Programmierblatt 2

Finite Differenzen, Laplace und Poisson Gleichung

Bearbeitende Studierende:

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Anmerkungen: Alle Aufgaben wurden bereits mit der freiwilligen aber sinnvollen Forderung nach Erweiterung bzw. Generalisierbarkeit implementiert

Aufgabe 1 (FD-Gitter)

In dieser Aufgabe wird das Gitter $\Omega = ((0,3)^2\backslash([2,3) \times (0,1]))\backslash((0,1] \times [2,3))$

Die Grundlegende Idee ist die kleinstmögliche Basis Ω_R zu schaffen, sodass $\bar{\Omega}_h \subset \Omega_R$ und ein rechteckiges Würfelgitter ist. Dann reicht es aus diesem allgemeinen Gitter kleinere Rechtecke "auszuschneiden", und man kann jedes beliebige gewünschte Gebiet erhalten.

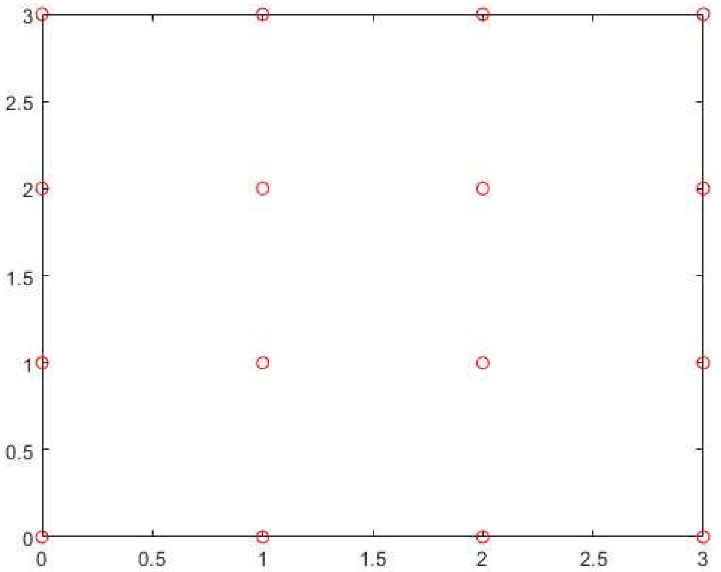
```
% omeg.o = [0,0]
% omeg.x = 3;
% omeg.y = 3;
% omeg.h = 1.5;
%
% Omega = genRectMesh2D(omeg);
% Omega = Omega.pointMat;
% plot(Omega(:,1), Omega(:,2),'ro');
%
%
%
% figure;
% plot(omeg.o(1),omeg.o(2),'ro');
% axis = [omeg.o(1),omeg.o(1)+omeg.x, omeg.o(2), omeg.o(2)+omeg.y];
% hold on
% nOmega = size(Omega,1);
% for k=1:nOmega
%     color = ones(3,1)*(k/nOmega);
%     plot(Omega(k,1),Omega(k,2),'Color',color,"Marker",'o')
%
%     drawnow
%     pause(0.025)
% end
% hold off
```

```
%Testing some shit
h = [1, 0.5, 0.25, 0.125, 0.1, 0.05, 0.02]
```

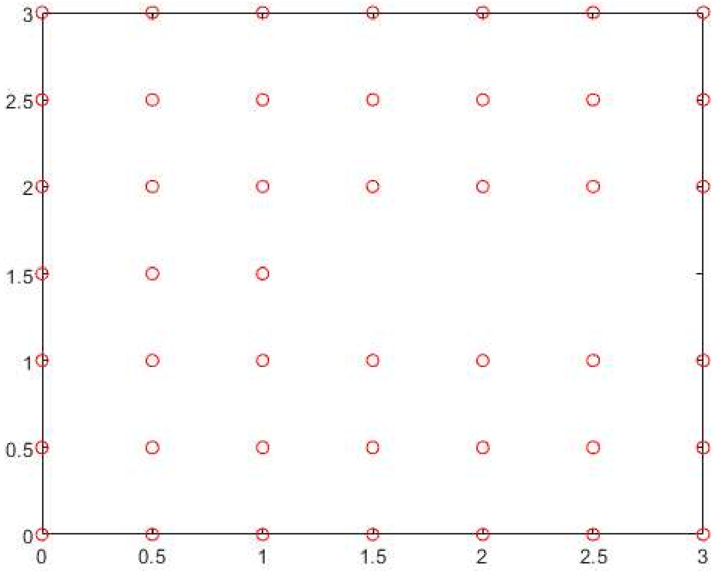
```
h = 1×7
    1.0000    0.5000    0.2500    0.1250    0.1000    0.0500    0.0200
```

```
t = zeros(numel(h),1);
for k = 1:numel(h)
    tic
omeg.o = [0,0];
omeg.x = 3;
omeg.y = 3;
omeg.h = h(k);
O1 = genRectMesh2D(omeg);
omeg.x = 2;
omeg.y = 1;
omeg.o = [1, 1];
O2 = genRectMesh2D(omeg);

O = meshSubtract(O1,O2);
O = O.pointMat;
figure
plot(O(:,1), O(:,2),'ro');
t(k) = toc;
end
```



0
0.5
1
1.5
2
2.5
3
0
0.5
1
1.5
2
2.5
3



0
0.5
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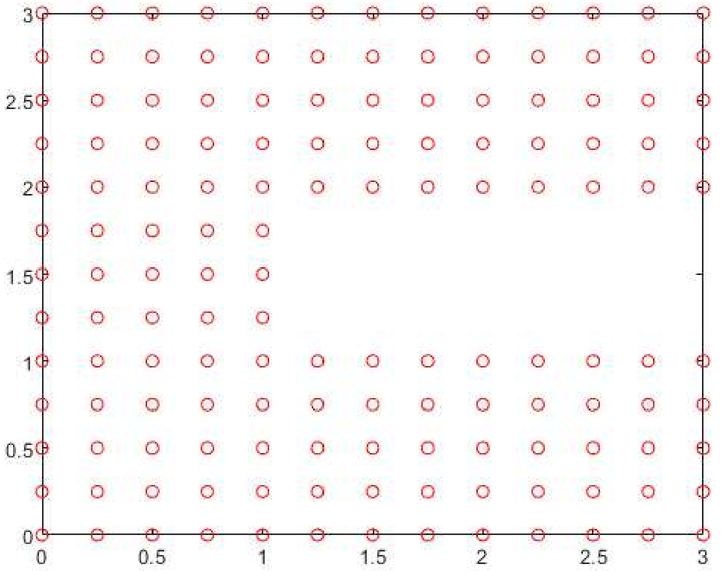
3



0
0.5
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1.5
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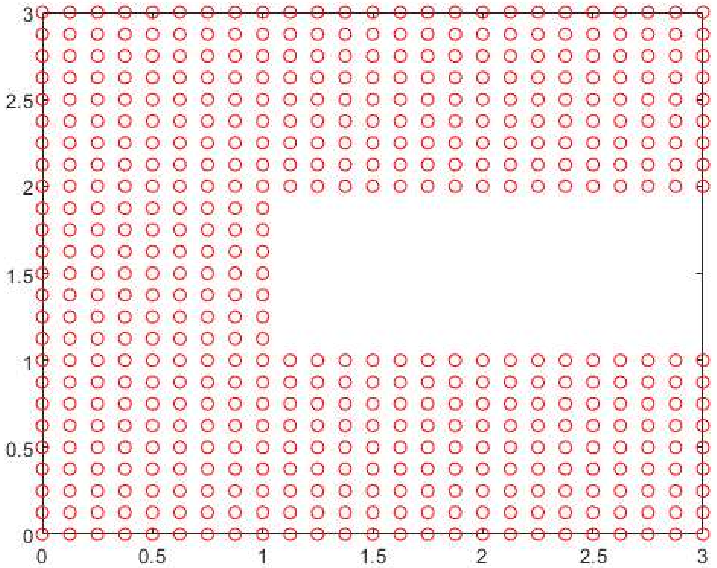
2.5
3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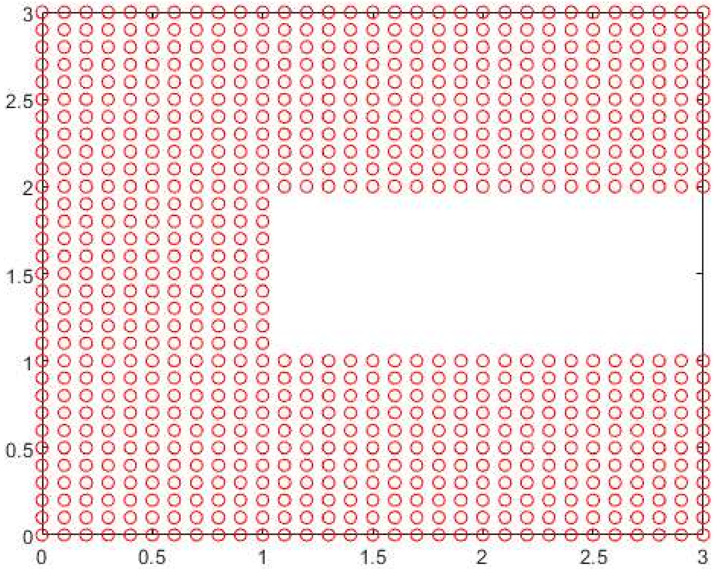
0
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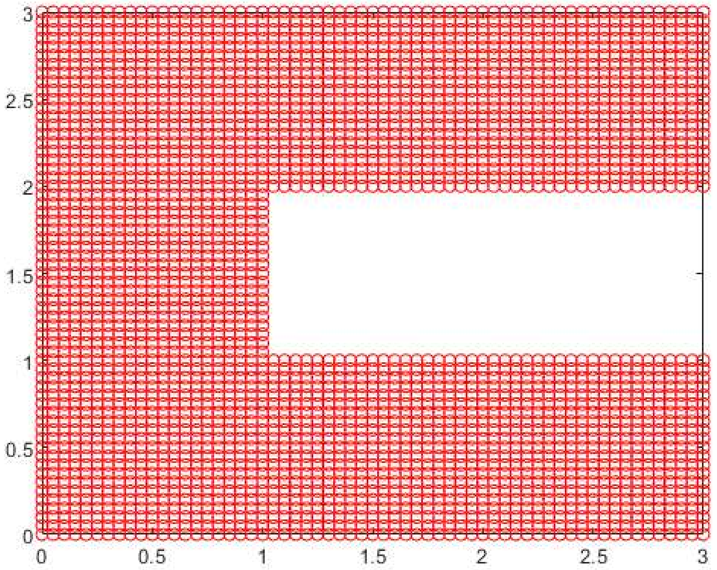
0
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2
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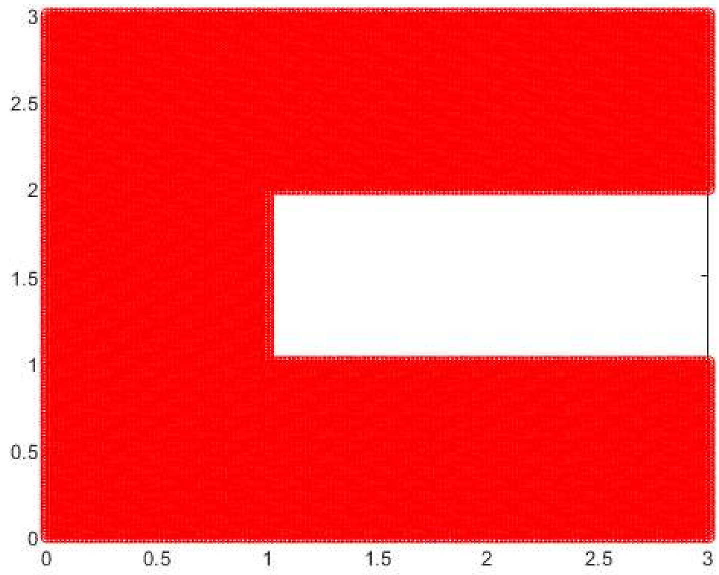
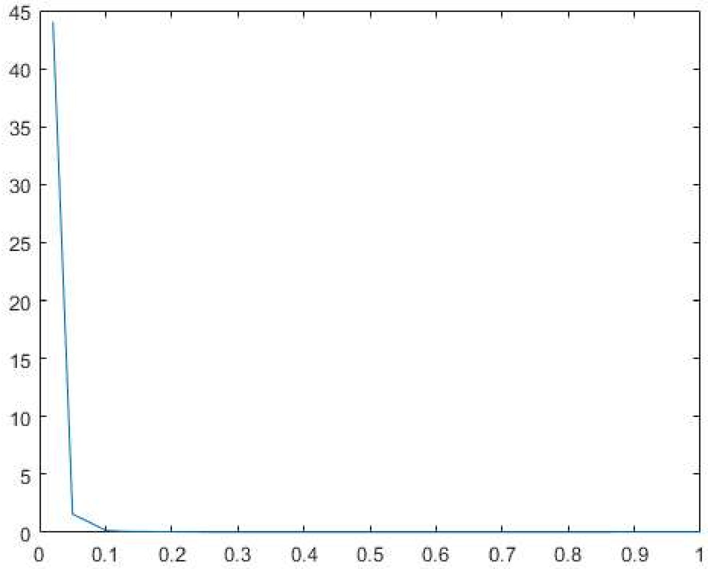
3



```
plot(h,t)
```



```
function OmegaHBar = genRectMesh2D(omeg)
%Input is a struct specifying the origin the X-Length, the Y-Length and the
%Meshsize h
    if mod(omeg.x,omeg.h) ~= 0 || mod(omeg.y,omeg.h) ~= 0
        error("X-Length or Y-Length are not applicable with the Meshsize of h");
    end

    nX = round(omeg.x/omeg.h);
    nY = round(omeg.y/omeg.h);
    N = (nX+1).*(nY+1);
    OmegaHBar = omeg;
    OmegaHBar.pointMat = zeros(N,2);
    for kY = 0:nY
        for kX = 0:nX
            OmegaHBar.pointMat(kY*(nX+1) + (kX + 1),:) = [omeg.o(1) + kX *omeg.h, omeg.o(2) + omeg.y - kY * omeg.h];
        end
    end
end
```

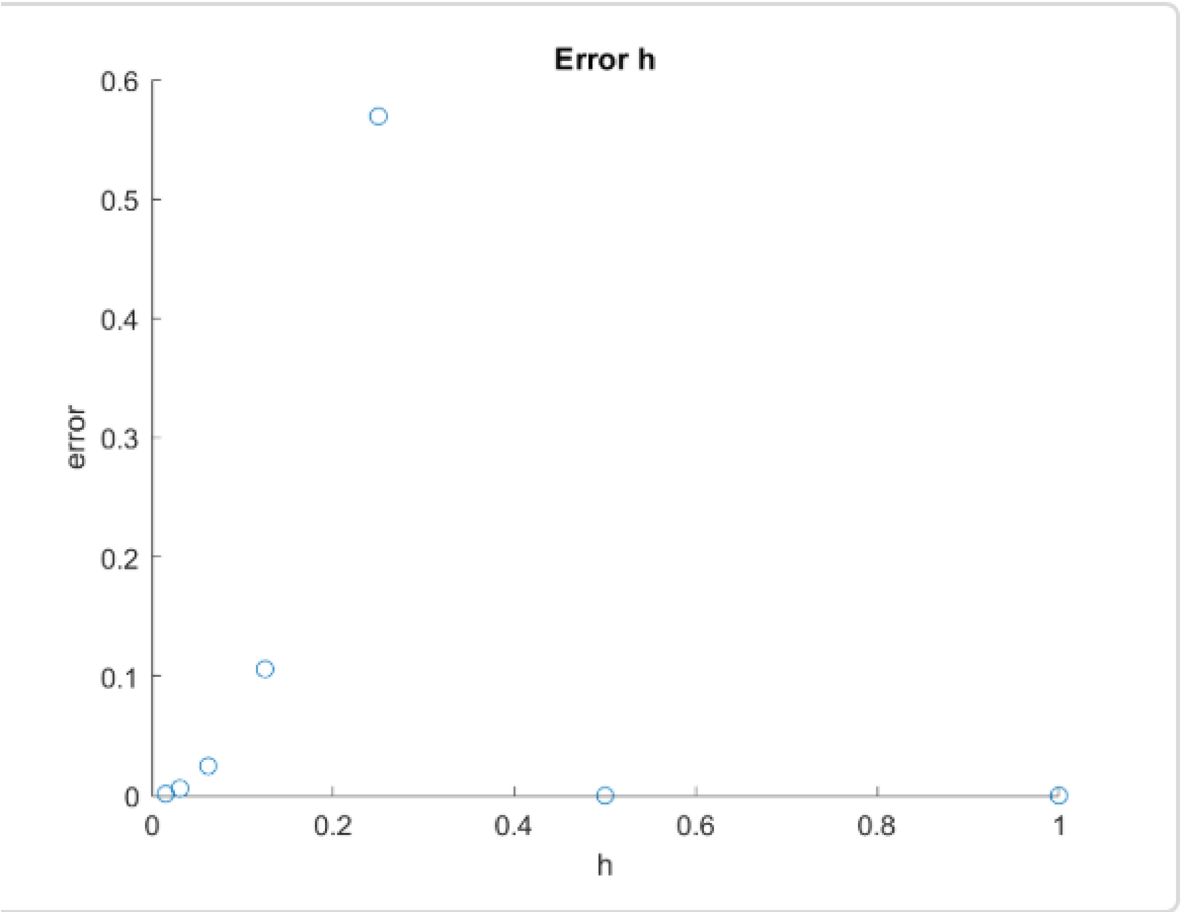
```
function OmegaHBar = meshSubtract(Omega, subOmega)
%Subtracts a rectangular mesh from an arbitrary cubic Mesh while keeping the
%edges
if subOmega.h ~= Omega.h
    error("Meshsizes h are different");
end
```

```
h = subOmega.h;

if mod(subOmega.o(1)-Omega.o(1), h) ~= 0 || mod(subOmega.o(2)-Omega.o(2), h) ~= 0
    error("Origins are not a multiple of h appart from each other")
end

%Subtracting inner rectangle without the border
innerPoints.x = subOmega.x - 2*h;
innerPoints.y = subOmega.y - 2*h;
innerPoints.o = subOmega.o + [h, h];
innerPoints.h = h;


innerPoints = genRectMesh2D(innerPoints);
Omega.pointMat = meshSubtractWithEdge(Omega.pointMat,innerPoints.pointMat);
border = meshSubtractWithEdge(subOmega.pointMat, innerPoints.pointMat);
pM = Omega.pointMat;


%For every Element on the border check, if it is isolated or not - if remove
for k = 1:2
idx = zeros(size(border,1));
keep = true(size(pM,1),1);
for k1 = 1:size(border,1)
    bP = border(k1,:);
    %Search index of borderPoint bP in pM
    %and if there is a
    neighX = false; %neighbour
    neighY = false;
    for k2 = 1:size(pM)
        if equalEps(bP, pM(k2,:))
            idx(k1) = k2;
        elseif equalEps(bP + [0 h],pM(k2,:))
            neighY = true;
        elseif equalEps(bP + [0 -h], pM(k2,:))
            neighY = true;
        end
    end
    if (idx(k1) > 2) && (idx(k1) < size(pM,1))

        if equalEps((bP + [-h 0]), pM(idx(k1)-1,:)) || equalEps((bP + [h 0]), pM(idx(k1)+1,:)) %lexikografisch
            neighX = true;
        end

        if ~neighX || ~neighY
        keep(idx(k1)) = false;
        end
    end
end
pM = pM(keep,:);
end
Omega.pointMat = pM;
OmegaHBar = Omega;


end
```

```
function Omega = meshSubtractWithEdge(pM, pMSub)
%Input is the point Matrix of a area and an area you want to subtract it
%from - output is the point Matrix of pM without pMSub
%Could be implemented faster
isIn = zeros(size(pM,1),1);
for k1 = 1:size(pM,1)
    for k2 = 1:size(pMSub,1)
        if equalEps(pM(k1,:),pMSub(k2,:))
            isIn(k1) = 1;
            break;
        end
    end
end
Omega = pM(~logical(isIn),:);
end
```

```
function equalEps = equalEps(x, y)
    if norm(x-y) < 1e4*eps(min(norm(x),norm(y)))
```

```
        equalEps = true;
    else
        equalEps = false;
    end
end

% function findNeighbor
```


Error h
error
h