National University of Computer & Emerging Sciences, Karachi

Linear Algebra **Assignment- 01**
Submission Date: **September 20th, 2025**

Question 1: Do the three lines $x - 4y = 1$, $2x - y = -3$, and $-x - 3y = 4$ have a common point of intersection? Explain.

Question 2: Find an equation involving g, h, and k that makes this augmented matrix correspond to a consistent system:

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix}$$

Question 3: Consider the following system of equations

$$\begin{aligned} &w + x + y + z = 6 \\ &w + y + z = 4 \\ &w + y = 2 \end{aligned}$$

a) List the leading variables.
b) List the free variables.
c) The general solution of system is?
d) Suppose that a fourth equation $-2w - 2y = -3$ is included in the system. What is the solution of the resulting system?

Question 4: Consider the following two systems of equations

$$x + y + z = 6$$

$$x + 2y + 2z = 11$$

$$2x + 3y - 4z = 3$$

And

$$x + y + z = 7$$

$$x + 2y + 2z = 10$$

$$2x + 3y - 4z = 3$$

a) Solve both systems simultaneously by applying Gauss-Jordan Method.
b) Solve both systems using inversion algorithm.

Question 5: For the matrix

$$C = \begin{bmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 1 & 0 & a \end{bmatrix}$$

a) Find $det(C)$ as a function of a.
b) For which values of a is C invertible?
c) Interpret these values of a in terms of whether a **linear system of equations with coefficient matrix C** has unique, no, or infinitely many solutions.

Question 6: A transformation $T: R^2 \to R^2$ defined by

$$T(x, y) = (x + 2y,\ y)$$

a) Find the **matrix representation** of T.
b) Interpret geometrically: does this transformation represent a **rotation, reflection, scaling, or shear**? Justify.
c) Suppose T is applied repeatedly to a square image (with corners at $(0,0), (1,0), (1,1), (0,1)$). What happens to the shape of the image after many applications?

Question 7: How should the coefficients a, b, and, c be chosen so that the system

$$\begin{aligned} ax + by - 3z &= -3 \\ -2x - by + cz &= -1 \\ ax + 3y - cz &= -3 \end{aligned}$$

has the solution $x = 1$, $y = -1$, and $z = 2$.

Question 8: A linear system whose coefficient matrix has a pivot position in every row must be consistent. Explain why this must be so.

Question 9: For the matrices

$$\begin{bmatrix} 5 & 3 & 2 \\ 3 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \text{ and } \begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$$

a) Find inverse using inversion algorithm and verify, if exists.
b) Find the solution of the system $Ax = 0$ for both matrices.
c) Find the solution of the system $Ax = b$, $b = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$ for both matrices.

Question 10: Write a Python program to find the solution of a system of linear equations, by using Gauss Jordan Method and Inversion Algorithm.

Question 11: Let

$u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$, $c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$, and define a transformation $T: R^2 \to R^3$ by

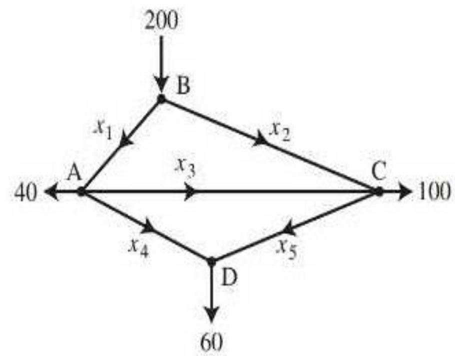
$$T(x) = (x_1 - 3x_2, 3x_1 + 5x_2, -x_1 + 7x_2)$$

a) Find the standard matrix A.
b) Find $T(\,u)$, the image of u under the transformation T.
c) Find an x in R^2 whose image under T is b.
d) Is there more than one x whose image under T is b?
e) Determine if c is in the range of the transformation T.

Question 12: Find the standard matrix A for the linear transformation $T: R^2 \to R^2$ for which

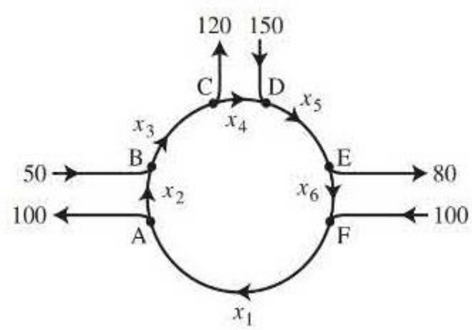
$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\-2\end{bmatrix}, T\left(\begin{bmatrix}2\\3\end{bmatrix}\right) = \begin{bmatrix}-2\\5\end{bmatrix}.$$

Question 13:



a) Find the general traffic pattern in the freeway network shown in the figure. (Flow rates are in cars/minute.)
b) Describe the general traffic pattern when the road whose flow is x_4 is closed.
c) When $x_4 = 0$, what is the minimum value of x_1?

Question 14: Intersections in England are often constructed as one-way "roundabouts," such as the one shown in the figure. Assume that traffic must travel in the directions shown. Find the general solution of the network flow. Find the smallest possible value for x_6? ·



Question 15: Use row reduction to show that

$$\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{vmatrix} = (b-a)(c-a)(c-b)$$