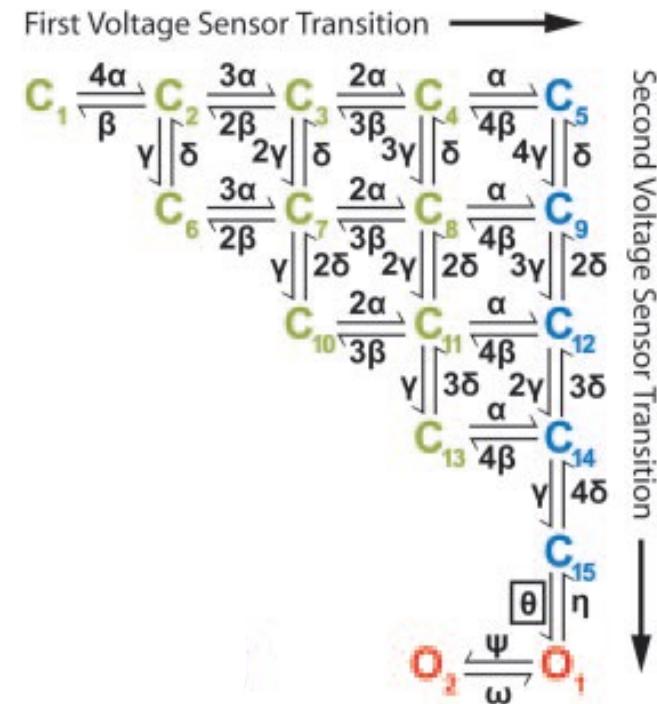


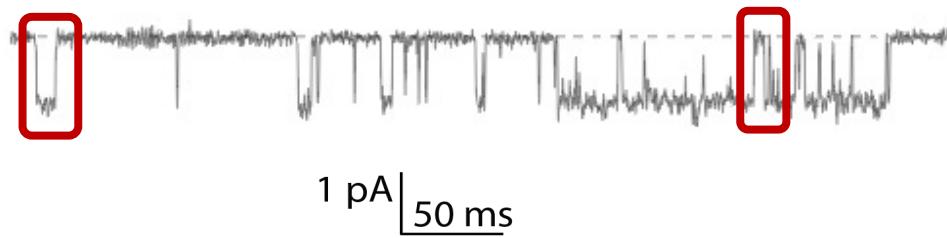
L5: Practical aspects of building channel and cell models

Reconstructing excitability

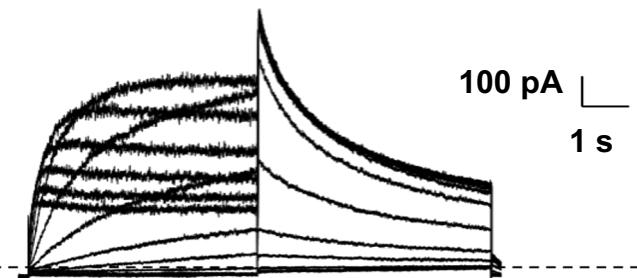
- **Experimental data sources**
 - Single-channel recordings
 - Single-channel analysis
 - Whole-cell recordings
- **Major cardiac ion current models**
 - Hodgkin-Huxley type models
 - Generalised Markov models



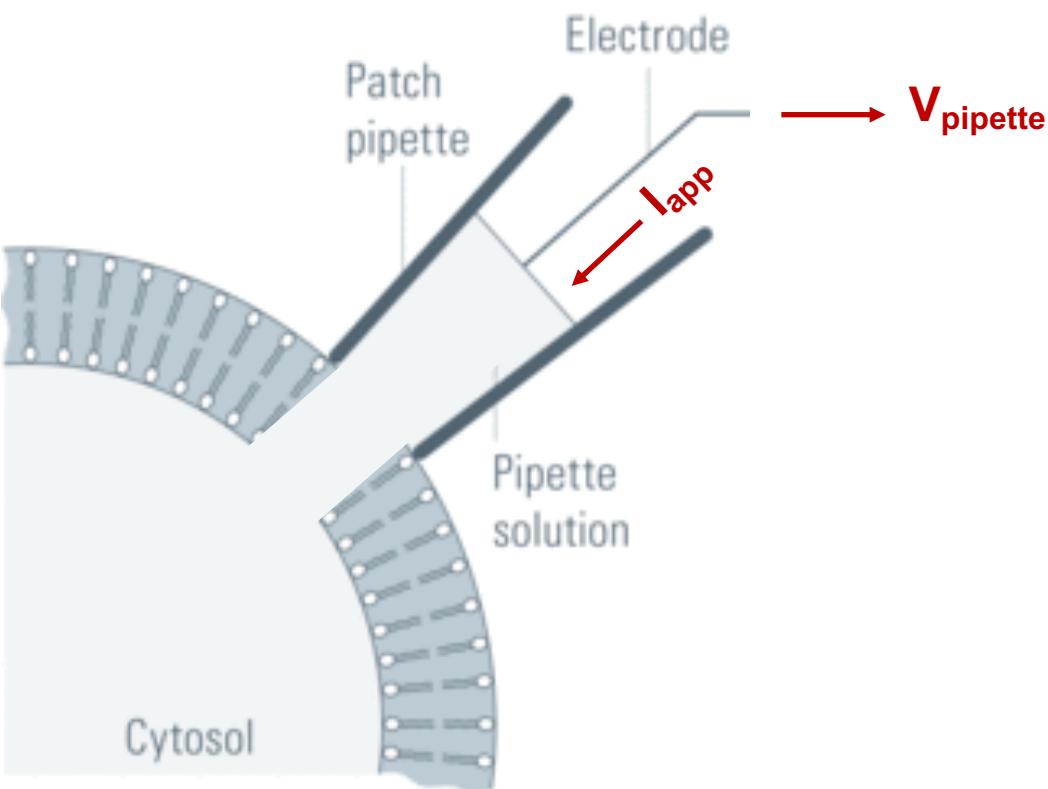
Single-channel



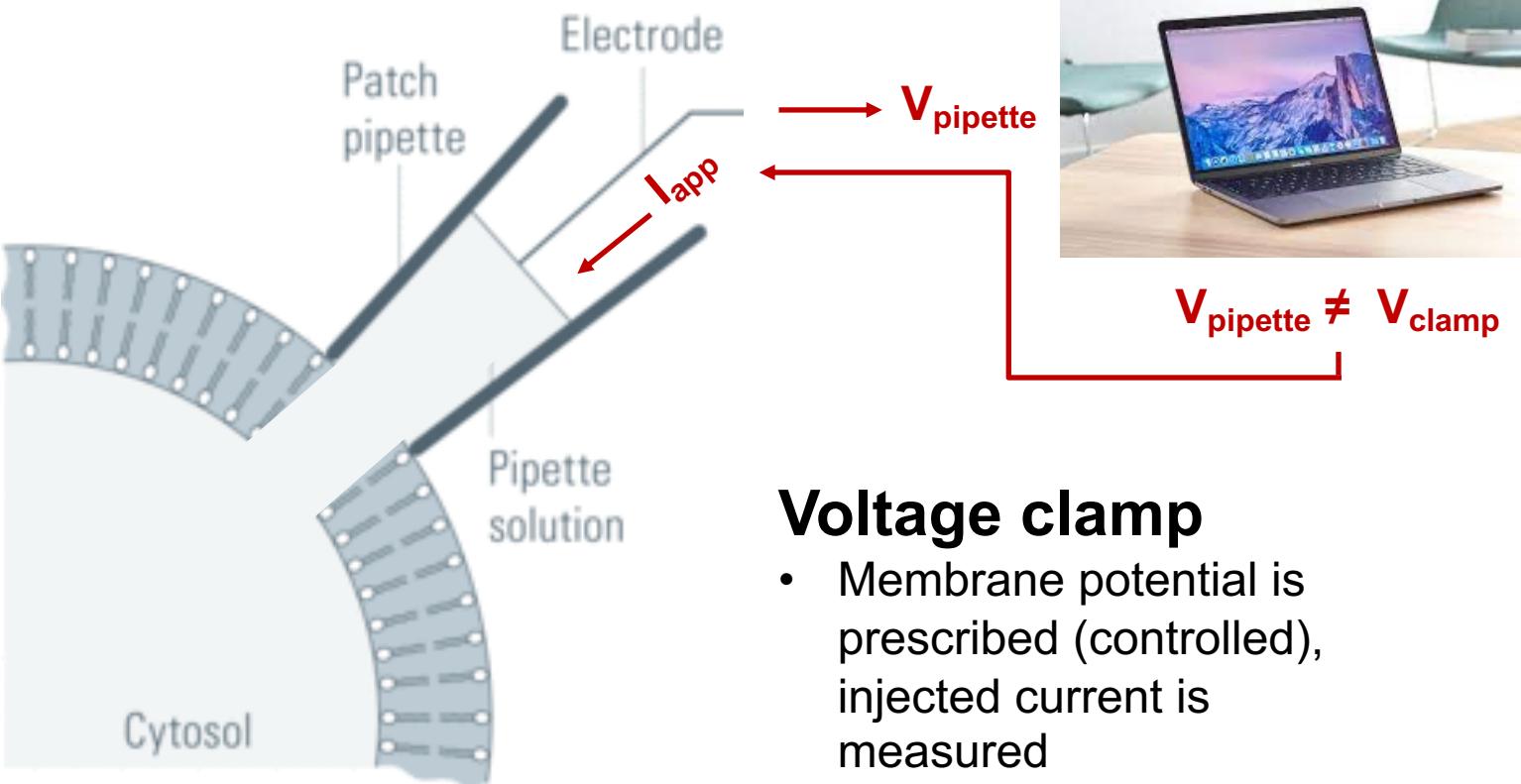
Whole-cell



Microelectrode recording preparations



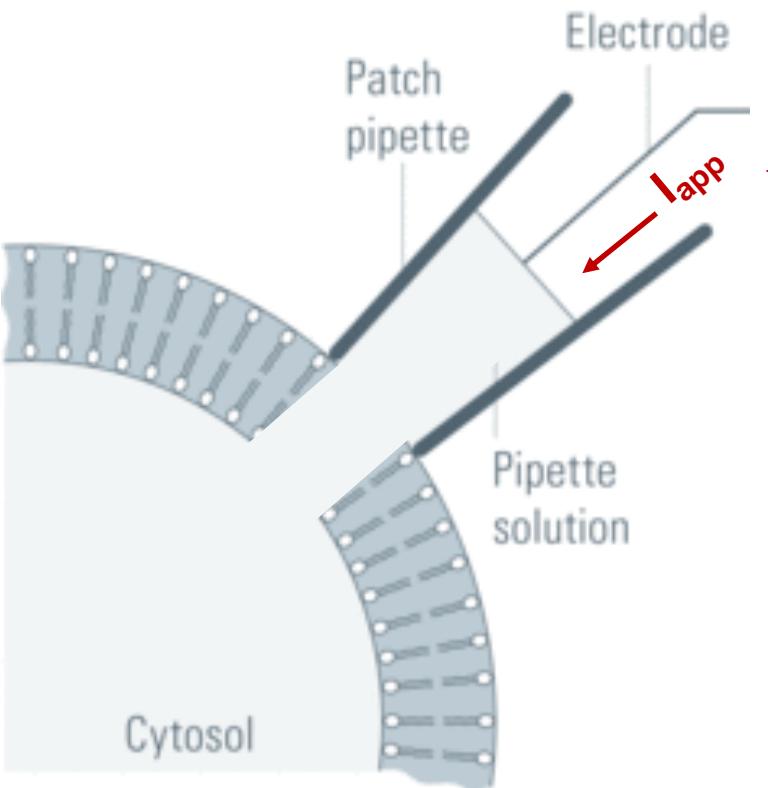
Microelectrode recording preparations



Voltage clamp

- Membrane potential is prescribed (controlled), injected current is measured

Microelectrode recording preparations

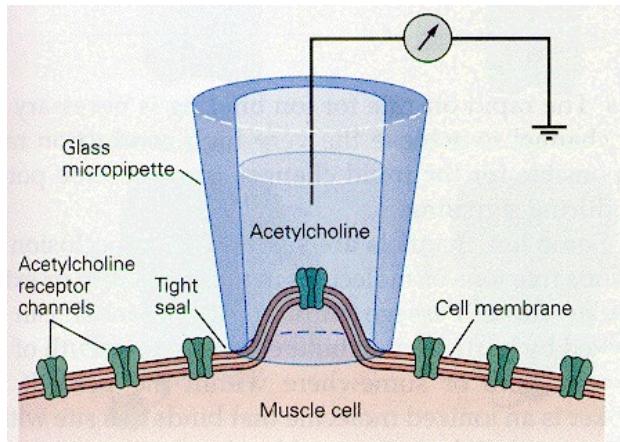


Current clamp

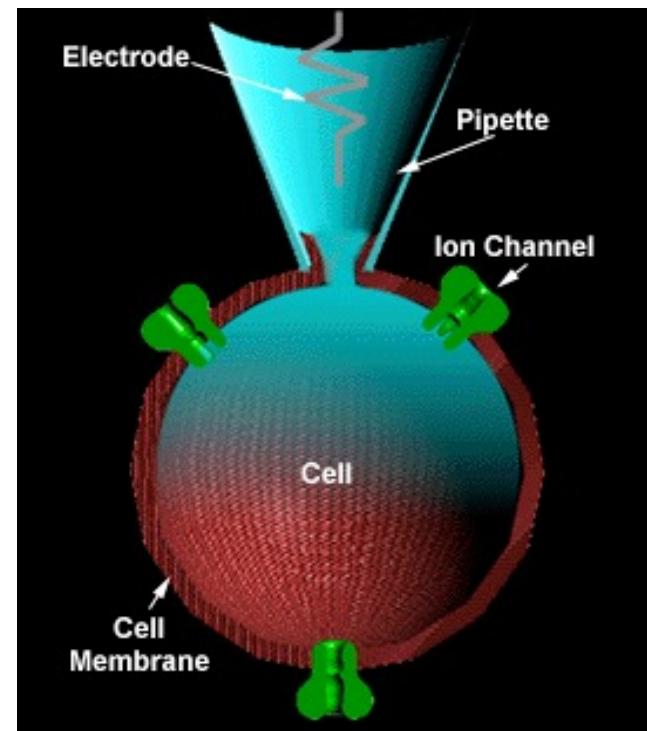
- Current injection is prescribed, membrane potential is measured

Patch-clamp recording configurations

Single-channel

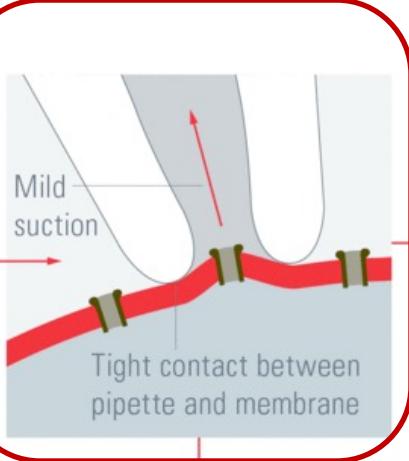
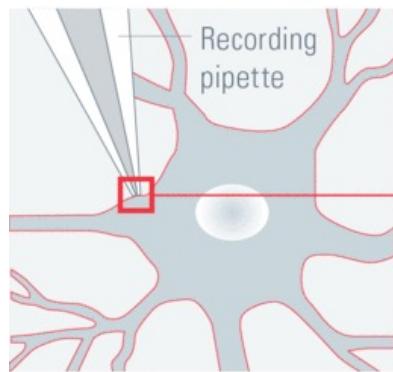


Whole-cell

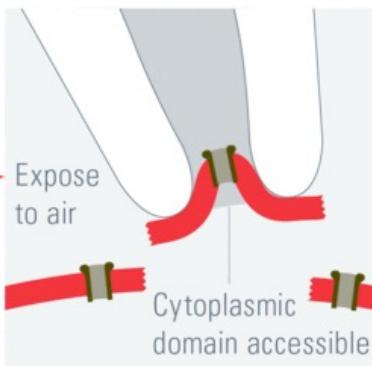


Patch-clamp recording configurations

Cell-attached recording

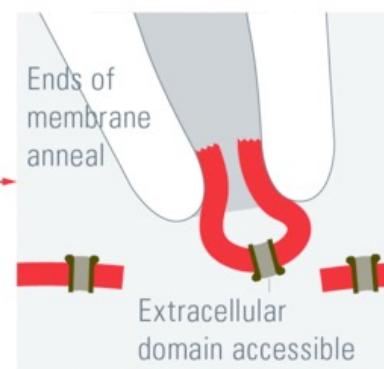
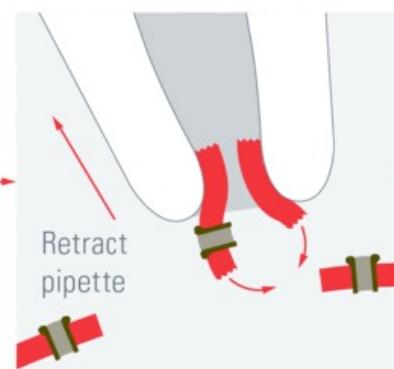
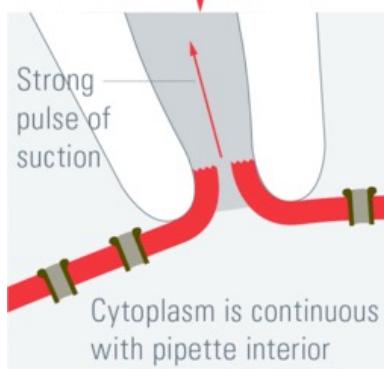


Inside-out recording



Single channel:

- Cell attached

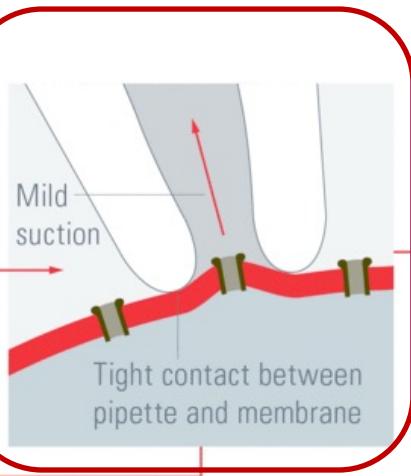
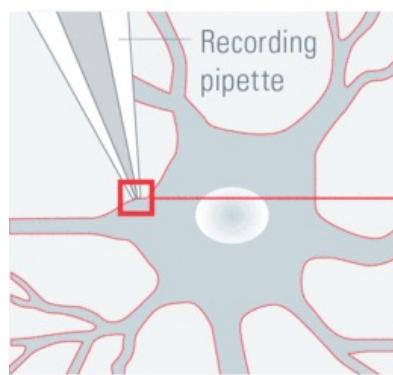


Whole-cell recording

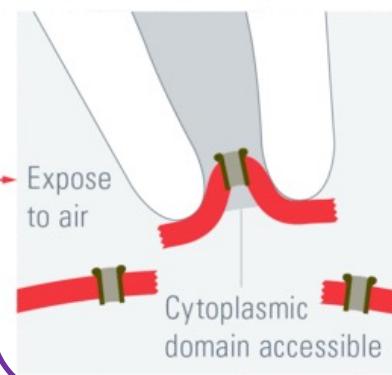
Outside-out recording

Patch-clamp recording configurations

Cell-attached recording

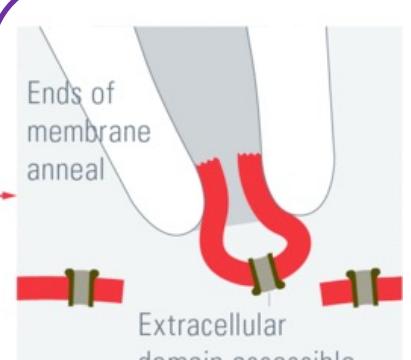
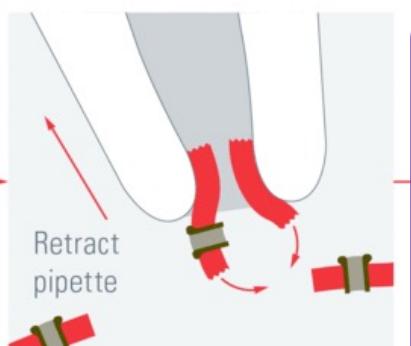
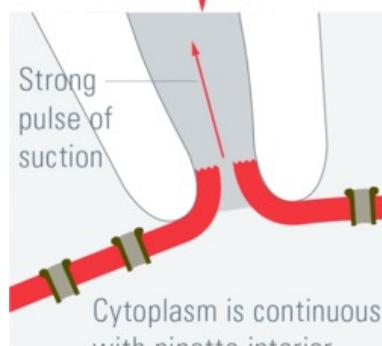


Inside-out recording



Single channel:

- Cell attached
- Excised patch
 - Inside-out
 - Outside-out

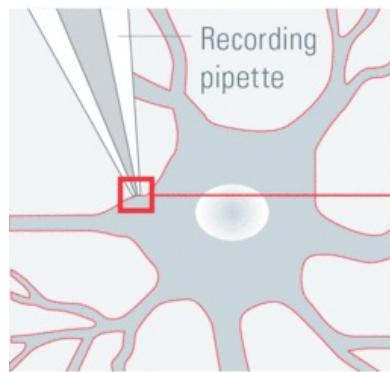


Whole-cell recording

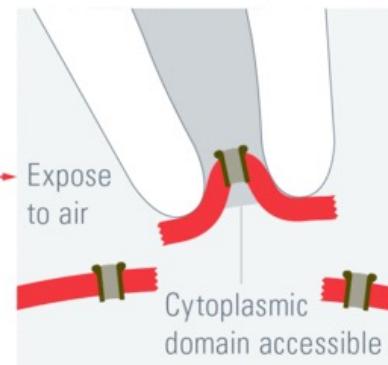
Outside-out recording

Patch-clamp recording configurations

Cell-attached recording

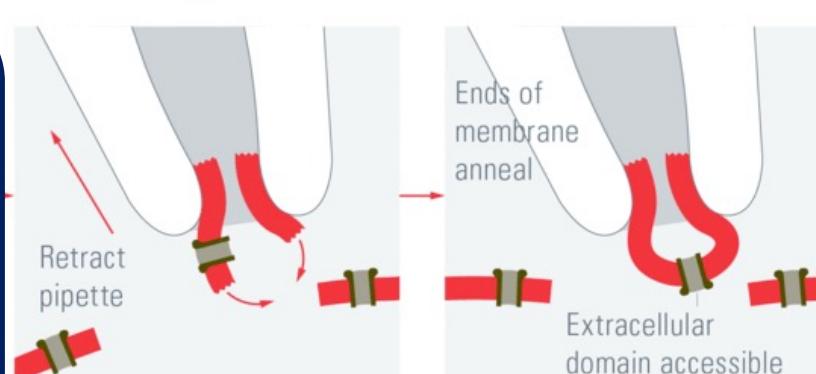
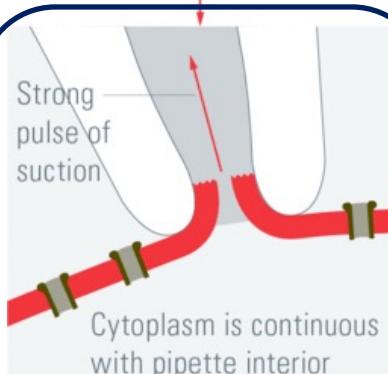


Inside-out recording



Whole-cell:

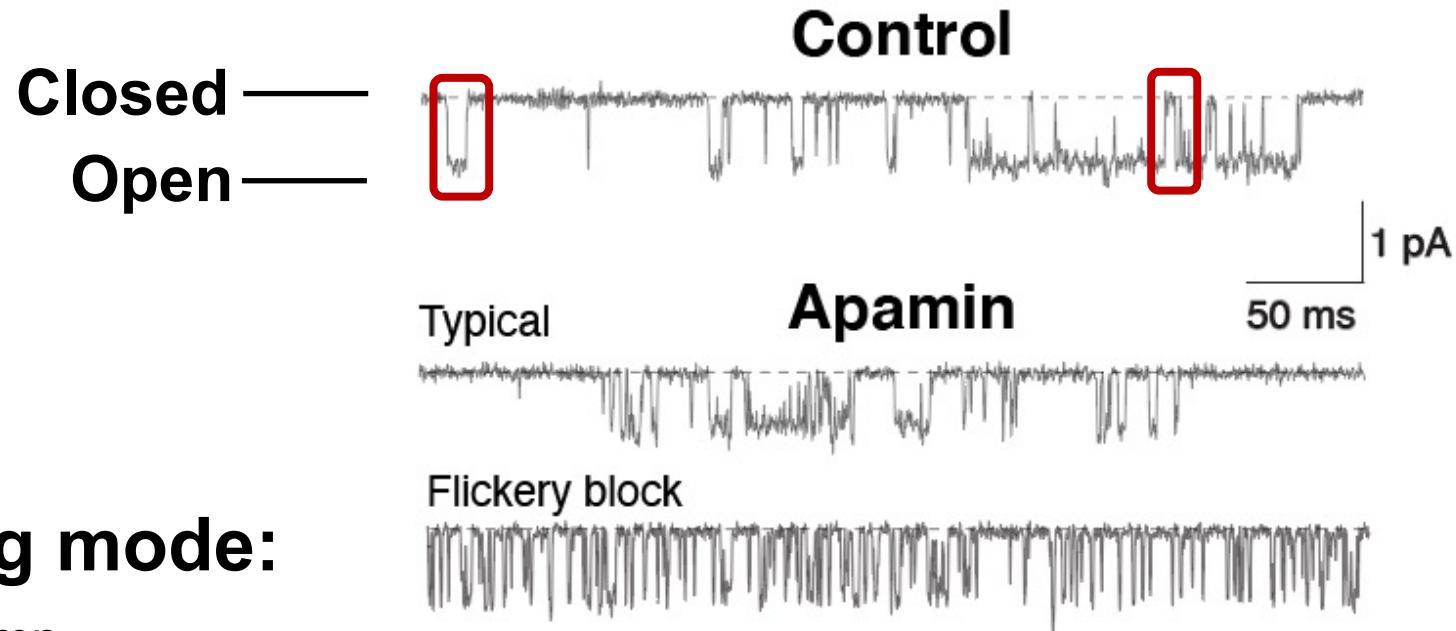
- Ruptured (suction)
- Perforated (ionophore)



Whole-cell recording

Outside-out recording

Single-channel recordings



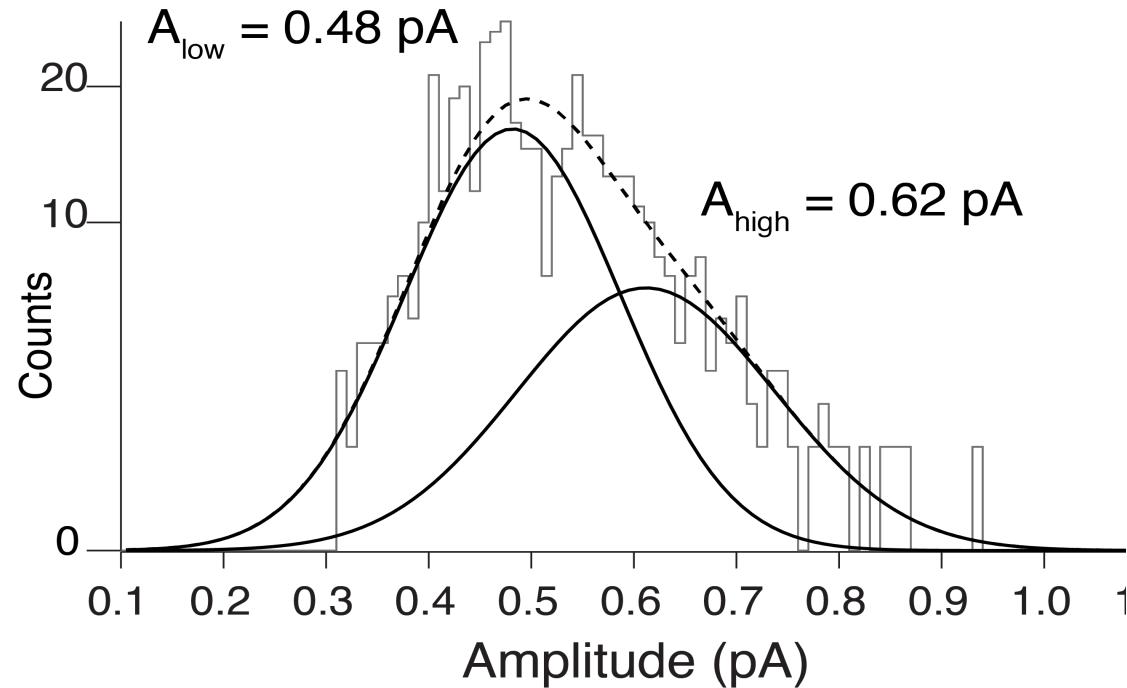
Recording mode:

- Voltage clamp

Event characteristics:

- Amplitude
- Open dwell time
- Closed dwell time

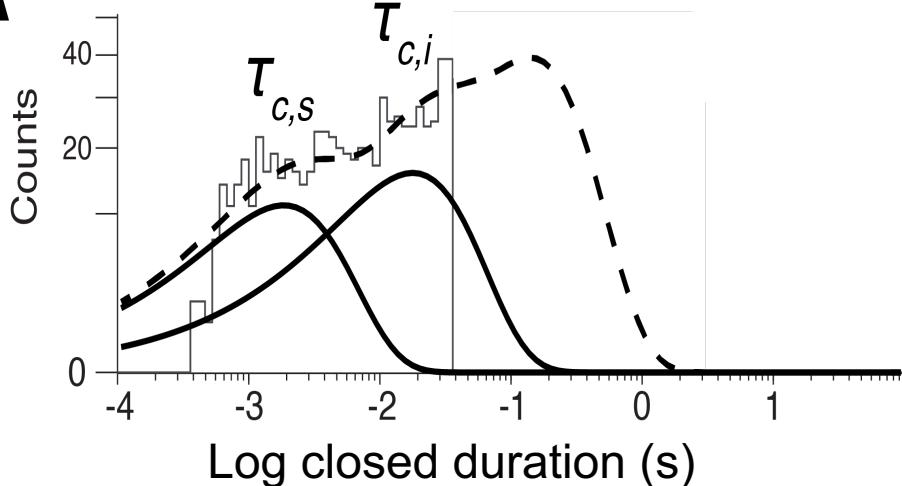
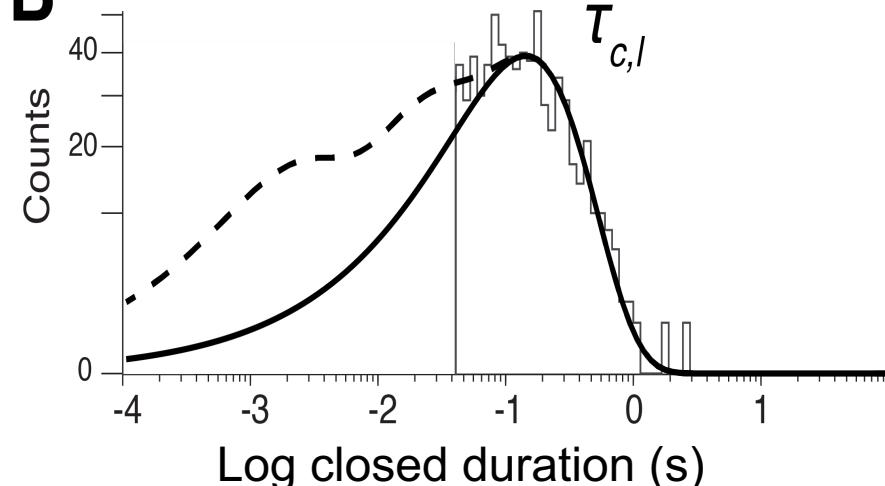
Single-channel recordings



Event characteristics:

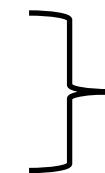
- Amplitude (Gaussian)

Single-channel recordings

A**B**

Event characteristics:

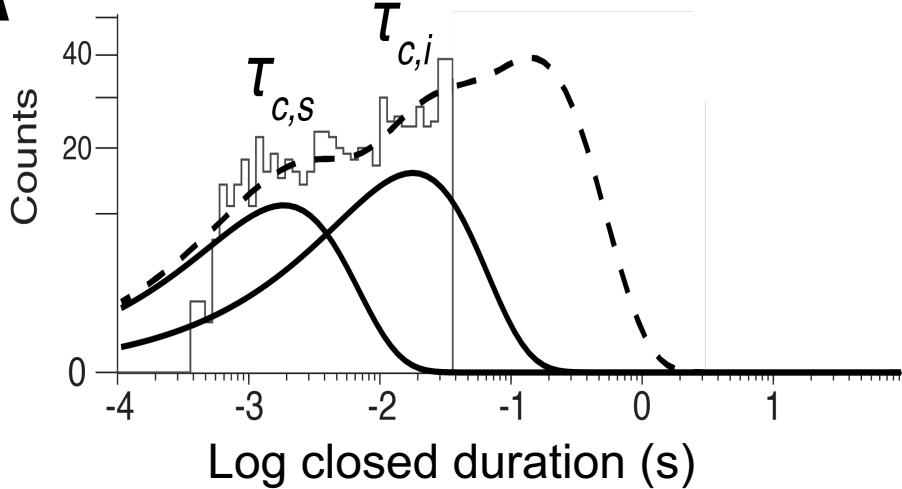
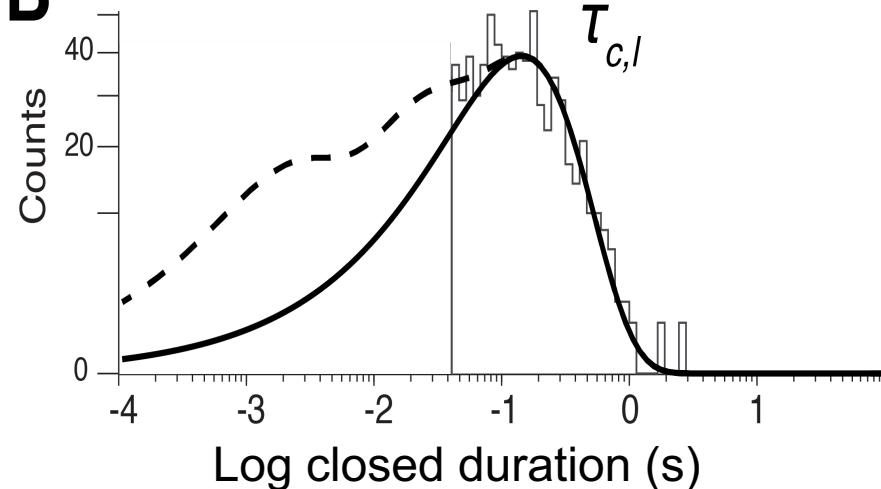
- Open dwell time
- Closed dwell time



Mass action transition process,
exponential in time:

$$p_i(t) = \frac{dP_i(t)}{dt} = K_i e^{-K_i t}$$

Single-channel recordings

A**B**

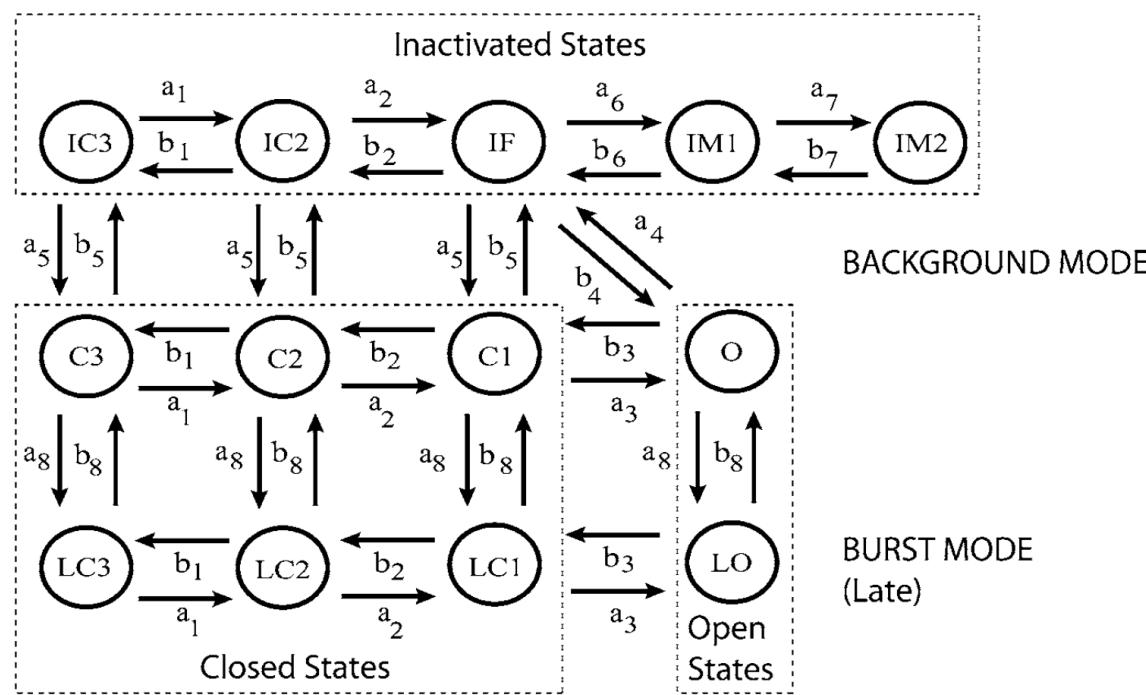
Event characteristics:

- How many components are there?
- What do those components mean for model-building?

Single-channel recordings

Event characteristics:

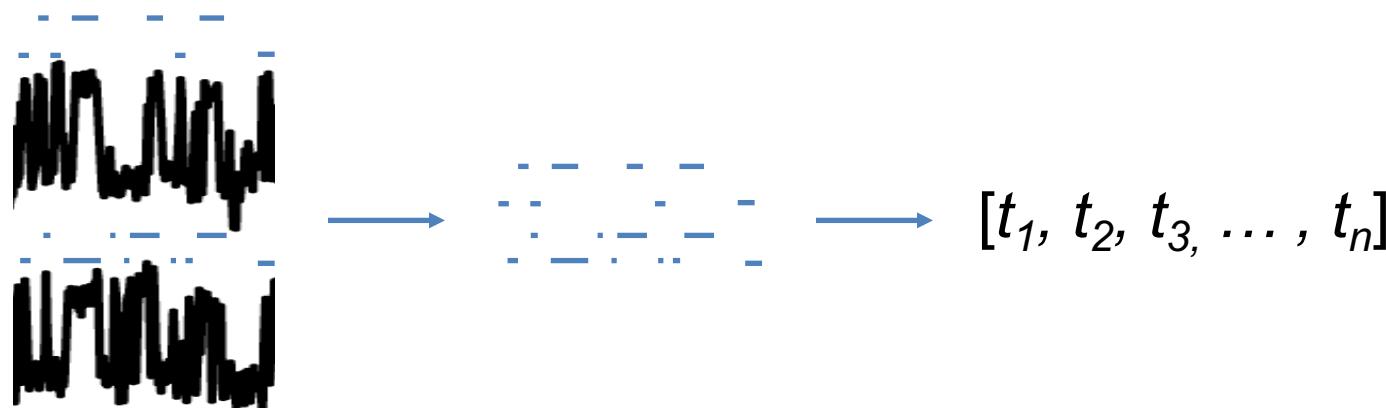
- How many components a
- What do those components mean for model-building?



Single-channel recordings

Event characteristics:

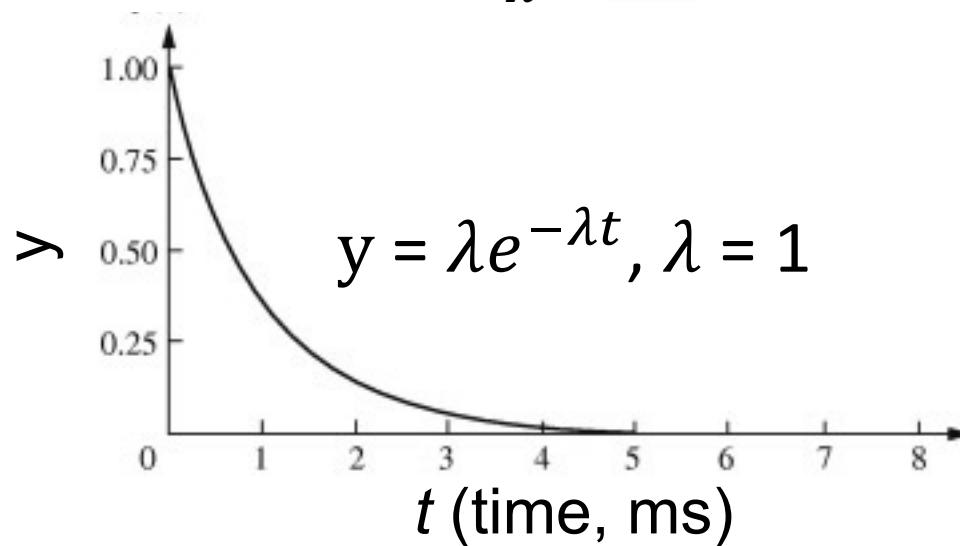
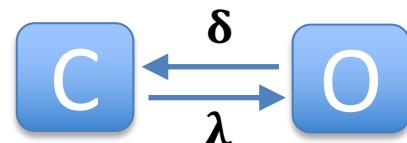
- **How many components are there?**
 - We assume an exponential distribution for each single step process



Single-channel recordings

Event characteristics:

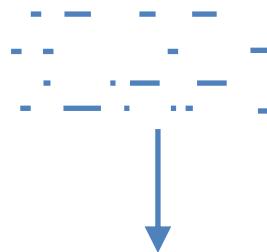
- How many components are there?
 - Example: closed dwell times in a simple single step gating process



Single-channel recordings

Event characteristics:

- How many components are there?
 - Looking at a simple single exponential gating process



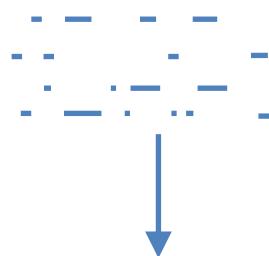
$[t_1, t_2, t_3, \dots, t_n]$

$$y = \lambda e^{-\lambda t}$$

Single-channel recordings

Event characteristics:

- How many components are there?
 - Looking at a simple single exponential gating process



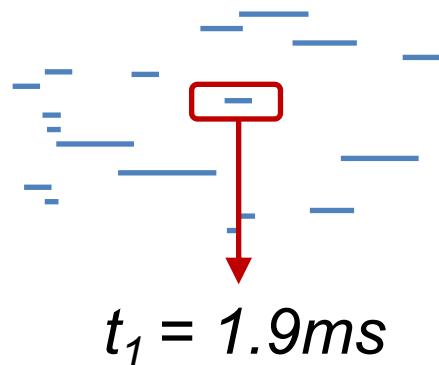
$[t_1, t_2, t_3, \dots, t_n]$

$$y = \lambda e^{-\lambda t} \longrightarrow \arg \max_{\lambda} L(\lambda | t_1, t_2, t_3, \dots, t_n)$$

Single-channel recordings

Event characteristics:

- How many components are there?
 - Likelihood of $t_1 = 1.9 \text{ ms}$ is simply the PDF value at 1.9 ms

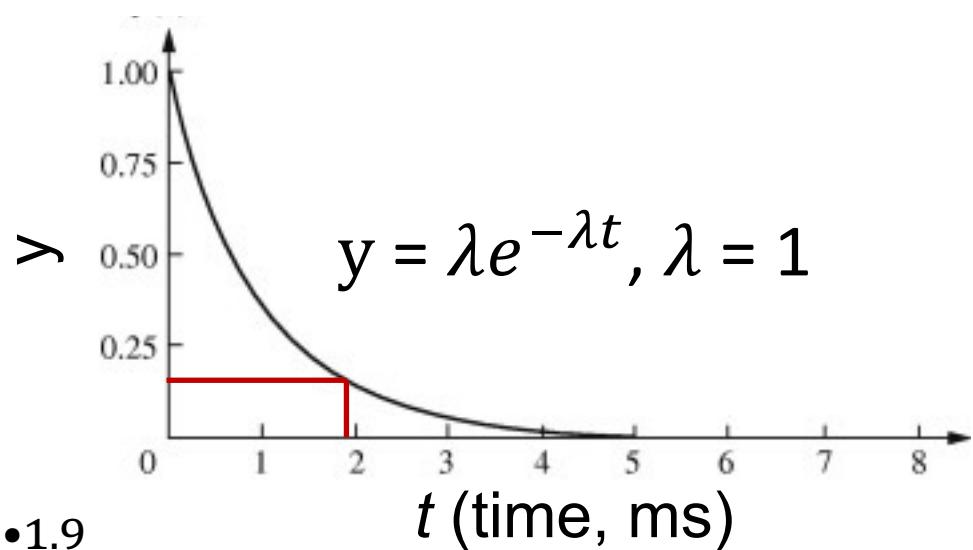
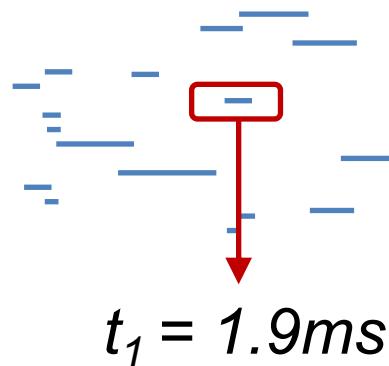


$$L(t_1|\lambda) = y = \lambda e^{-\lambda t}$$

Single-channel recordings

Event characteristics:

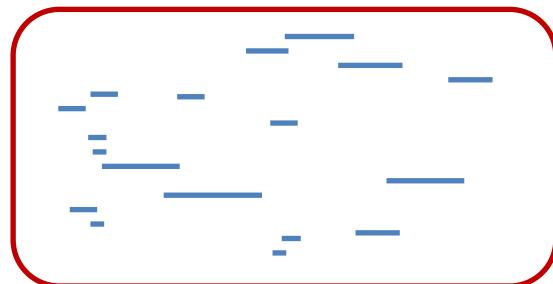
- How many components are there?
 - Likelihood of $t_1 = 1.9 \text{ ms}$ is simply the PDF value at 1.9 ms



$$L(\lambda = 1 | t_1 = 1.9) = y = 1e^{-1 \cdot 1.9}$$

Single-channel recordings

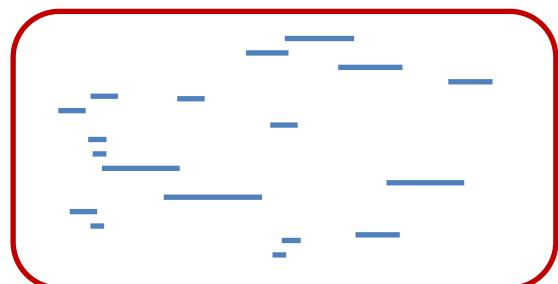
Now for all t



$$\begin{aligned}L(\lambda | t_1, t_2, t_3, \dots, t_n) &= \prod_{t=1}^n \lambda e^{-\lambda t} \\&= \lambda^n [e^{-\lambda(t_1+t_2+t_3+\dots+n)}]\end{aligned}$$

Single-channel recordings

Now for all t



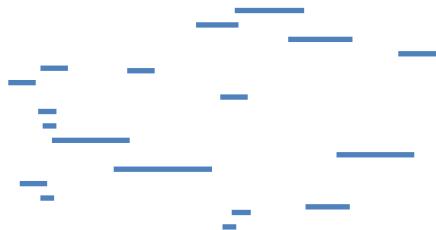
$$\begin{aligned} L(\lambda | t_1, t_2, t_3, \dots, t_n) &= \prod_{t=1}^n \lambda e^{-\lambda t} \\ &= \lambda^n [e^{-\lambda(t_1+t_2+t_3+\dots+n)}] \end{aligned}$$

Gradient-based method (assumes convexity):

$$0 = \frac{d}{d\lambda} L(\lambda | t_1, t_2, t_3, \dots, t_n) = \frac{d}{d\lambda} \lambda^n [e^{-\lambda(t_1+t_2+t_3+\dots+n)}]$$

Single-channel recordings

Now for all t



$$\begin{aligned} L(\lambda | t_1, t_2, t_3, \dots, t_n) &= \prod_{t=1}^n \lambda e^{-\lambda t} \\ &= \lambda^n [e^{-\lambda(t_1+t_2+t_3+\dots+tn)}] \end{aligned}$$

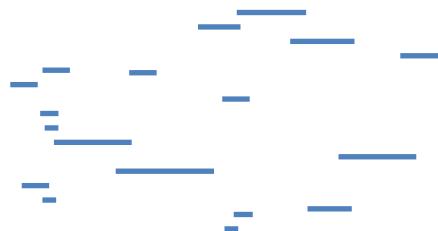
Gradient-based method (assumes convexity):

$$0 = \frac{d}{d\lambda} L(\lambda | t_1, t_2, t_3, \dots, t_n) = \frac{d}{d\lambda} \lambda^n [e^{-\lambda(t_1+t_2+t_3+\dots+tn)}]$$

$$\text{log likelihood estimator} = \frac{d}{d\lambda} n \ln \lambda - \lambda(t_1 + t_2 + t_3 + \dots + t_n)$$

Single-channel recordings

Now for all t



$$\begin{aligned}
 L(\lambda | t_1, t_2, t_3, \dots, t_n) &= \prod_{t=1}^n \lambda e^{-\lambda t} \\
 &= \lambda^n [e^{-\lambda(t_1 + t_2 + t_3 + \dots + n)}]
 \end{aligned}$$

Gradient-based method (assumes convexity):

$$0 = \frac{d}{d\lambda} L(\lambda | t_1, t_2, t_3, \dots, t_n) = \frac{d}{d\lambda} \lambda^n [e^{-\lambda(t_1 + t_2 + t_3 + \dots + n)}]$$

$$\text{log likelihood estimator} = \frac{d}{d\lambda} n \ln \lambda - \lambda(t_1 + t_2 + t_3 + \dots + n)$$

$$= n \frac{1}{\lambda} - \frac{(t_1 + t_2 + t_3 + \dots + t_n)}{n}$$

$$\text{MLE estimate of } \lambda = \frac{(t_1 + t_2 + t_3 + \dots + t_n)}{(t_1 + t_2 + t_3 + \dots + t_n)}$$

Single-channel recordings

Multiple kinetic components:

$$L(\lambda_1, \lambda_2, \dots, \lambda_m; a_1, a_2, \dots, a_m | t_1, t_2, t_3, \dots, t_n) = \\ \prod_{t=1}^n [a_1 \lambda_1 e^{-\lambda_1 t} + a_2 \lambda_2 e^{-\lambda_2 t} + \dots + a_m \lambda_m e^{-\lambda_m t}]$$

$MLE(m) > MLE(m-1)$?

Single-channel recordings

Multiple kinetic components:

$$L(\lambda_1, \lambda_2, \dots, \lambda_m; a_1, a_2, \dots, a_m | t_1, t_2, t_3, \dots, t_n) = \prod_{t=1}^n [a_1 \lambda_1 e^{-\lambda_1 t} + a_2 \lambda_2 e^{-\lambda_2 t} + \dots + a_m \lambda_m e^{-\lambda_m t}]$$

$MLE(m) > MLE(m-1)$?

likelihood ratio test: $\vec{t} \stackrel{\text{def}}{=} [t_1, t_2, t_3, \dots, t_n]$

Single-channel recordings

Multiple kinetic components:

$$L(\lambda_1, \lambda_2, \dots, \lambda_m; a_1, a_2, \dots, a_m | t_1, t_2, t_3, \dots, t_n) = \\ \prod_{t=1}^n [a_1 \lambda_1 e^{-\lambda_1 t} + a_2 \lambda_2 e^{-\lambda_2 t} + \dots + a_m \lambda_m e^{-\lambda_m t}]$$

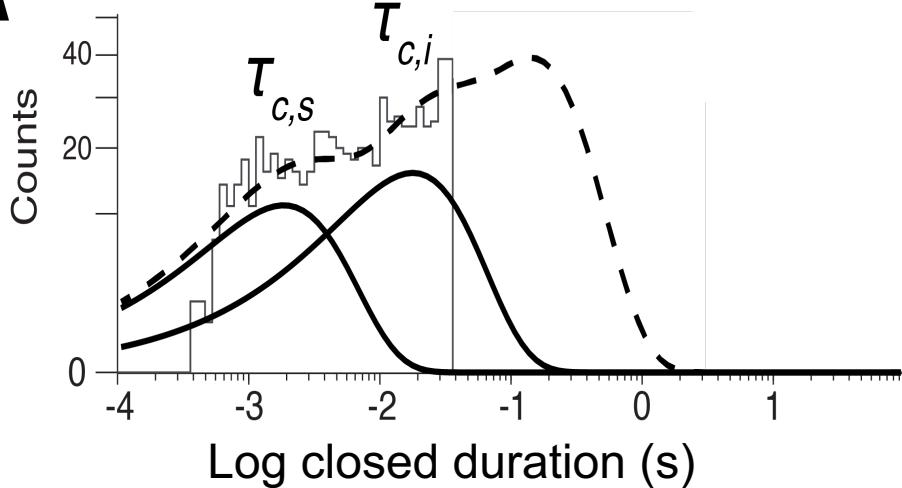
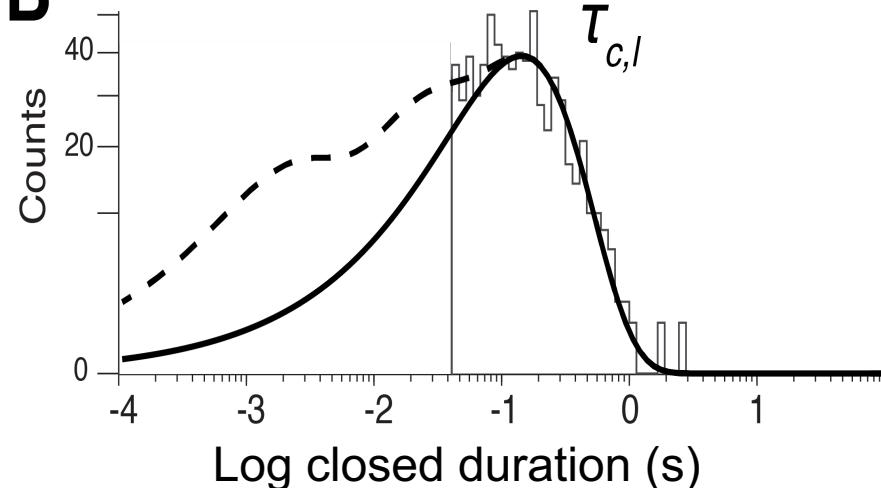
$MLE(m) > MLE(m-1)$?

likelihood ratio test: $\vec{t} \stackrel{\text{def}}{=} [t_1, t_2, t_3, \dots, t_n]$

(log transformed data)

$$\Lambda(\vec{t}) = -2 \ln \left[\frac{MLE(m-1)}{MLE(m)} \right], \quad \text{or} \quad \Lambda(\vec{t}) = -2[\ell(m-1) - \ell(m)]$$

Single-channel recordings

A**B**

Event characteristics:

- How many components are there?
- **What do those components mean?**

Single-channel recordings

Event characteristics:

- **What do these components mean?**

From L4:

$$E(T_i) = \int_0^{\infty} tp_i(t)dt = \frac{1}{K_i}$$

Single-channel recordings

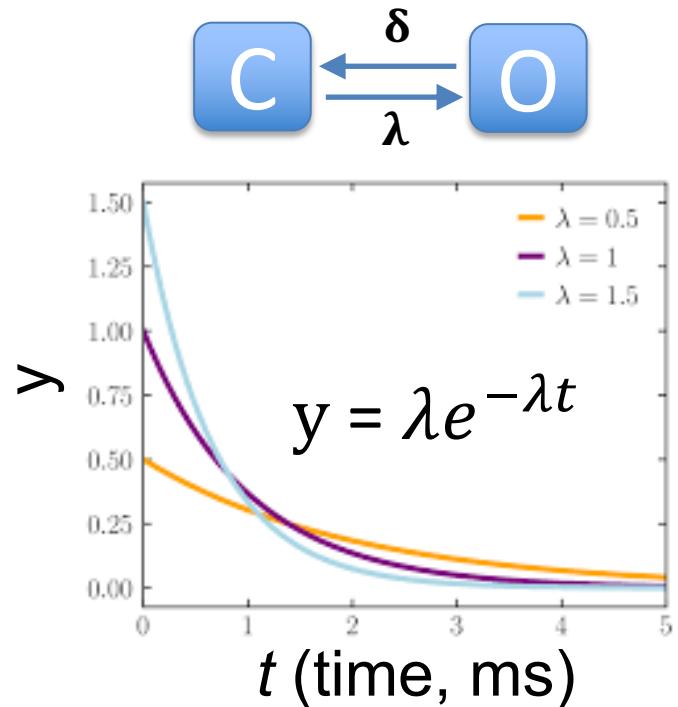
Event characteristics:

- What do these components mean?

From L4:

$$E(T_i) = \int_0^\infty t p_i(t) dt = \frac{1}{K_i}$$

$$\begin{aligned} E(t_i) &= \int_0^\infty t \lambda_i e^{-\lambda_i t} dt, \quad (i = 1, \dots, m) \\ &= \frac{1}{\lambda_i} \\ &= \tau_i \\ &= \text{mean dwell time of } i\text{th component} \end{aligned}$$



Single-channel recordings

Event characteristics:

- What do these components mean?

Some conceptual exercises:

- You have data exhibiting 2 distinct closed dwell times and 1 open dwell time. Try to draw a model of this channel.

Single-channel recordings

Event characteristics:

- What do these components mean?

Some conceptual exercises:

- You have data exhibiting 2 distinct closed dwell times and 1 open dwell time. Try to draw a model of this channel.
- How many open states does it have?

Single-channel recordings

Event characteristics:

- What do these components mean?

Some conceptual exercises:

- You have data exhibiting 2 distinct closed dwell times and 1 open dwell time. Try to draw a model of this channel.
- How many open states does it have?
- How many closed states does it have?

Single-channel recordings

Event characteristics:

- What do these components mean?

Some conceptual exercises:

- You have data exhibiting 2 distinct closed dwell times and 1 open dwell time. Try to draw a model of this channel.
- How many open states does it have?
- How many closed states does it have?
- Does it include an inactive state?

Single-channel recordings

Event characteristics:

- What do these components mean?

Some conceptual exercises:

- You have data exhibiting 2 distinct closed dwell times and 1 open dwell time. Try to draw a model of this channel.
- How many open states does it have?
- How many closed states does it have?
- Does it include an inactive state?
- Try to repeat these questions for a channel with 3 closed dwell times and 2 open dwell times

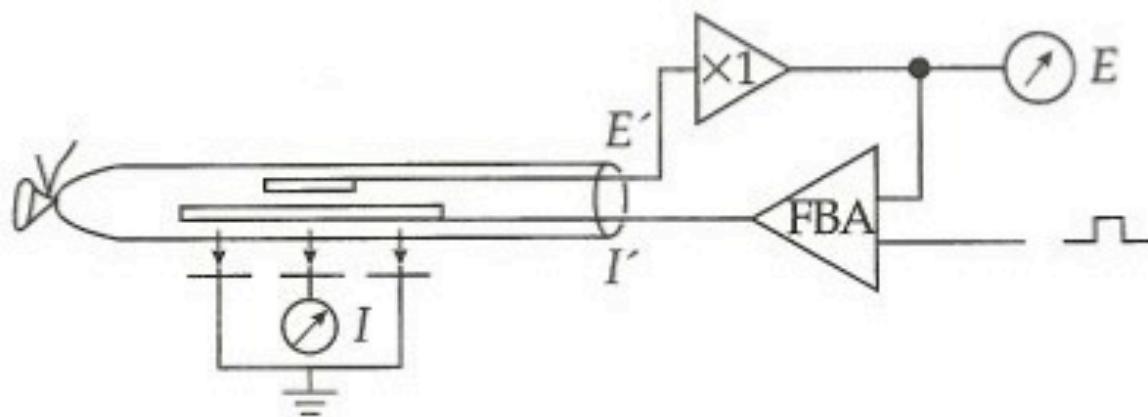
Take a break



Whole-cell recordings

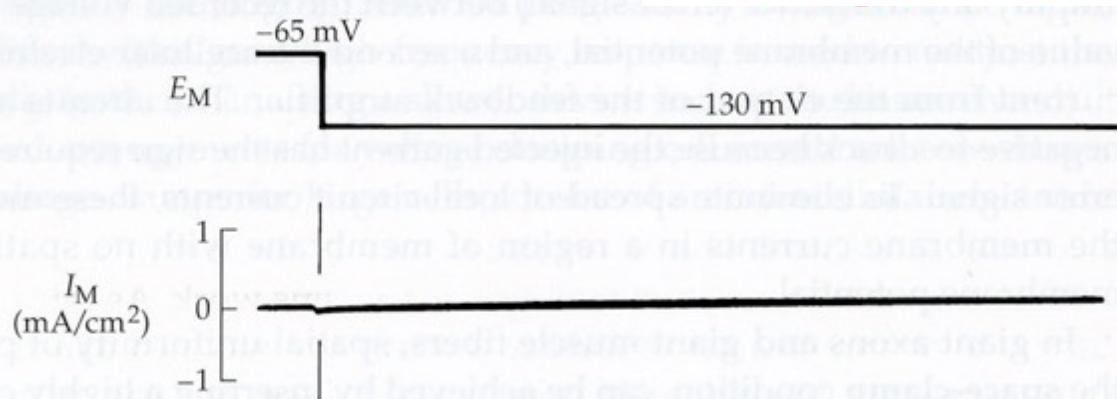
- Hodgkin and Huxley voltage clamp

(A) AXIAL WIRE

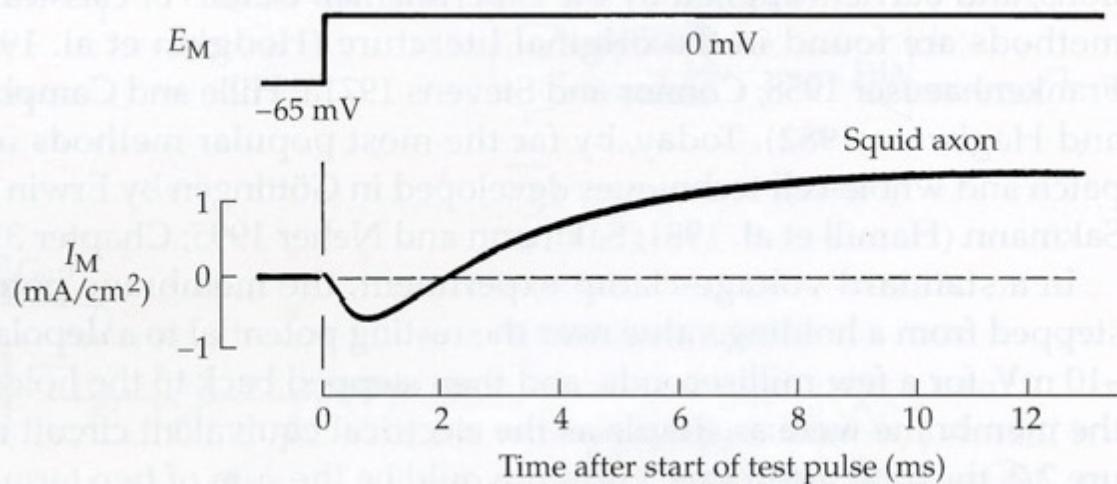


Whole-cell recordings: Hodgkin and Huxley

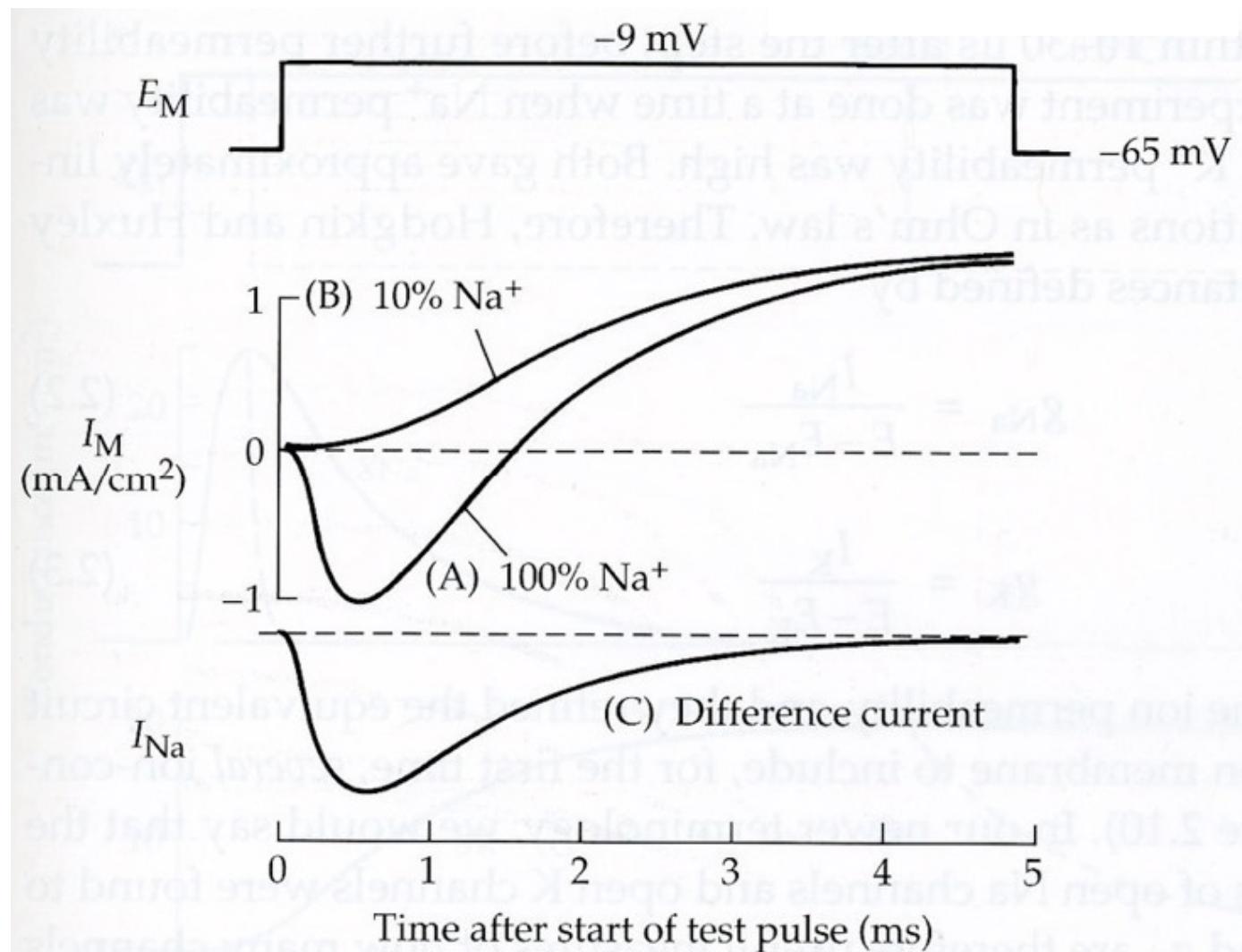
(A) HYPERPOLARIZATION



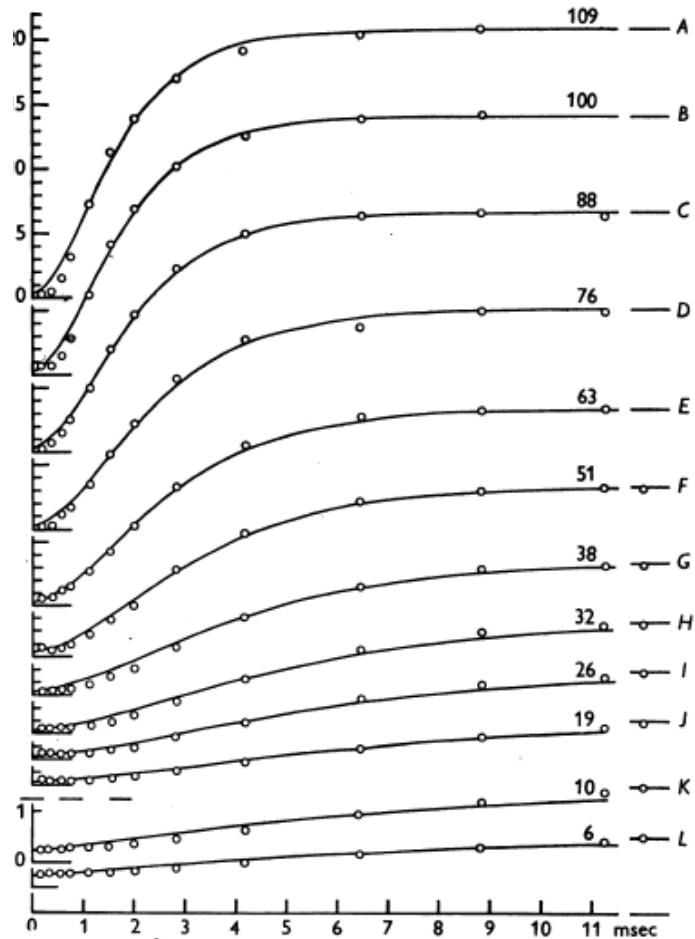
(B) DEPOLARIZATION



Whole-cell recordings: Hodgkin and Huxley



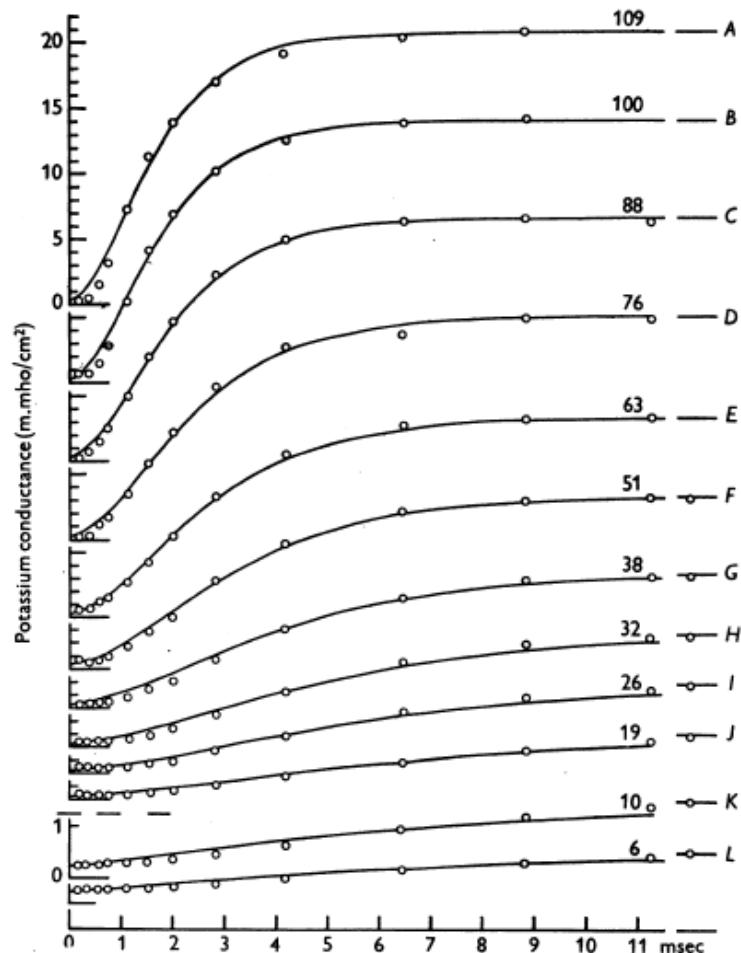
Whole-cell recordings: K⁺ current (I_K)



I_K measured at increasingly positive holding potentials

- *What information can you retrieve from these traces?*

Whole-cell recordings: K⁺ current (I_K)

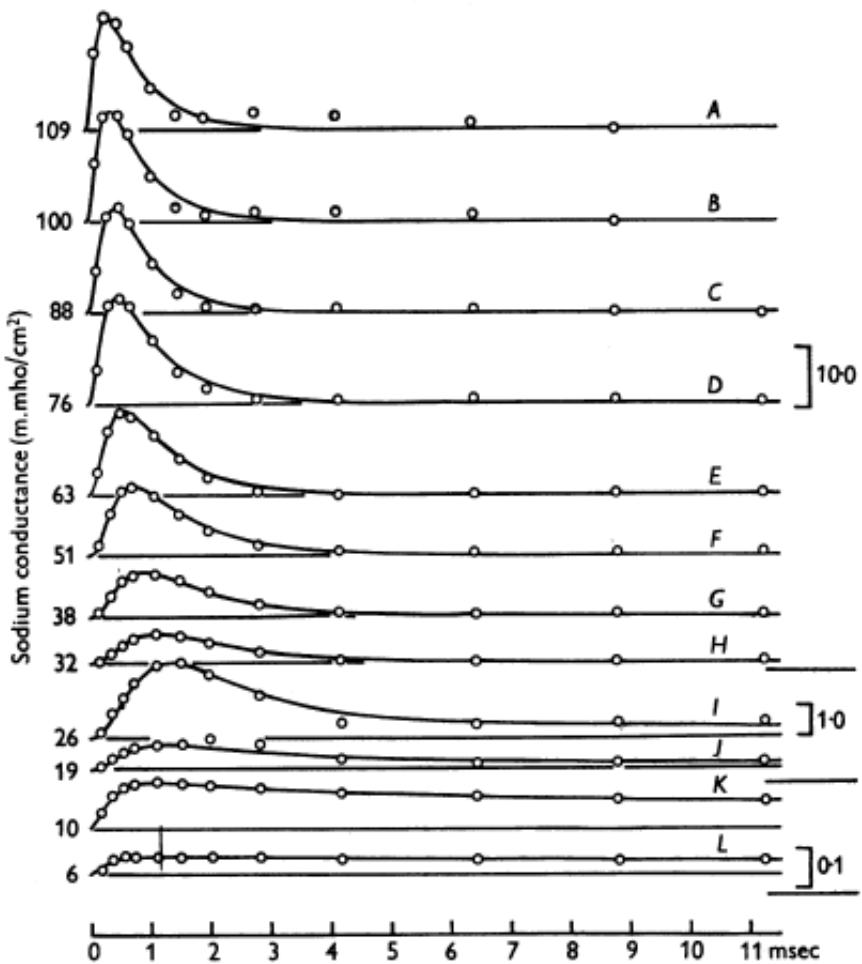


I_K measured at increasingly positive holding potentials

- *What information can you retrieve from these traces?*
1. Peak (equilibrium) current
 2. Kinetics of current activation
 3. Voltage dependence
 4. Inactivation?
 5. Deactivation?

$$g_K = \bar{g}_K n^4, \text{ with } \frac{dn}{dt} = \alpha(v)(1 - n) - \beta(v)n$$

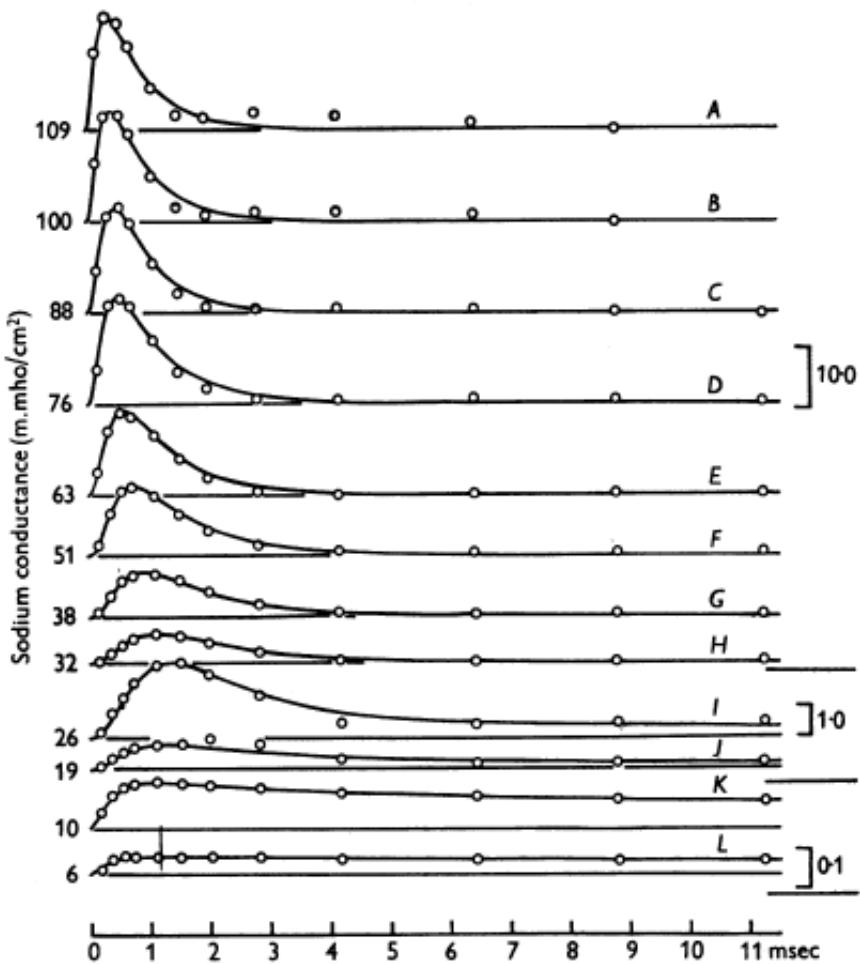
Whole-cell recordings: Na^+ current (I_{Na})



I_{Na} measured at increasingly positive holding potentials

- *What information can you retrieve from these traces?*

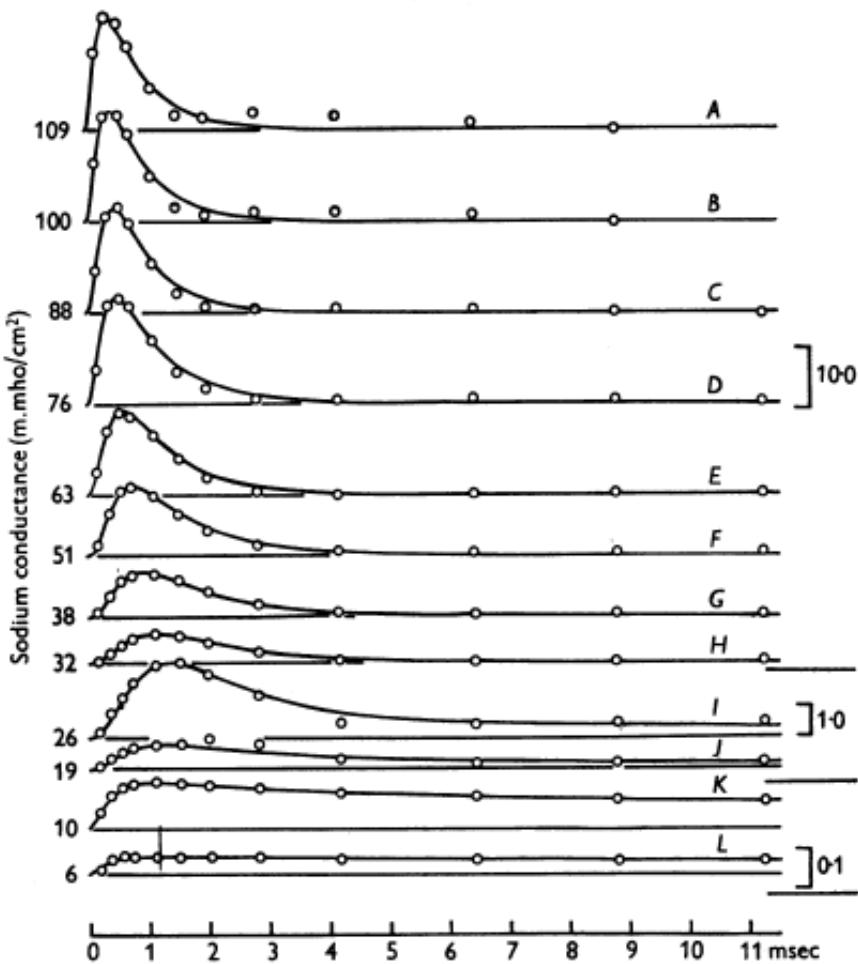
Whole-cell recordings: Na^+ current (I_{Na})



I_{Na} measured at increasingly positive holding potentials

- *What information can you retrieve from these traces?*
 1. Peak current
 2. Kinetics of current activation
 3. Kinetics of inactivation
 4. Voltage dependencies

Whole-cell recordings: Na^+ current (I_{Na})



I_{Na} measured at increasingly positive holding potentials

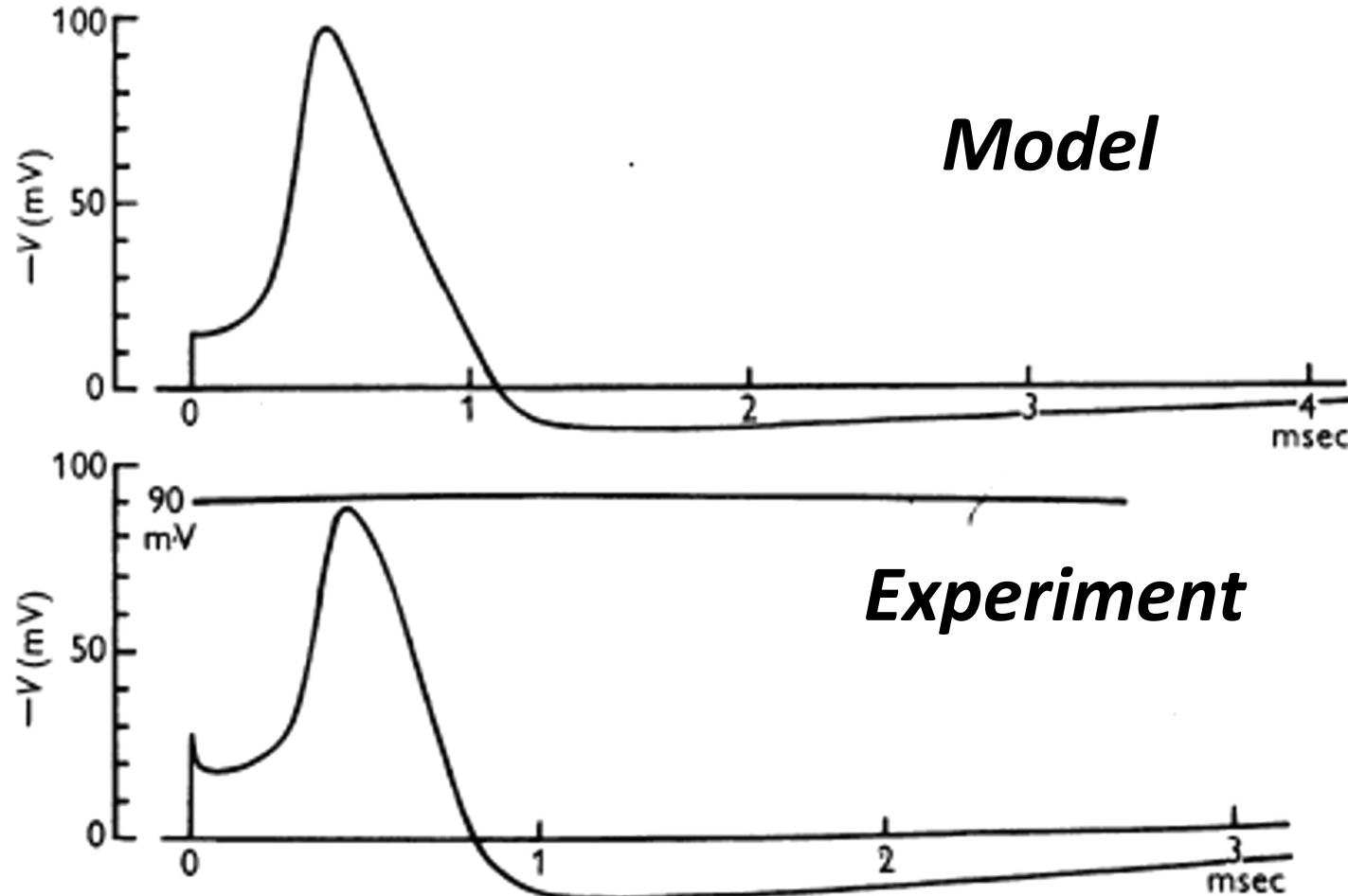
- *What information can you retrieve from these traces?*
1. Peak current
 2. Kinetics of current activation
 3. Kinetics of inactivation
 4. Voltage dependencies

$$\frac{dg_{\text{Na}}}{dt} = \bar{g}_{\text{Na}} m^3 h$$

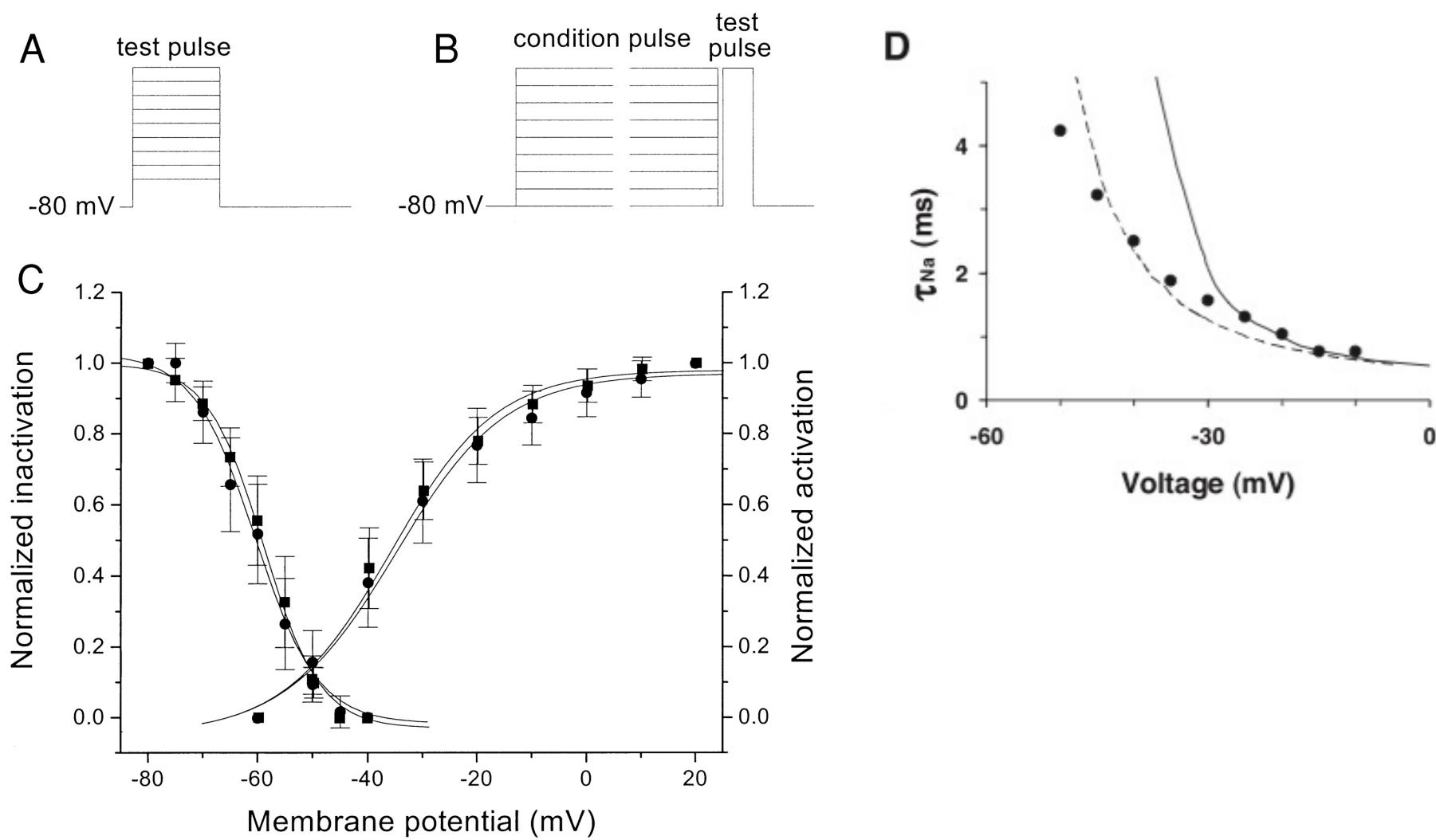
$$\frac{dm}{dt} = \alpha(1 - m) - \beta m,$$

$$\frac{dh}{dt} = \gamma(1 - h) - \delta h,$$

Whole-cell recordings: Reconstructed AP

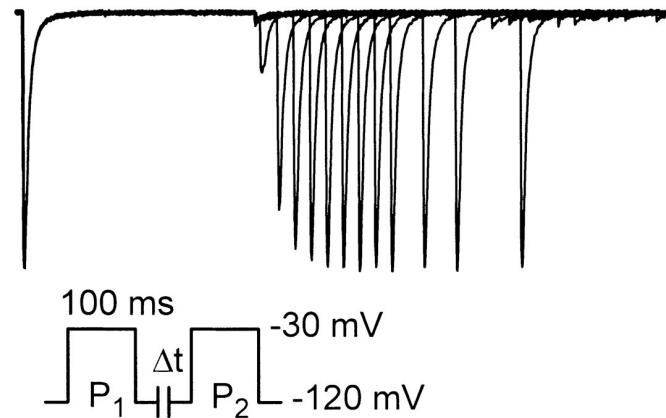


Conventional protocols: activation and inactivation

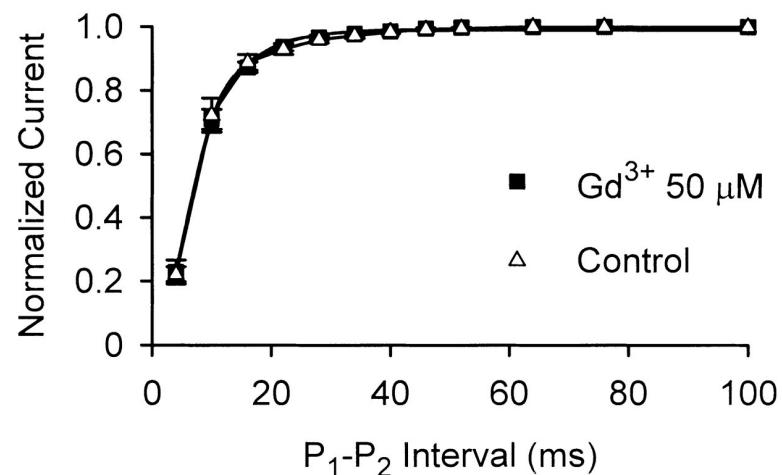


Conventional protocols: recovery from inactivation

A

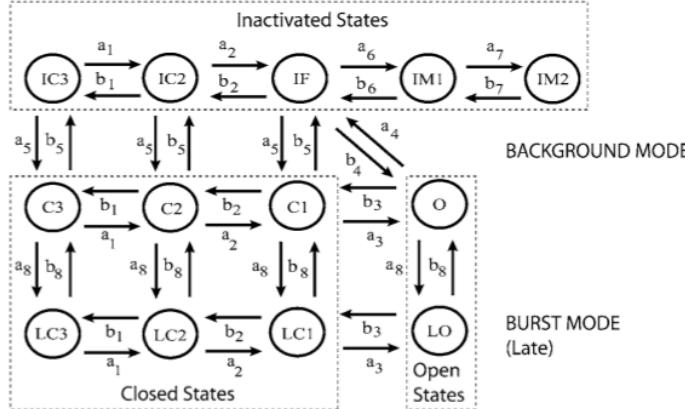


B



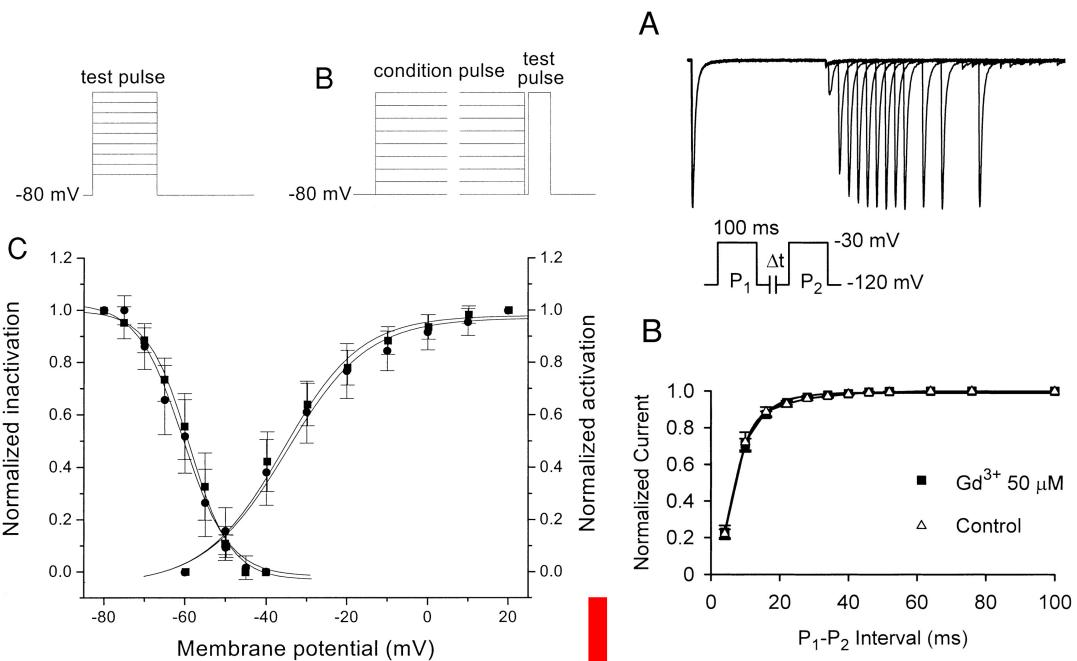
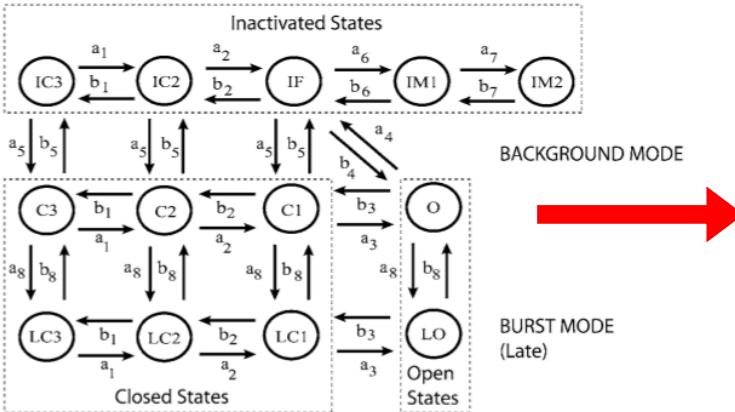
Conventional protocols: combinatorial optimization

A priori constraints



Conventional protocols: combinatorial optimization

A priori constraints

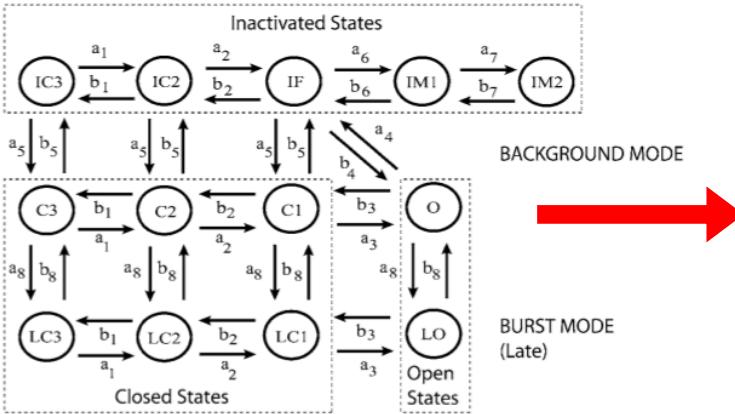


*Cost function
- Error estimate*



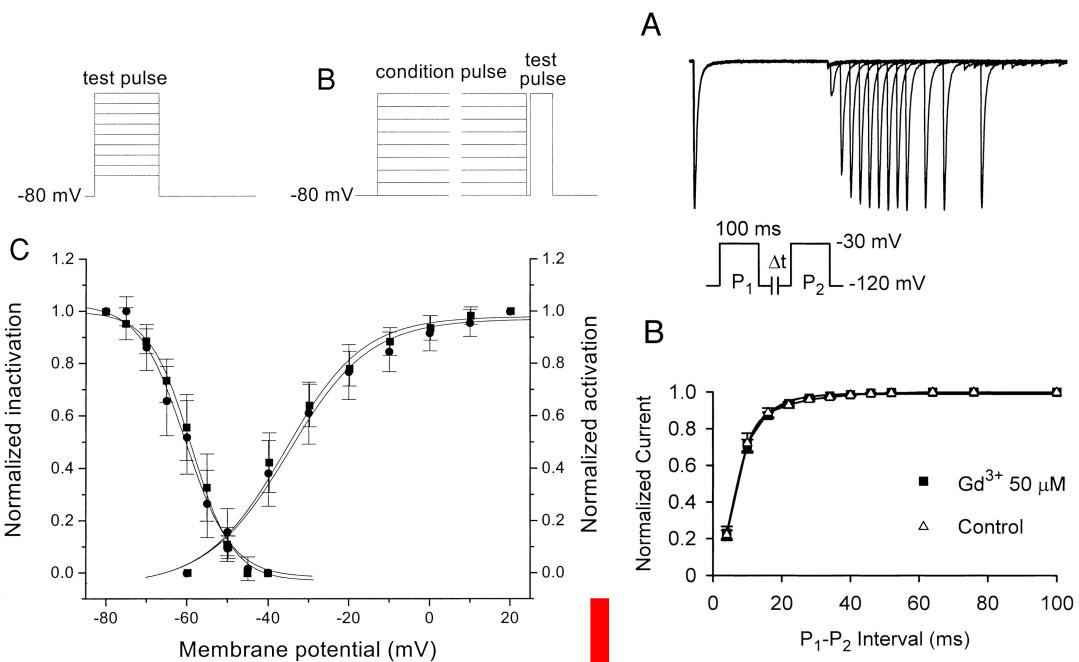
Conventional protocols: combinatorial optimization

A priori constraints



BURST MODE
(Late)

BACKGROUND MODE



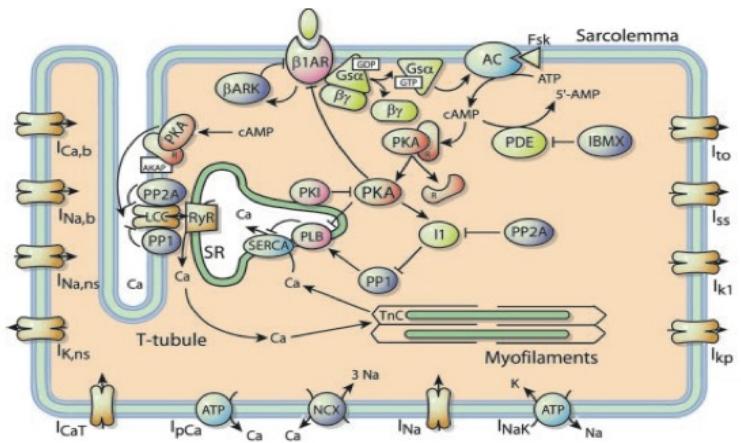
*Cost function
- Error estimate*



*Optimization algorithm
- Parameter reconfiguration*



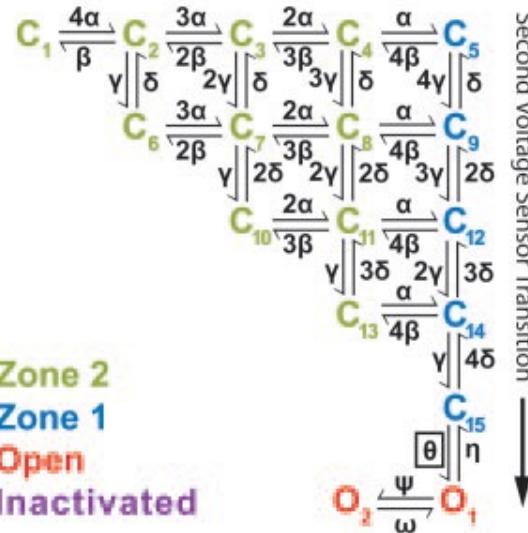
L5: Practical aspects of building channel and cell models



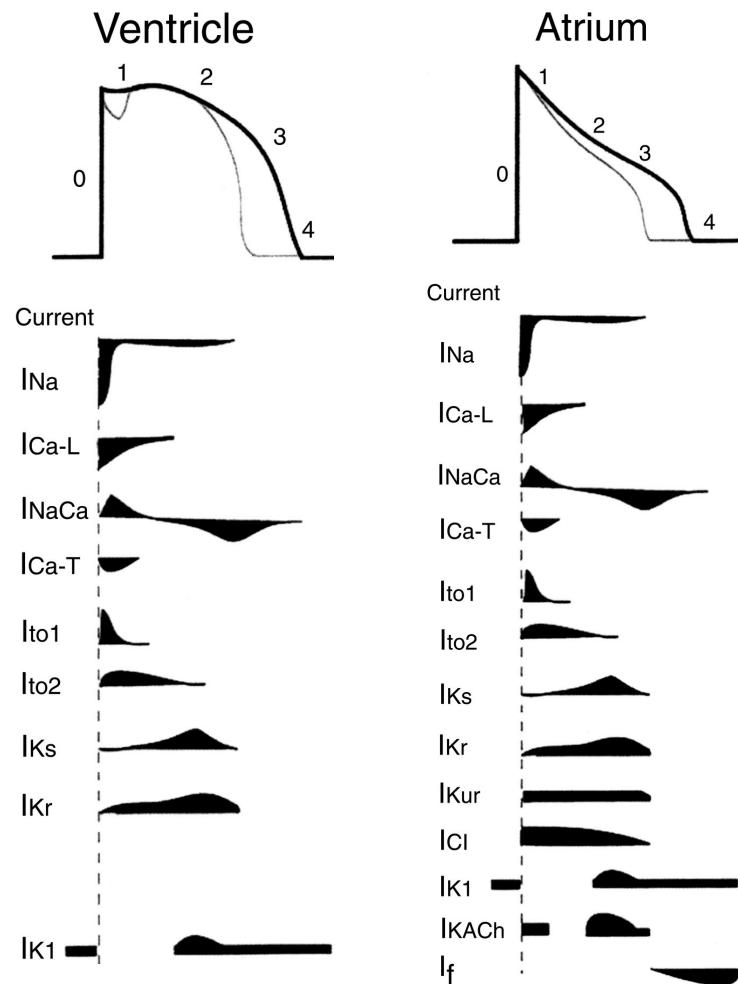
- Reconstructing excitability
 - Experimental data source
 - Single-channel recordings
 - Single-channel analysis
 - Whole-cell recordings
 - Major cardiac ion current models
 - Hodgkin-Huxley type models
 - Generalised Markov models

B. Model of I_{Ks}

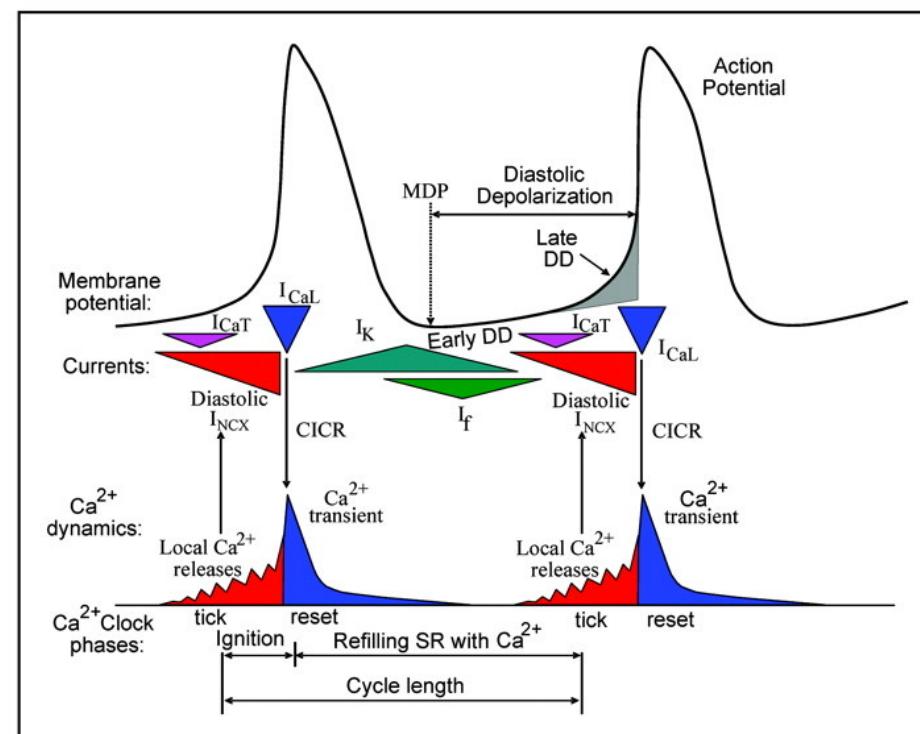
First Voltage Sensor Transition →



Major cardiac ion currents



Sinus Node



Sodium current (I_{Na}) models

Hodgkin and Huxley (1952)

$$\frac{dg_{Na}}{dt} = \bar{g}_{Na} m^3 h$$

$$\frac{dm}{dt} = \alpha(1 - m) - \beta m,$$

$$\frac{dh}{dt} = \gamma(1 - h) - \delta h,$$

Alpha subunit

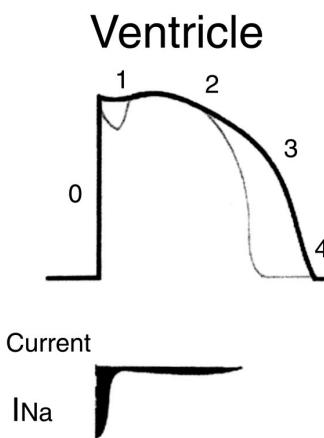
GENE: *SCN5a*

PROTEIN: *NaV1.5*

Beta subunit

GENE: *SCN1B*

PROTEIN: $\beta 1B$



Sodium current (I_{Na}) models

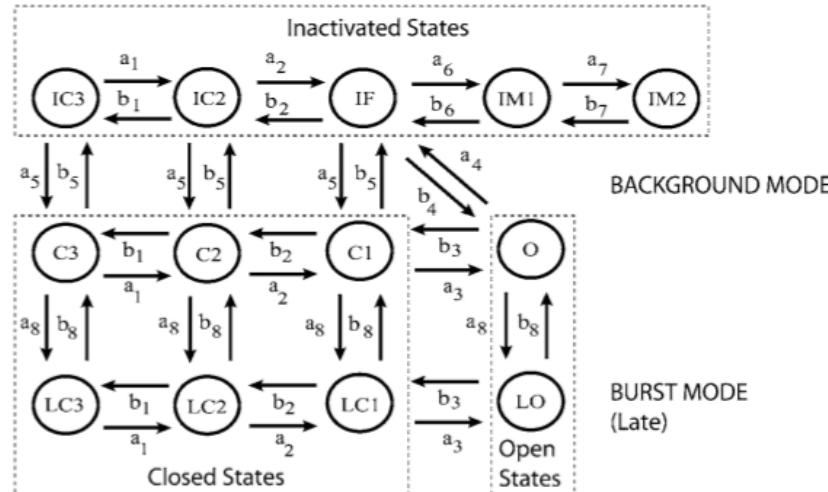
Hodgkin and Huxley (1952)

$$\frac{dg_{Na}}{dt} = \bar{g}_{Na} m^3 h$$

$$\frac{dm}{dt} = \alpha(1 - m) - \beta m,$$

$$\frac{dh}{dt} = \gamma(1 - h) - \delta h,$$

Clancy and Rudy (1999)



Alpha subunit

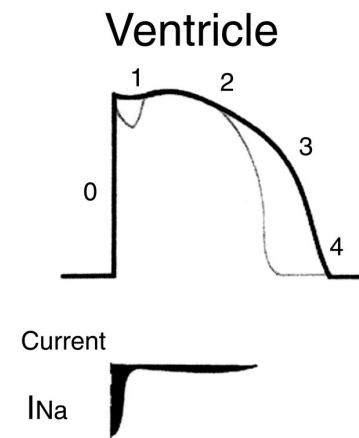
GENE: SCN5a

PROTEIN: NaV1.5

Beta subunit

GENE: SCN1B

PROTEIN: β1B

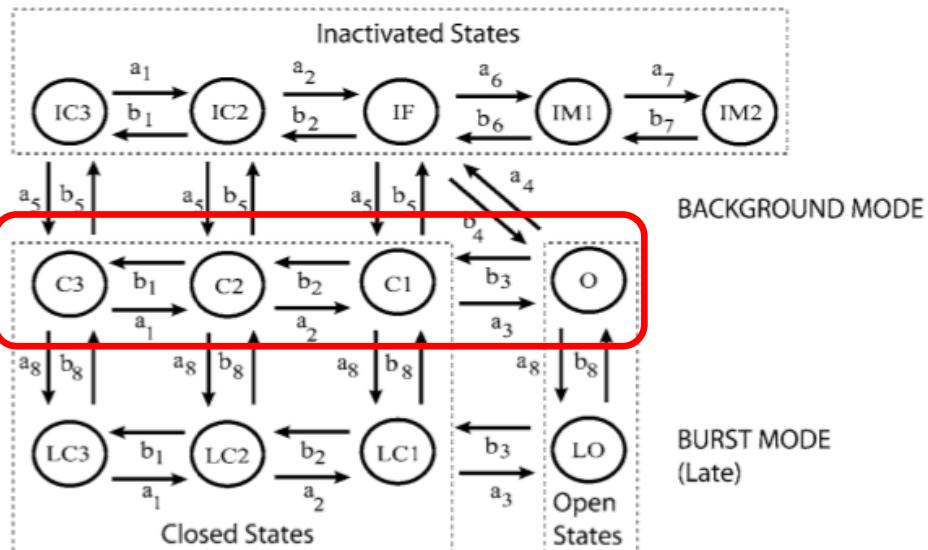


Sodium current (I_{Na}) models

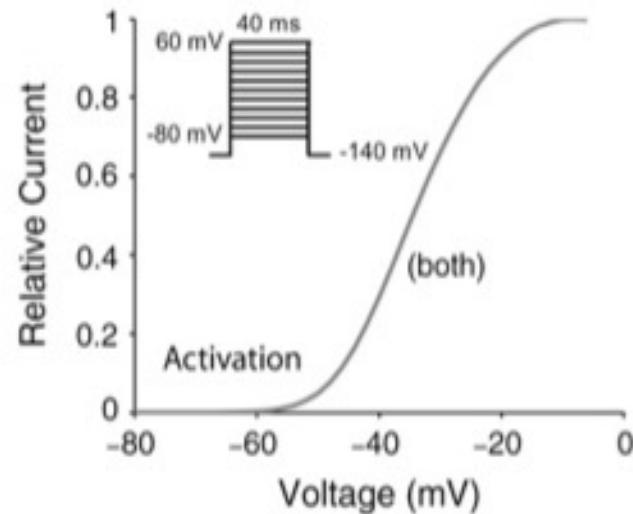
$$\frac{dg_{Na}}{dt} = \bar{g}_{Na} m^3 h$$

$$\frac{dm}{dt} = \alpha(1 - m) - \beta m,$$

$$\frac{dh}{dt} = \gamma(1 - h) - \delta h,$$



Steady-state activation

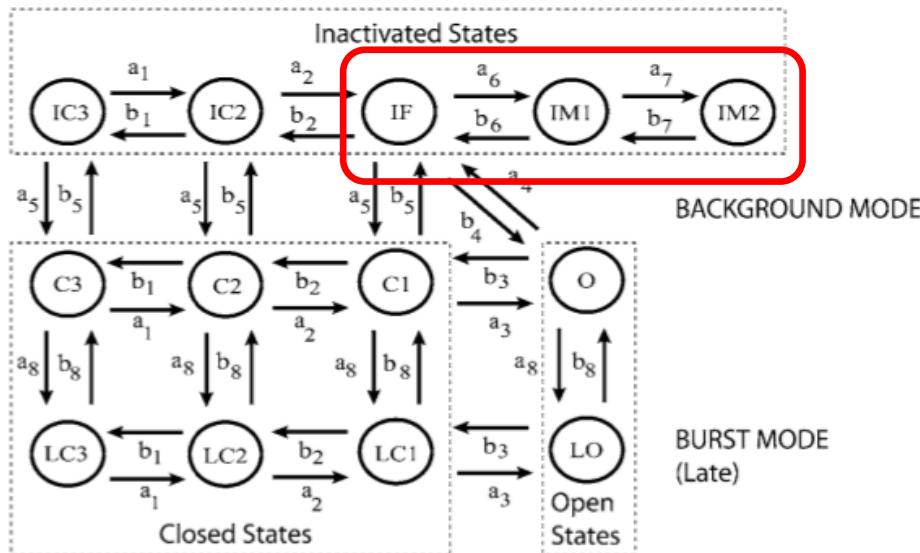


Sodium current (I_{Na}) models

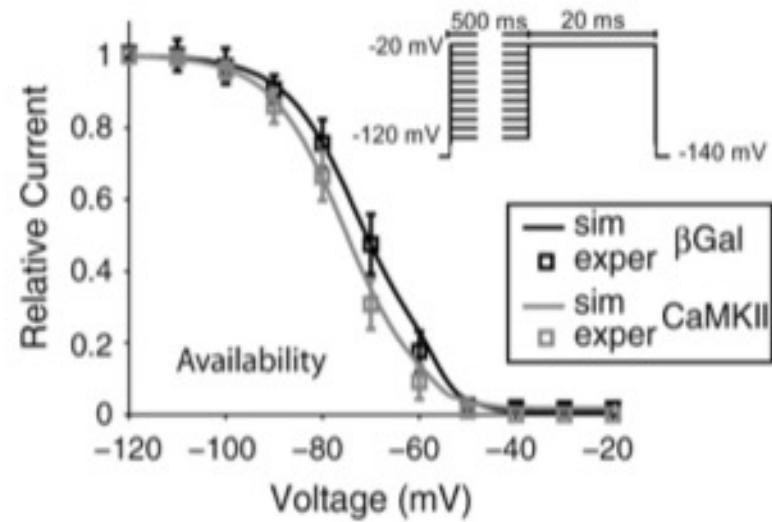
$$\frac{dg_{Na}}{dt} = \bar{g}_{Na} m^3 h$$

$$\frac{dm}{dt} = \alpha(1 - m) - \beta m,$$

$$\frac{dh}{dt} = \gamma(1 - h) - \delta h,$$



Steady-state inactivation

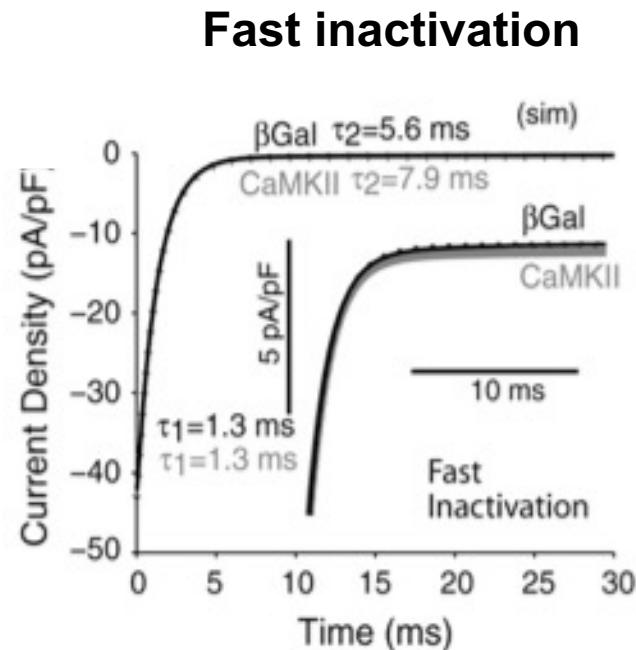
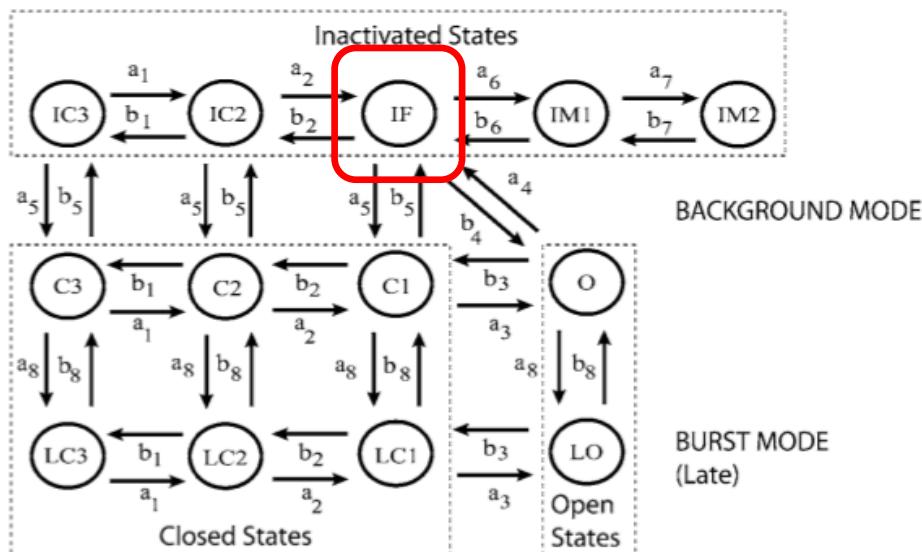


Sodium current (I_{Na}) models

$$\frac{dg_{Na}}{dt} = \bar{g}_{Na} m^3 h$$

$$\frac{dm}{dt} = \alpha(1 - m) - \beta m,$$

$$\frac{dh}{dt} = \gamma(1 - h) - \delta h,$$

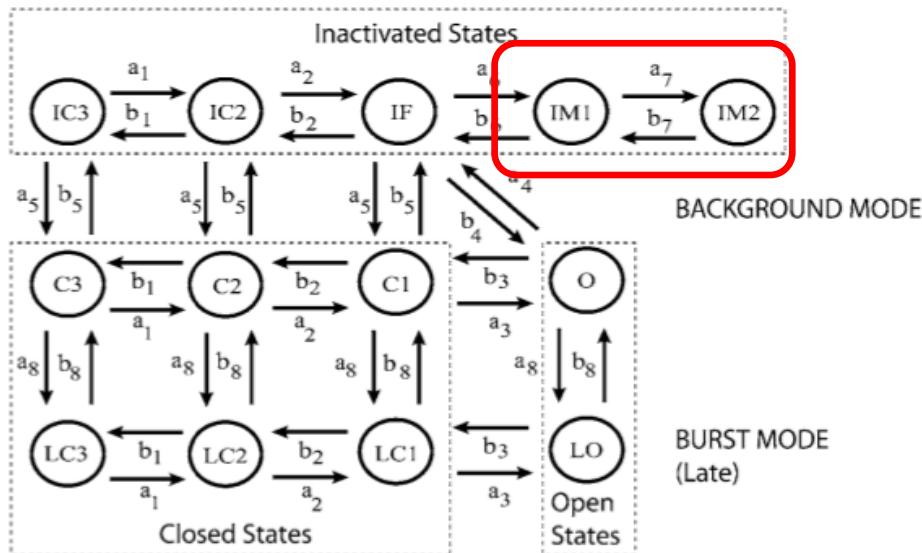


Sodium current (I_{Na}) models

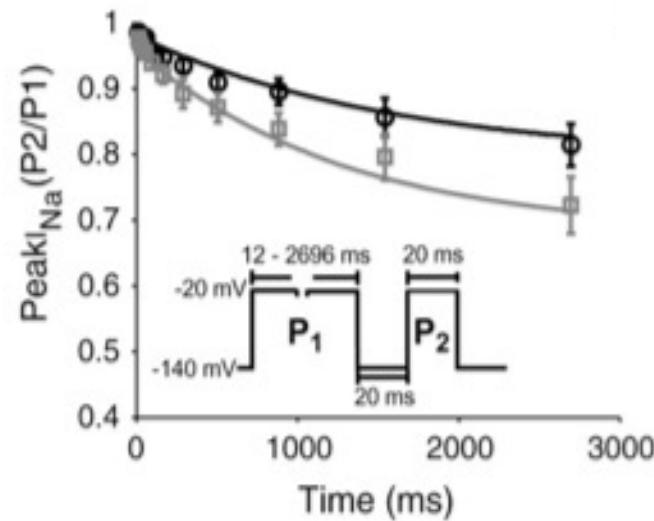
$$\frac{dg_{Na}}{dt} = \bar{g}_{Na} m^3 h$$

$$\frac{dm}{dt} = \alpha(1 - m) - \beta m,$$

$$\frac{dh}{dt} = \gamma(1 - h) - \delta h,$$



Intermediate inactivation

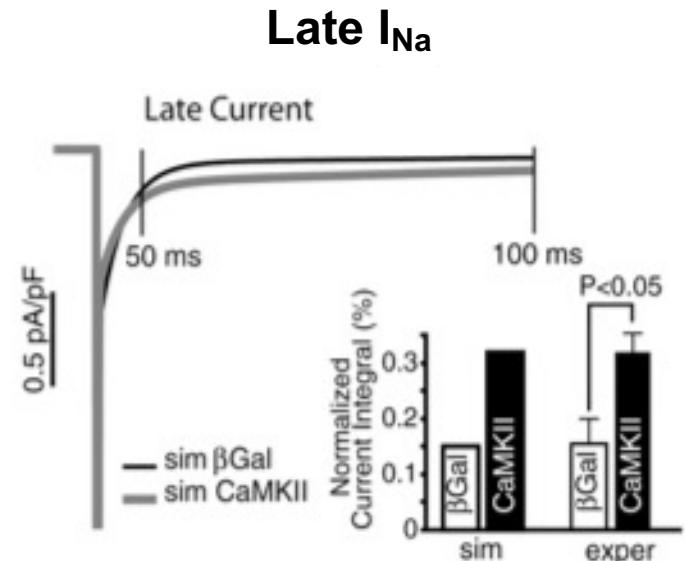
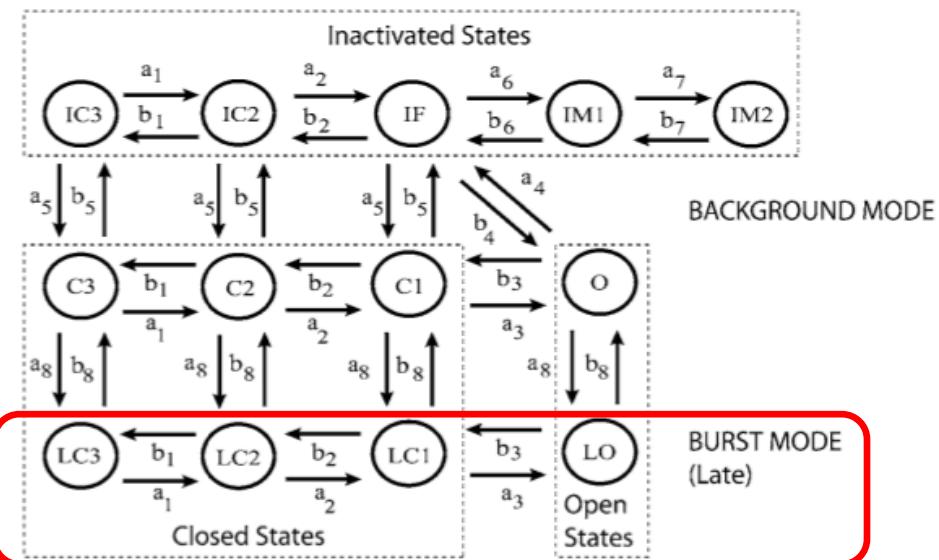


Sodium current (I_{Na}) models

$$\frac{dg_{Na}}{dt} = \bar{g}_{Na} m^3 h$$

$$\frac{dm}{dt} = \alpha(1 - m) - \beta m,$$

$$\frac{dh}{dt} = \gamma(1 - h) - \delta h,$$



L-type calcium current (I_{CaL}) models

Alpha subunit

GENE: CACNA1C

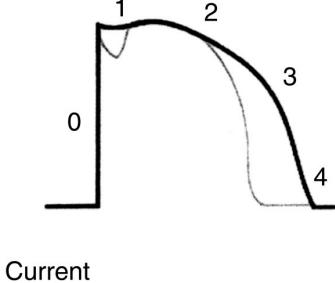
PROTEIN: CaV1.2

Beta subunit

GENE: CACN2B

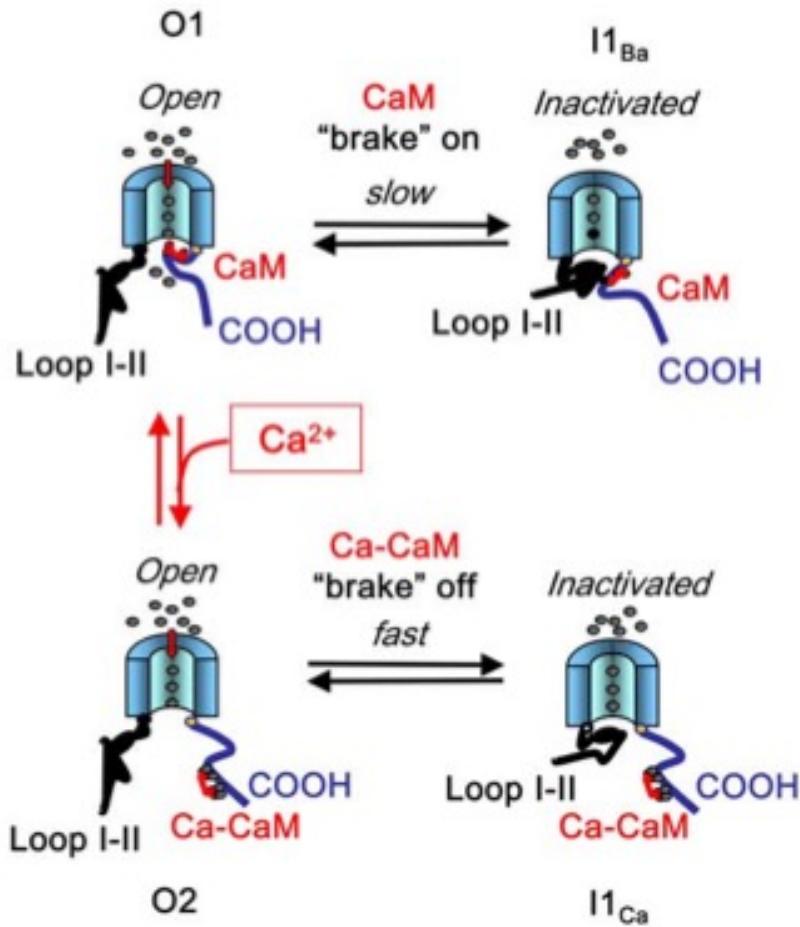
PROTEIN: $\beta 2A$

Ventricle



L-type calcium current (I_{CaL}) models

Mahajan (2008)



Alpha subunit

GENE: CACNA1C

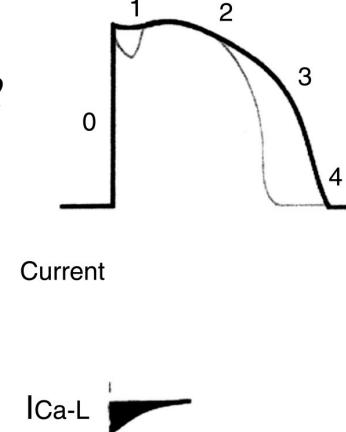
PROTEIN: *CaV1.2*

Beta subunit

GENE: CACN2B

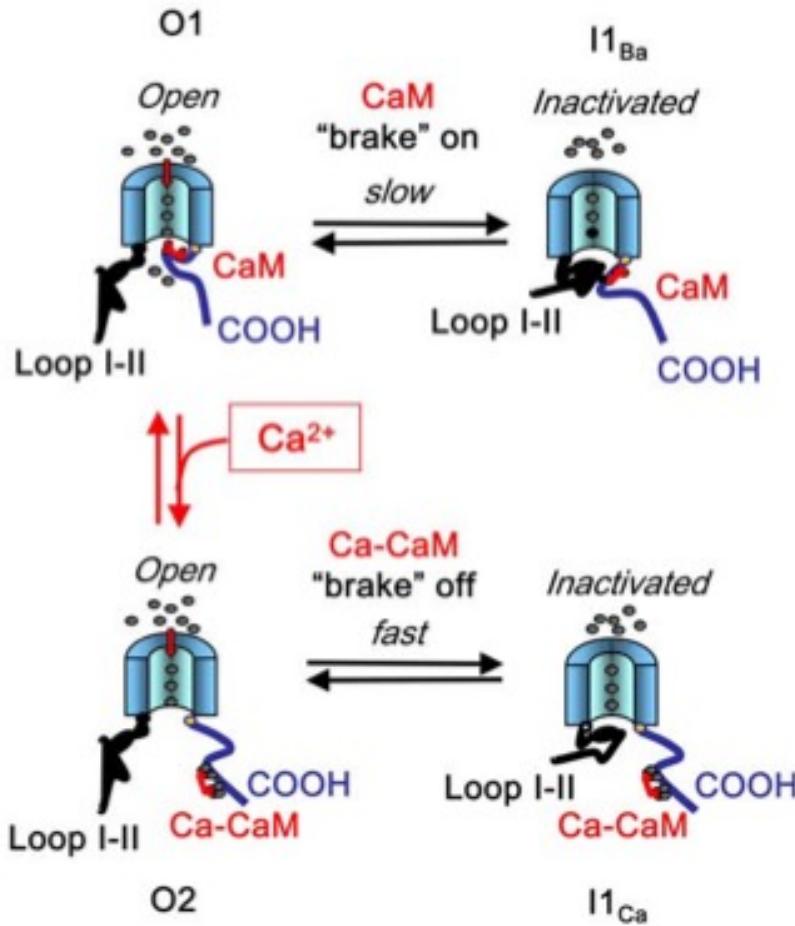
PROTEIN: $\beta 2A$

Ventricle



L-type calcium current (I_{CaL}) models

Mahajan (2008)



Alpha subunit

GENE: CACNA1C

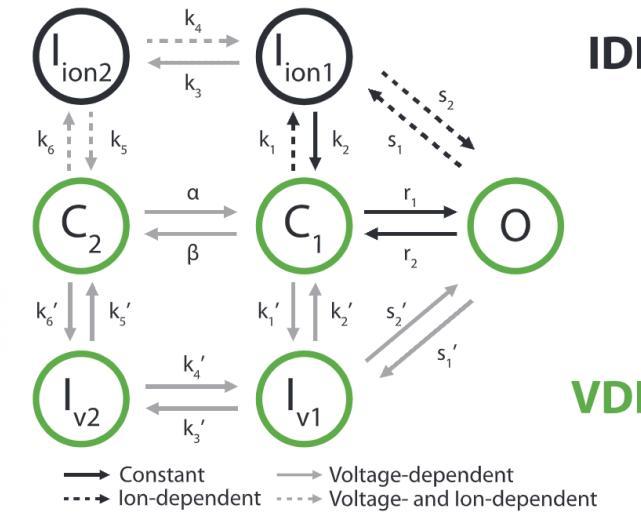
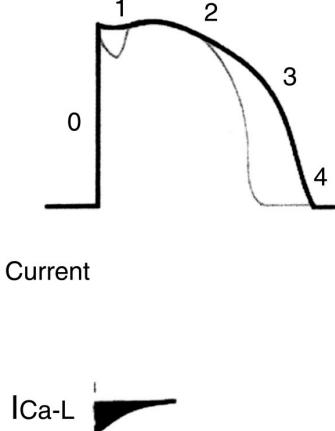
PROTEIN: *CaV1.2*

Beta subunit

GENE: CACN2B

PROTEIN: $\beta 2A$

Ventricle



L-type calcium current (I_{CaL}) models

Hodgkin Huxley (Shannon 2004)

$$\bar{I}_s = P_s z_s^2 V \frac{F^2}{RT} \frac{\gamma_{Si}[S]_{SL} \exp\left(\frac{z_s VF}{RT}\right) - \gamma_{So}[S]_o}{\exp\left(\frac{z_s VF}{RT}\right) - 1}$$

$$IP_s = d(f)f_{Ca}Q_{CaL}\bar{I}_s$$

Alpha subunit

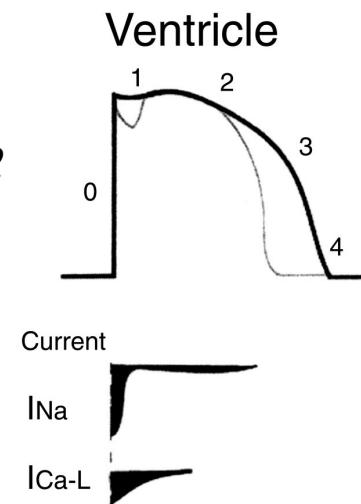
GENE: CACNA1C

PROTEIN: CaV1.2

Beta subunit

GENE: CACN2B

PROTEIN: $\beta 2A$



L-type calcium current (I_{CaL}) models

Hodgkin Huxley (Shannon 2004)

$$\bar{I}_S = P_S z_s^2 V \frac{F^2}{RT} \frac{\gamma_{Si}[S]_{SL} \exp\left(\frac{z_s VF}{RT}\right) - \gamma_{So}[S]_o}{\exp\left(\frac{z_s VF}{RT}\right) - 1}$$

$$IP_S = d(f) f_{Ca} Q_{CaL} \bar{I}_S$$

$$d_\infty = 1 / (1 + \exp(-(V + 14.5)/6.0))$$

$$f_\infty = \frac{1}{1 + \exp\left(\frac{(V + 35.06)}{3.6}\right)} + \frac{0.6}{1 + \exp\left(\frac{(50 - V)}{20}\right)}$$

$$f_{Ca} = 1 - f_{CaB}$$

$$\frac{df_{CaB}}{dt} = 1.7 [Ca]_C (1 - f_{CaB}) - 11.9 f_{CaB}$$

Alpha subunit

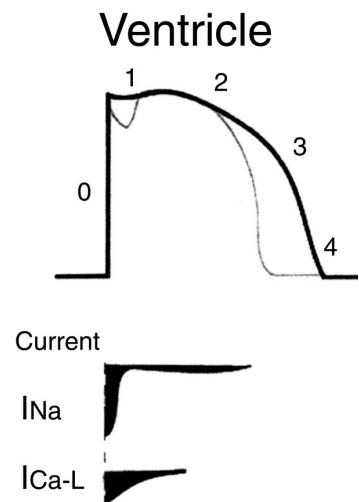
GENE: CACNA1C

PROTEIN: CaV1.2

Beta subunit

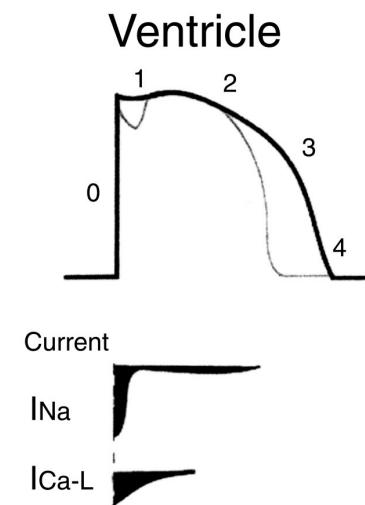
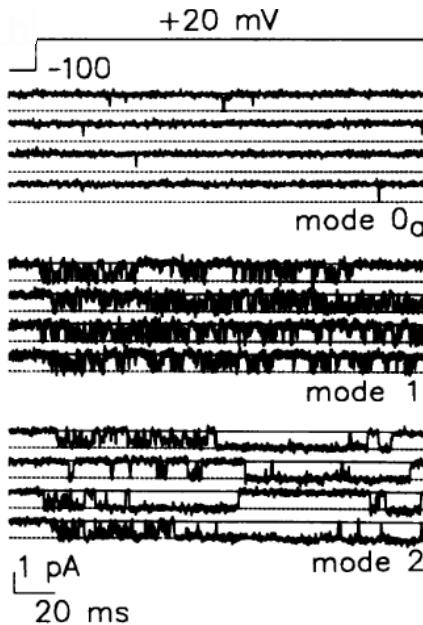
GENE: CACN2B

PROTEIN: $\beta 2A$



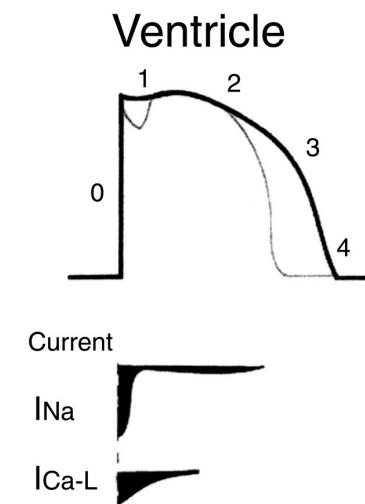
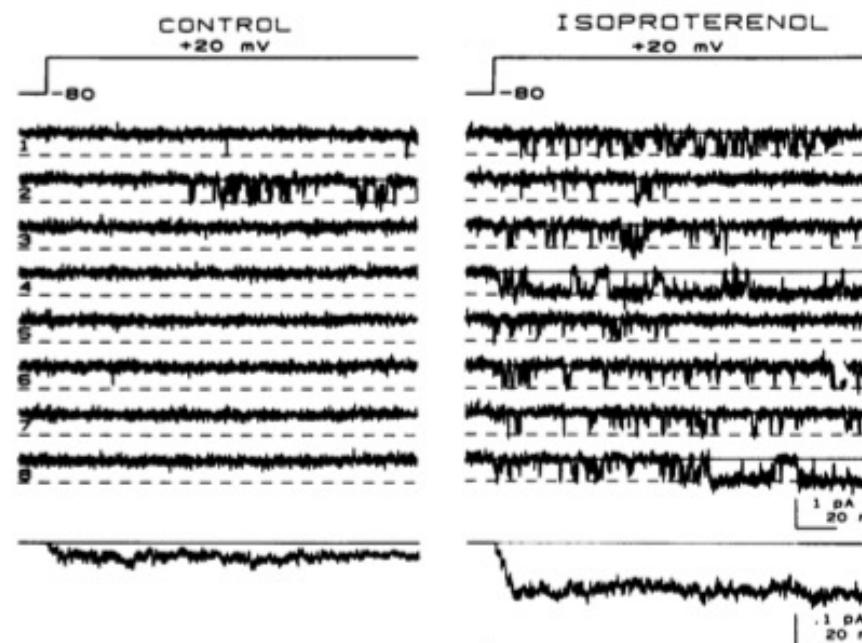
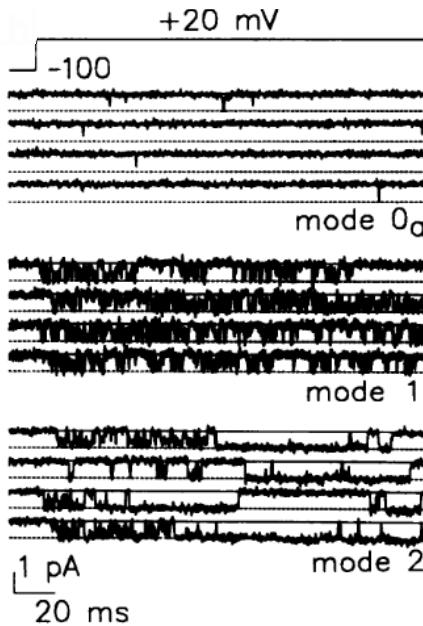
L-type calcium current (I_{CaL}) models

Modal gating is key property for I_{CaL} regulation



L-type calcium current (I_{CaL}) models

Modal gating is key property for I_{CaL} regulation



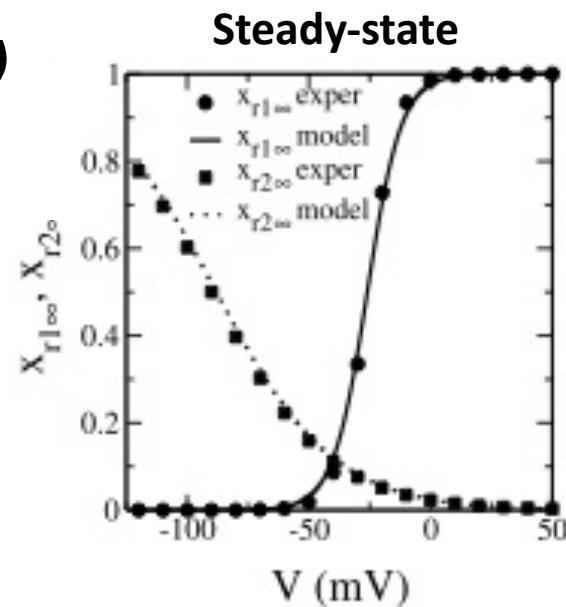
Rapidly activating delayed rectifier (I_{Kr}) models

Shannon et al. (2004)

$$I_{Kr} = \bar{G}_{IKr} X_r R_r (V - E_K)$$

$$\bar{G}_{IKr} = 0.03 \left(\frac{[K]_o}{5.4} \right)^{0.5}$$

Romero et al. (2015)



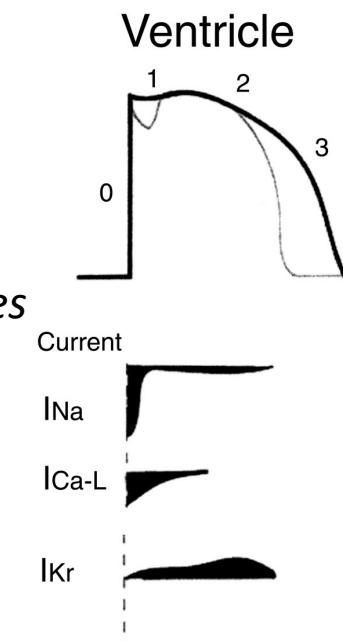
Alpha subunit

GENE: KCNH2

PROTEIN: HERG

Beta subunit

Multiple candidates



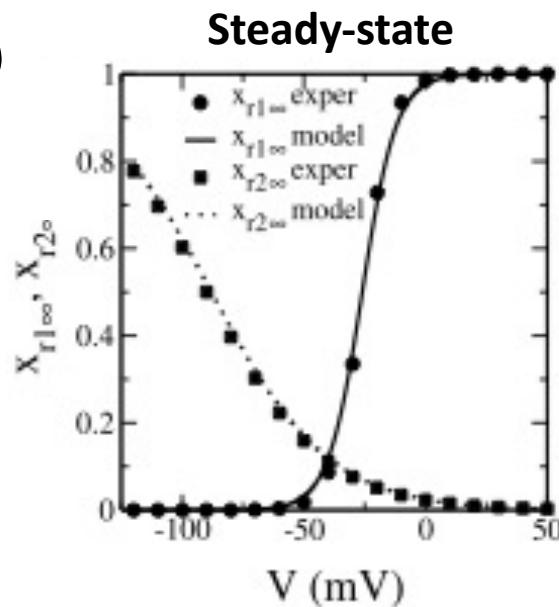
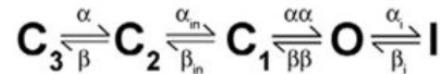
Rapidly activating delayed rectifier (I_{Kr}) models

Shannon et al. (2004)

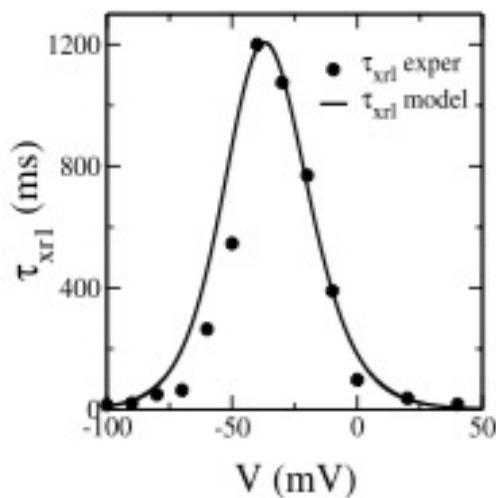
$$I_{Kr} = \bar{G}_{IKr} X_r R_r (V - E_K)$$

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Romero et al. (2015)



Slow activation



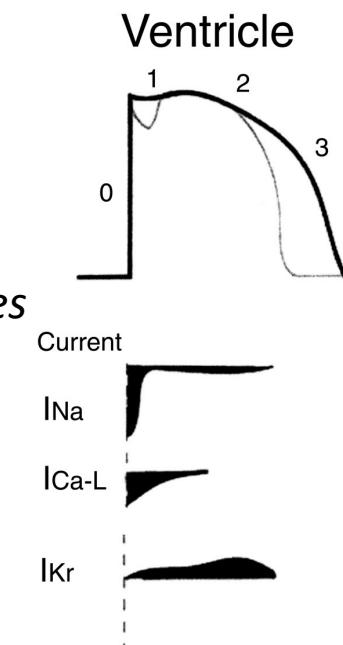
Alpha subunit

GENE: KCNH2

PROTEIN: HERG

Beta subunit

Multiple candidates



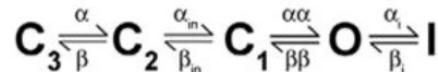
Rapidly activating delayed rectifier (I_{Kr}) models

Shannon et al. (2004)

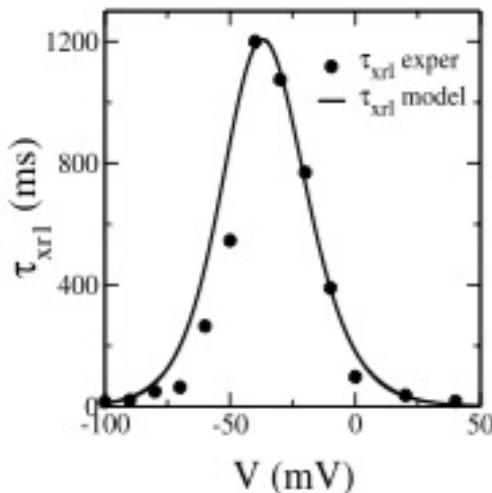
$$I_{Kr} = \bar{G}_{IKr} X_r R_r (V - E_K)$$

$$\bar{G}_{IKr} = 0.03 \left(\frac{[K]_o}{5.4} \right)^{0.5}$$

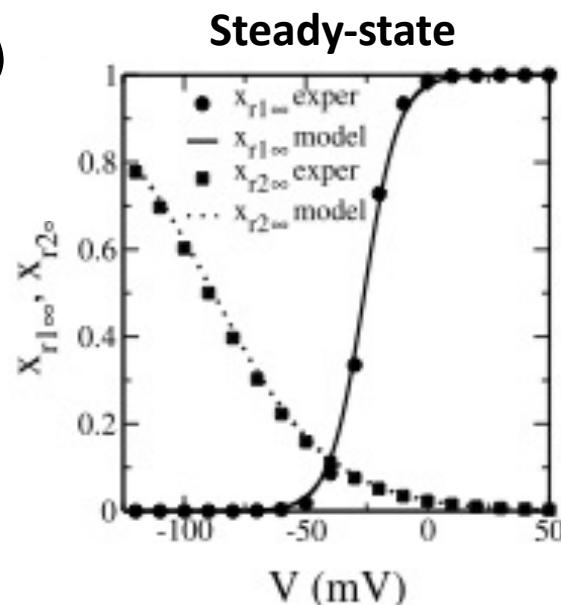
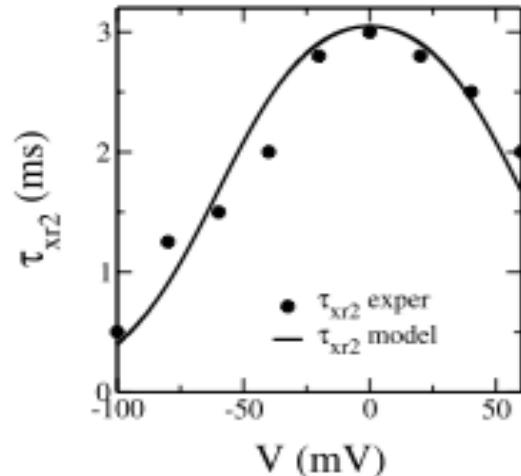
Romero et al. (2015)



Slow activation



Very fast inactivation



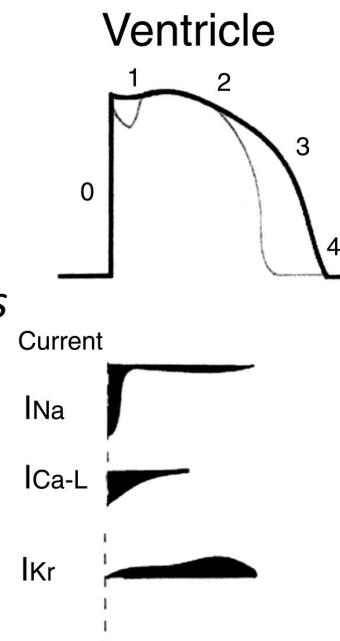
Alpha subunit

GENE: KCNH2

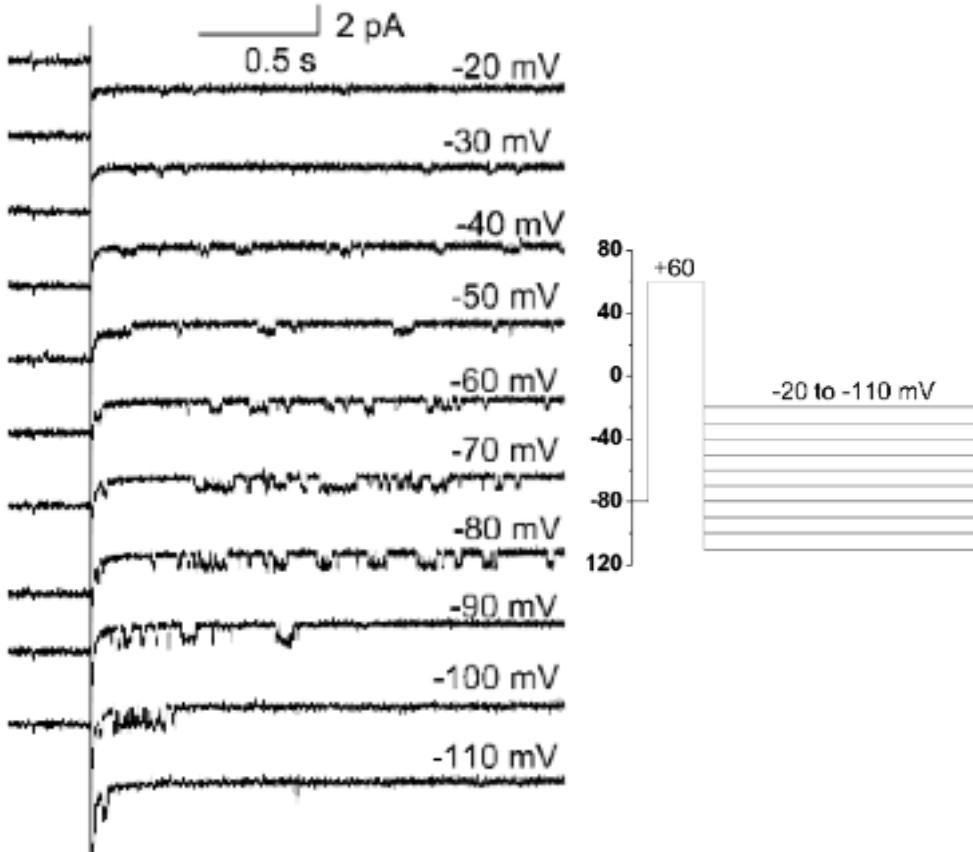
PROTEIN: HERG

Beta subunit

Multiple candidates



Rapidly activating delayed rectifier (I_{Kr}) models



Alpha subunit

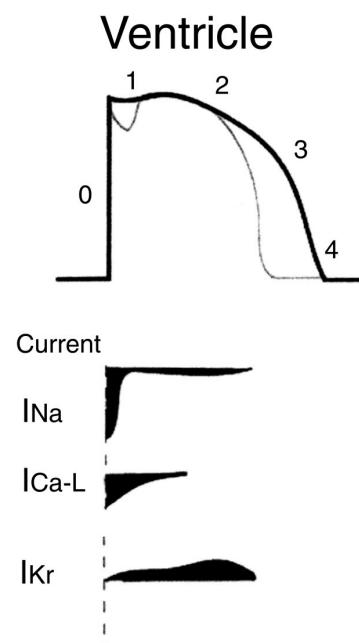
GENE: KCNH2

PROTEIN: HERG

Beta subunit

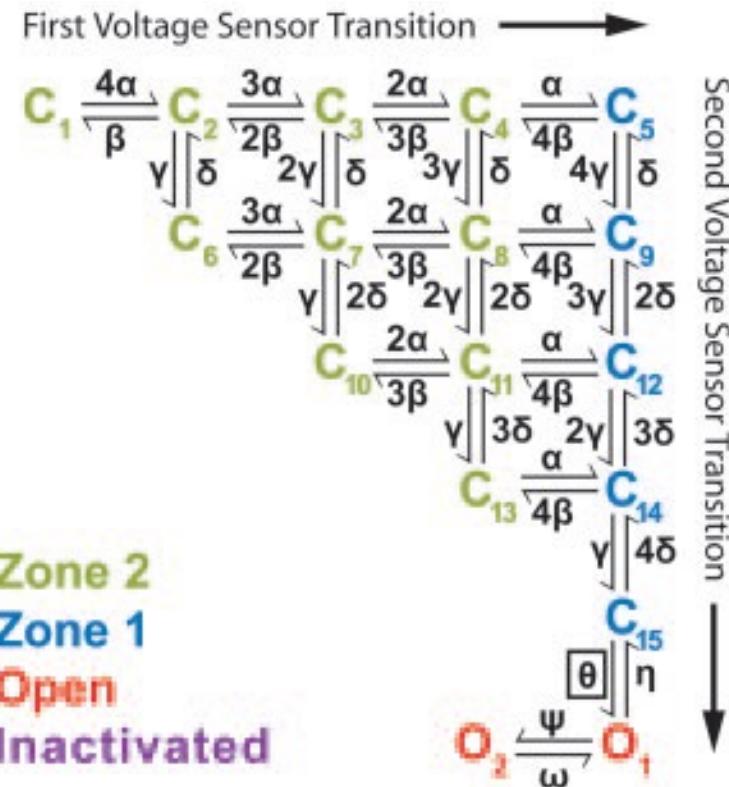
Unestablished

Multiple candidates



Slowly activating delayed rectifier (I_{Ks}) models

Silva et al. (2005)



Alpha subunit

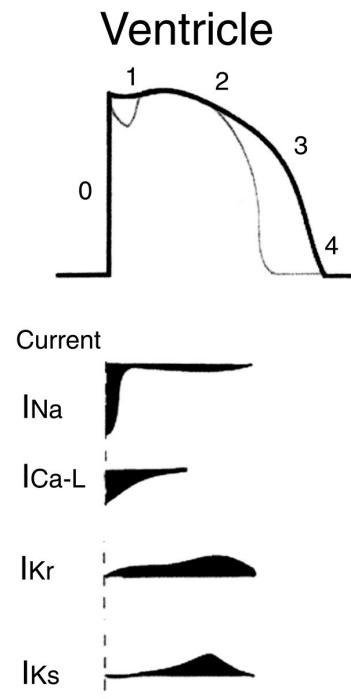
GENE: KCNQ1

PROTEIN: Kv7.1

Beta subunit

GENE: KCNE1

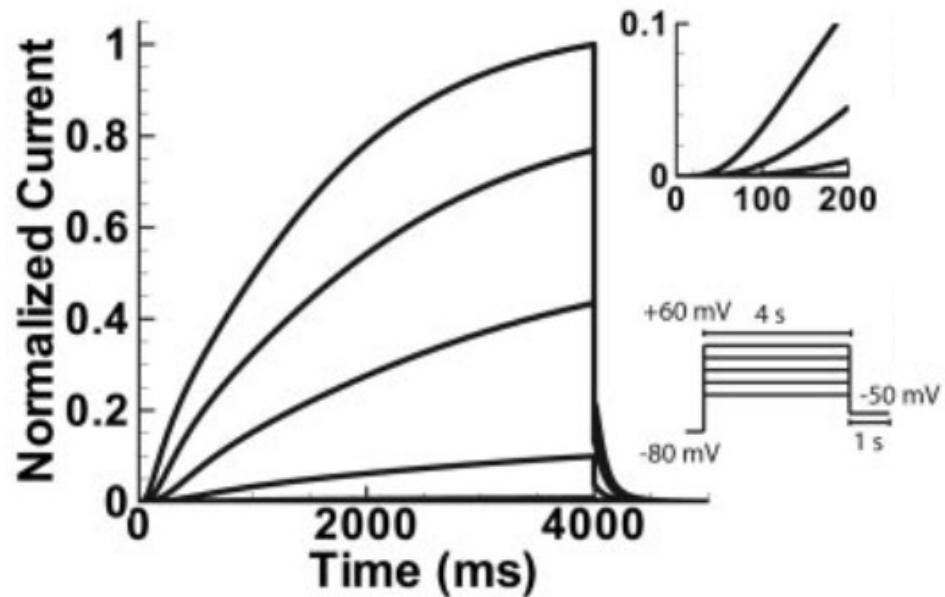
PROTEIN: minK



Slowly activating delayed rectifier (I_{Ks}) models

Silva et al. (2005)

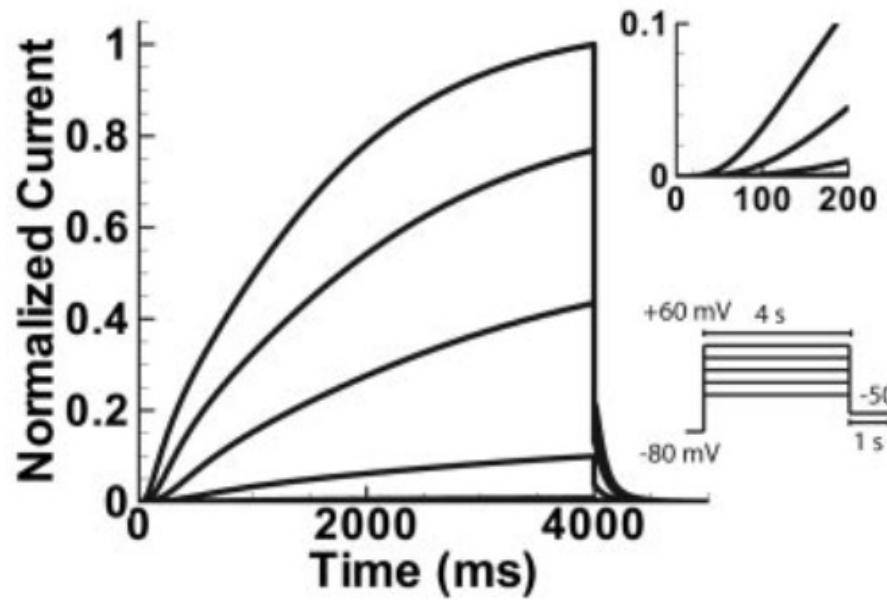
Human I_{Ks} I-V Relationship



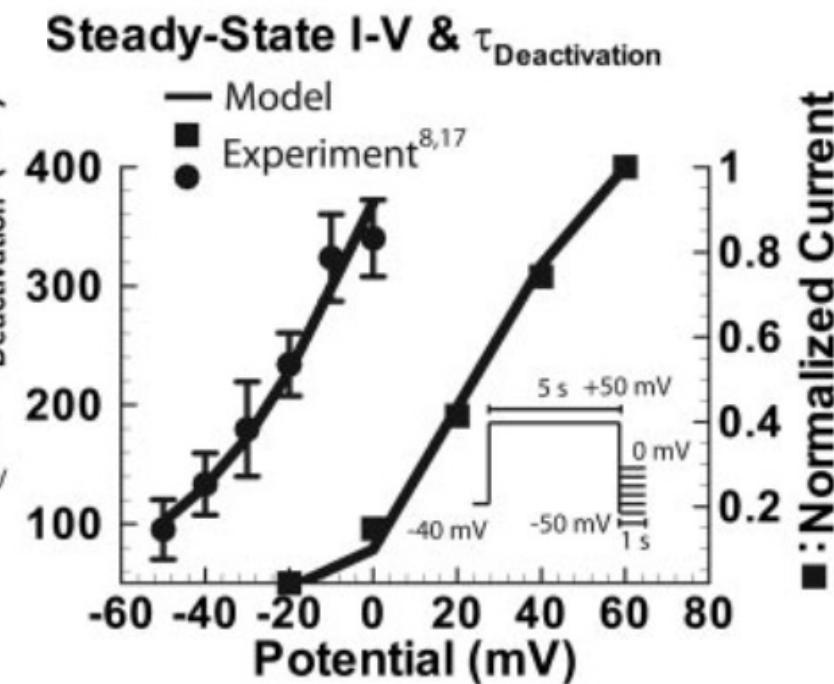
Slowly activating delayed rectifier (I_{Ks}) models

Silva et al. (2005)

Human I_{Ks} I-V Relationship

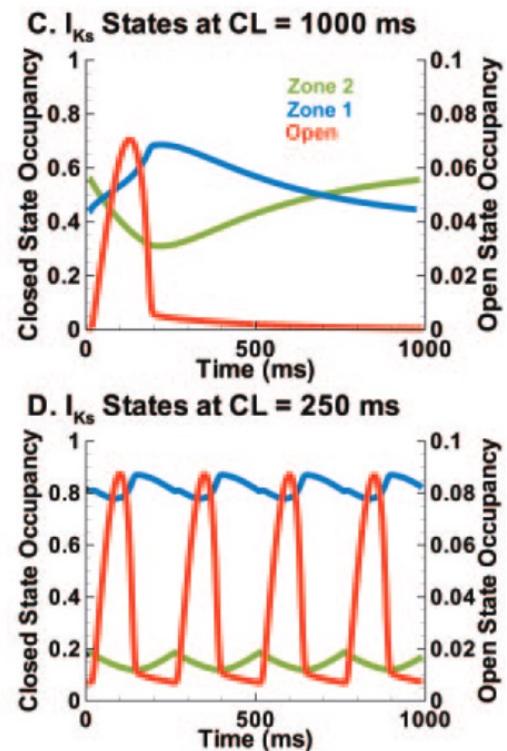
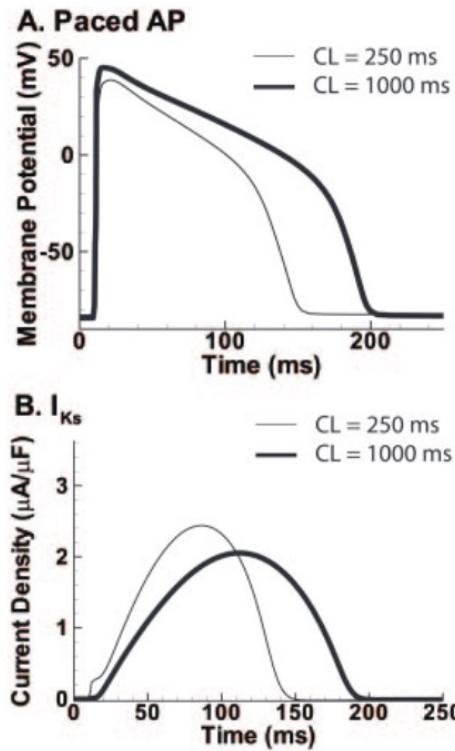


Steady-State I-V & $\tau_{\text{Deactivation}}$



Slowly activating delayed rectifier (I_{Ks}) models

Silva et al. (2005)



Alpha subunit

GENE: KCNQ1

PROTEIN: Kv7.1

Beta subunit

GENE: KCNE1

PROTEIN: minK

