Assignment 1

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P1

1. Let A_{SQ} be the area of the unit square and A_{QC} be the area of the quarter-circle. The probability of a uniformly chosen point in A_{SQ} lying in A_{QC} is then given by:
$$P(x \in A_{QC}) = \frac{A_{QC}}{A_{SQ}} = \frac{\pi}{4} \tag{1}$$
2. See attached Matlab files (without for-loops)
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4. Running the program gives the following solutions:

N	π_{min}	π_{max}	π_{mean}	$\Delta\pi$
100	2.60000	3.48000	3.14640	0.17077
1000	3.03600	3.24000	3.14304	0.04473
10000	3.10280	3.18200	3.14028	0.01555
100000	3.13092	3.15756	3.14226	0.00500
1000000	3.13752	3.14668	3.14169	0.00167
10000000	3.14047	3.14283	3.14164	0.00052

P2

1. Let $X_i \in \{"yes", "no"\}$ for $i = 1, ..., N$. The rule could be as follows:
$$\begin{cases} "yes" & \text{if } \sum_{i=1}^{N} \mathbb{1}\{X_i = "yes")\} \geq N/2 \\ "no", & \text{otherwise} \end{cases} \tag{2}$$

That is, we would choose "*yes*" if we encounter "*yes*" at least five times in all the X_i. In the particular case we would therefore choose "yes" as it occurs six times.

2. Yes, the rule would have to be modified as the probability of error in one of the two cases would be greater than the other one. Weighting the outcomes by the inverse of the error probabilities sounds like a good idea.