank of A = rank of A|b = 3, the system is consistent.

2. a) $A|b = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 24 \\ 0 & 3 & 7 & 17 \\ 2 & 4 & 4 & 6 \end{array}\right]$

b) 1) $R_3 \rightarrow R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 24 \\ 0 & 3 & 7 & 17 \\ 0 & 0 & 6 & -42 \end{array}\right]$$

2) $R_3 \rightarrow R_3/6$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 24 \\ 0 & 3 & 7 & 17 \\ 0 & 0 & 1 & -7 \end{array}\right]$$

3) $R_2 \rightarrow R_2 - 7R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 24 \\ 0 & 3 & 0 & 66 \\ 0 & 0 & 1 & -7 \end{array}\right]$$

4) $R_2 \rightarrow R_2/3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 24 \\ 0 & 1 & 0 & 22 \\ 0 & 0 & 1 & -7 \end{array}\right]$$

5) $R_1 \rightarrow R_1 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -20 \\ 0 & 1 & 0 & 22 \\ 0 & 0 & 1 & -7 \end{array}\right]$$

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6) $R_1 \rightarrow R_1 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -27 \\ 0 & 1 & 0 & 22 \\ 0 & 0 & 1 & -7 \end{array}\right]$$

c) $x = -27,\ y = 22,\ z = -7$

$(x, y, z) = (-27, 22, -7)$

3. a) $M|I = \left[\begin{array}{ccc|ccc} 1 & 0 & 7 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 6 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

b) & c) 1) $R_3 \rightarrow R_3 - 6R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 7 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & -41 & -6 & 0 & 1 \end{array}\right]$$

2) $R_3 \rightarrow -R_3/41$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 7 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 6/41 & 0 & -1/41 \end{array}\right]$$

3) $R_2 \rightarrow R_2 - 4R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 7 & 1 & 0 & 0 \\ 0 & 1 & 0 & -24/41 & 1 & 4/41 \\ 0 & 0 & 1 & 6/41 & 0 & -1/41 \end{array}\right]$$

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c) $R_1 \rightarrow R_1 - 7R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/41 & 0 & 7/41 \\ 0 & 1 & 0 & -24/41 & 1 & 4/41 \\ 0 & 0 & 1 & 6/41 & 0 & -1/41 \end{array}\right]$$

d) $$M^{-1} = \begin{bmatrix} -1/41 & 0 & 7/41 \\ -24/41 & 1 & 4/41 \\ 6/41 & 0 & -1/41 \end{bmatrix}$$

e) $$M \cdot M^{-1} = I$$

$$= \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 4 \\ 6 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1/41 & 0 & 7/41 \\ -24/41 & 1 & 4/41 \\ 6/41 & 0 & -1/41 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times -1/41)+(0\times -24/41)+(7\times 6/41) & (1\times 0)+(0\times 1)+(7\times 0) & (1\times 7/41)+(0\times 4/41)+(7\times -1/41) \\ (0\times -1/41)+(1\times -24/41)+(4\times 6/41) & (0\times 0)+(1\times 1)+(4\times 0) & (0\times 7/41)+(1\times 4/41)+(4\times -1/41) \\ (6\times -1/41)+(0\times -24/41)+(1\times 6/41) & (6\times 0)+(0\times 1)+(1\times 0) & (6\times 7/41)+(0\times 4/41)+(1\times -1/41) \end{bmatrix}$$

$$= \begin{bmatrix} -1/41+0+42/41 & 0+0+0 & 7/41+0-7/41 \\ 0-24/41+24/41 & 0+1+0 & 0+4/41-4/41 \\ -6/41+0+6/41 & 0+0+0 & 42/41+0-1/41 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Hence Verified!