COMP 331 Note

Week1 Lecture Note

Tools: Gurobi Optimizer

Course Intro Problem 1: Mortimer Middleman

Problem Description:

Motimer middleman is a diamond salesman, who is living depands sale diamonds, for **$900/carat** .If they run out of diamonds, they travel to Antwerp and buy a certain carats of diamonds. They will pay **$700/carat** to buy new diamonds. And whenever they place an order in Antwerp, they have to order **at least 100 carats** diamonds. And then they go back, and they have restocked their supply and restart selling diamonds. The cost of traveling to Antwerp is **$2000**(money cost) for **1 week**(time cost). The Middleman could sales **55 carats per week** on average. We want to figure out which point Mortimer should place a new order(when the stocks touch the threshold). Besides, they have to pay **$ 3.50/carat each week** for holding diamonds in their warehouse or banks.

Decisions:

1. Reorder point --At which level(thersolds) of stock does mortimer go to Antwerp and make a place in new order.
2. Reoreder quantity --How much diamonds should they reorder.

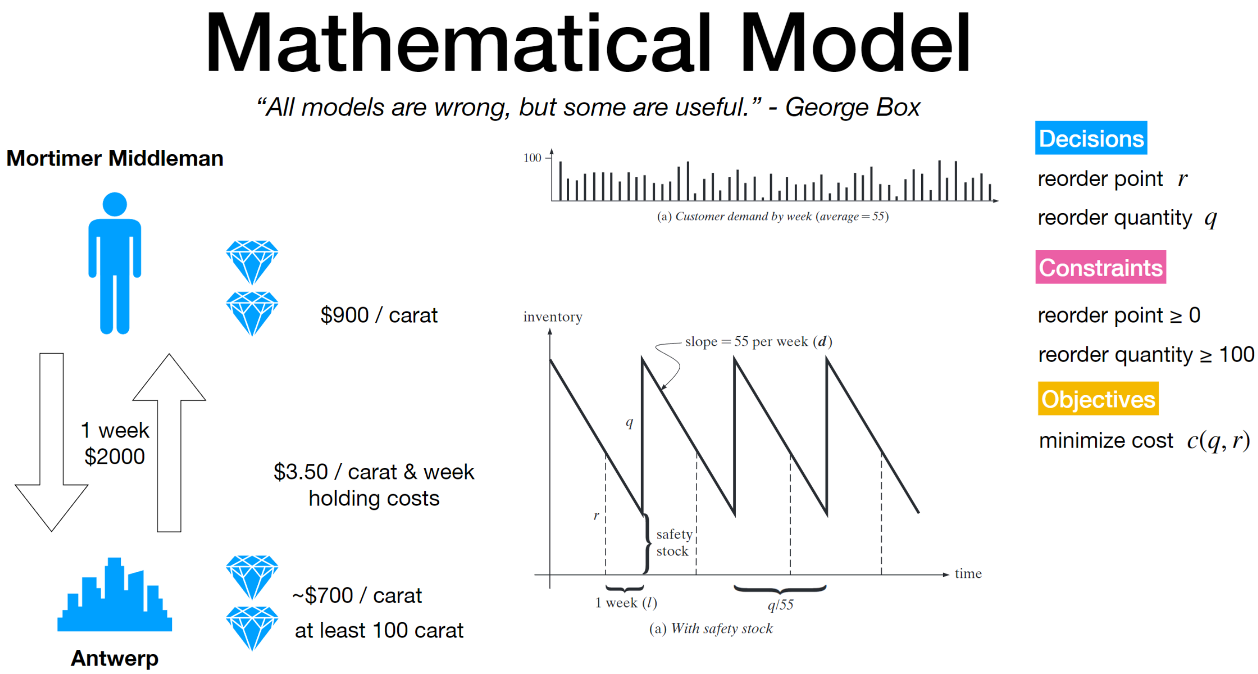
Constraints:

1. Reorder point $\geq$ 0
2. Reorder Quantity $\geq$ 100

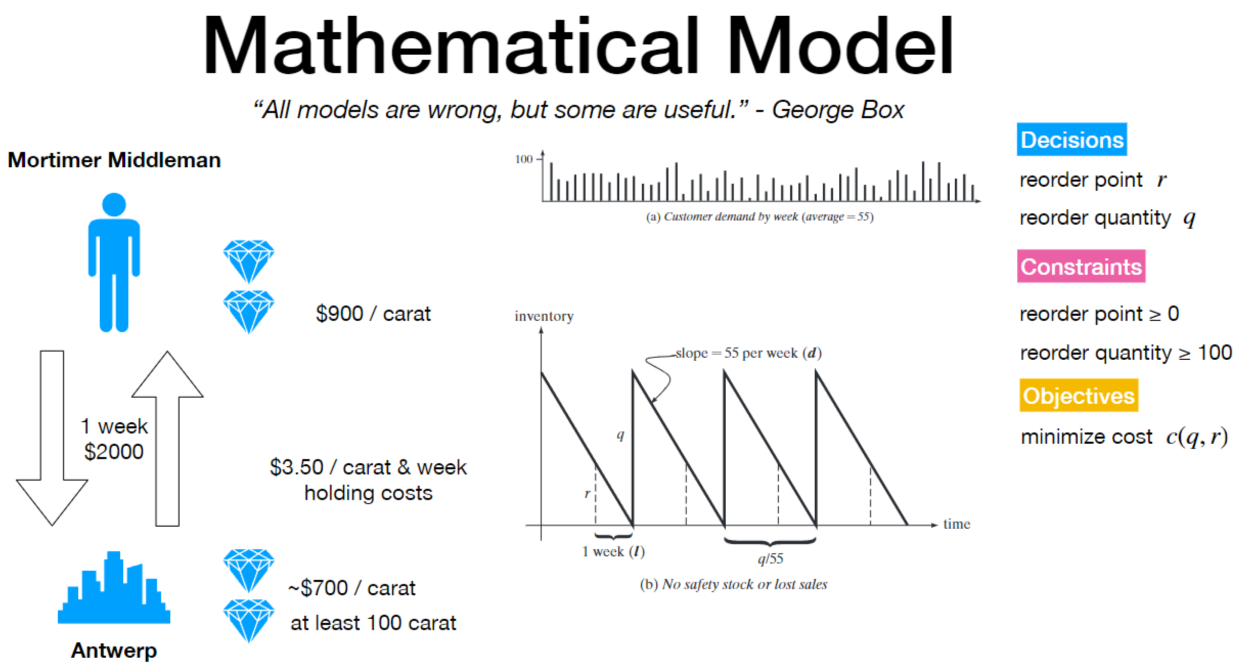
Objectives:

$minimize\ cost\ c(q, r)$

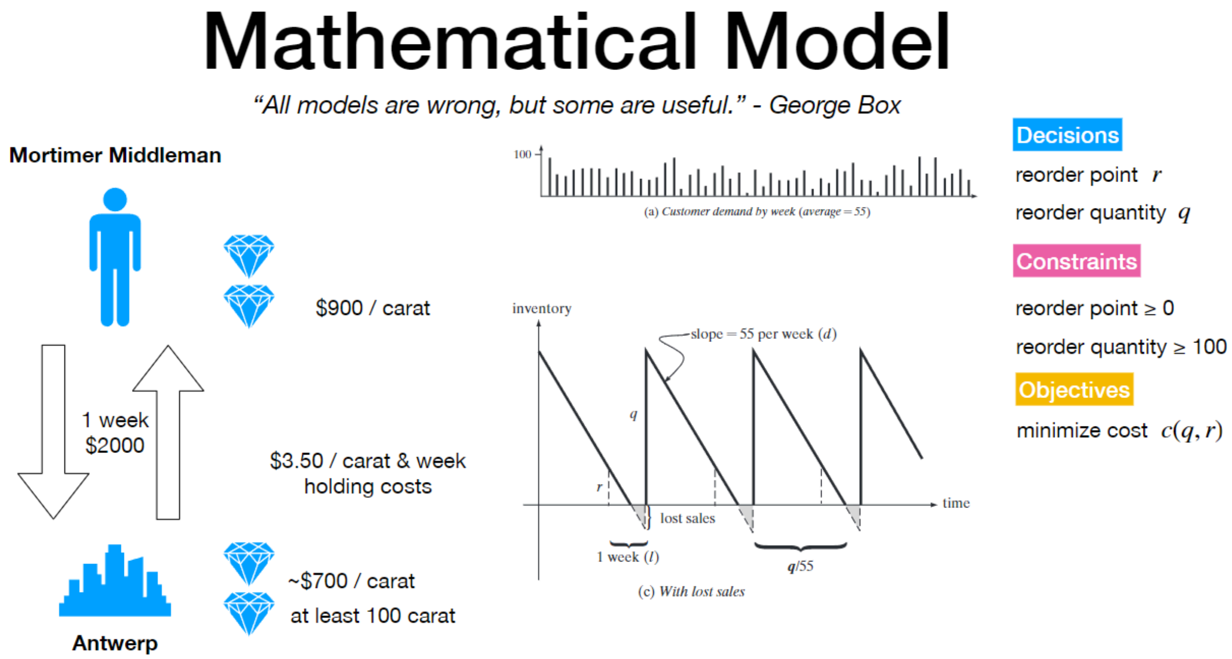
Some diagram to illustrate the mathematical model:



[Fig.1] With Safety Stock



[Fig.2] No Safety Stock & Loss Sales



[Fig.3] With Loss Sales

$Closed - Form\ Solution\ for\ the\ General\ Case$

$r := reorder\ point$
$q := reorder\ quantity$
$d := weekly\ demand$
$f := cost\ of\ replenishment$
$h := cost\ per\ carat\ per\ week\ for\ holding$
$l := lead\ time\ for\ replenishment$
$m := minimum\ order\ size$

$Sol:$

$firstly\ the\ no\ loss\ sales\ period\ is:$
$\frac{sales\ price - cost\ price}{storge\ cost} = \frac{900-700}{3.5} \approx 57.14$
$that\ means\ 1\ carat\ diamond\ will\ make\ loss\ at\ 57.14\ week\ after\ buy\ in.$

$The\ reorder\ point\ should\ be\ 55,\ that\ means\ when\ the\ sales\ man\ get\ back\ from\ Antwer$

By these analysis, we can got some values and constrains:

$r \geq 55$

$q \geq 100$

$The\ cost\ could\ be\ divided\ into\ 2\ part:\ Travel\ cost\ and\ Storage\ cost.$

$The\ First\ part\ is\ travel\ cost:$
$The\ times\ of\ travel\ is\ \frac{q}{55}$
$it's\ inverse\ could\ be\ considered\ as\ how\ many\ times\ should\ salesman\ travel.$
$So,\ we\ could\ use\ that\ to\ calculate\ travel\ cost\ per\ week.$
$Thus,\ the\ travel\ cost\ per\ week\ is:$
$c(r,q)_{Travel} = \frac{55}{q}2000$

$Now\ let's\ going\ to\ holding\ cost:$
$The\ safety\ stock\ is:\ r-55$
$Because\ the\ procedure\ is\ linear,\ thus,\ we\ could\ use\ a\ simple\ average\ method:$
$average = \frac{maxmum-minimum}{2}$
$Thus,\ we\ got\ the\ holding\ cost\ per\ week:$
$c(r,q)_{holding} = 3.5 * (\frac{(r-55+q)+(r-55)}{2})$

Organize the above formulas, we are able to constrate a optimization problem:

$minimise:\ c(r,q) = c(r,q)_{travel} + c(r,q)_{holding} = \frac{55}{q}2000 + 3.5 * (\frac{(r-55+q)+(r-55)}{2})$
$s.t.\ r^* = 55$
$\quad q \geq 100$

$substituting\ r^* = 55\ into\ objection,\ we\ got:$
$c(q) = \frac{55}{q}2000 + 3.5\frac{q}{2}$

$calculate\ the\ gradient:$
$\frac{\partial c(q)}{\partial q} = -55 * 2000 * q^2 + 3.5 * \frac{1}{2}$

$Let\ the\ gradient\ equals\ to\ 0,\ we\ got:$
$q^* \approx \pm 250$
$due\ to\ the\ constrains\ q^* = 250$
$c^* = 565$
$So,\ we\ got\ the\ optimal\ cost:\ \$565\ per\ week,\ and\ \$29380\ per\ year.$

Course Into Problem 2: Two Crude Petroleum

Problem Description:
Two Crude Petroleum runs a small refinery on the Texas coast. The refinery distills crude petroleum from **two sources, Saudi Arabia and Venezuela**, into three main products: **gasoline, jet fuel, and lubricants**.
The two crudes differ in chemical composition and thus yield different product mixes. **Each barrel of Saudi crude yields 0.3 barrel of gasoline, 0.4 barrel of jet fuel, and 0.2 barrel of lubricants**. On the other hand, **each barrel of Venezuelan crude yields 0.4 barrel of gasoline but only 0.2 barrel of jet fuel and 0.3 barrel of lubricants**. The remaining 10% of each barrel is lost to refining.
The crudes also differ in cost and availability. **Two Crude can purchase up to 9000 barrels per day from Saudi Arabia at $100 per barrel**. **Up to 6000 barrels per day of Venezuelan petroleum are also available at the lower cost of $75 per barrel** because of the shorter transportation distance.
Two Crude's contracts with independent distributors require it to **produce 2000 barrels per day of gasoline, 1500 barrels per day of jet fuel, and 500 barrels per day of lubricants**. How can these requirements be fulfilled most efficiently?

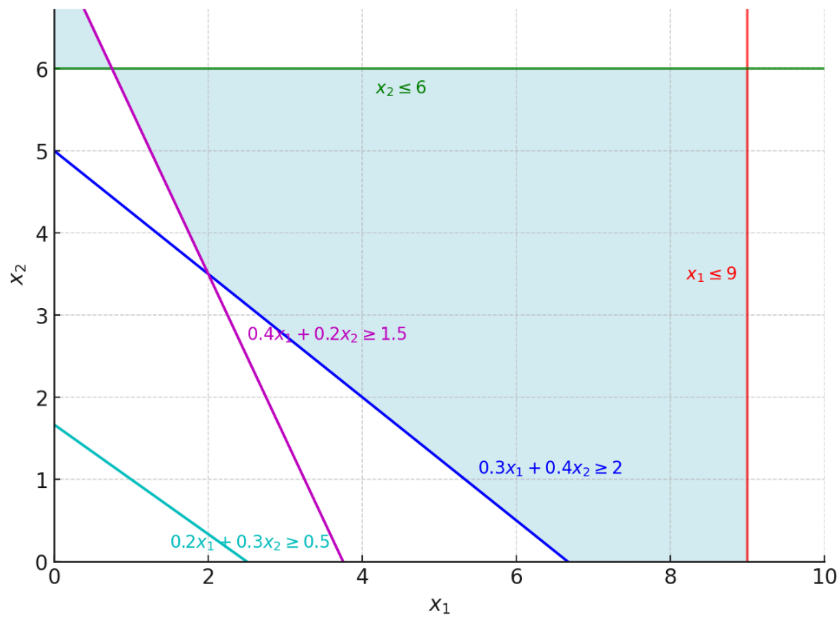
Decisions:
x_1 is barrel of Saudi crude(in 1000s).
x_2 is barrel of Venezuela crude(in 1000s).
$x_1,\ x_2\ \in \mathbb{R}$

	gas	jet fuel	lube	costs
Saudi:	0.3	0.4	0.2	100
Venezuela:	0.4	0.2	0.3	75
Demand:	2	1.5	0.5	

$$
\begin{aligned}
&Sol:\\
&min\ 100x_1 + 75x_2\\
&s.t.:\ 0 \le x_1 \le 9\\
&\qquad\ 0 \le x_2 \le 6\\
&\qquad\ 3x_1 + 4x_2 \ge 20
\end{aligned}
$$

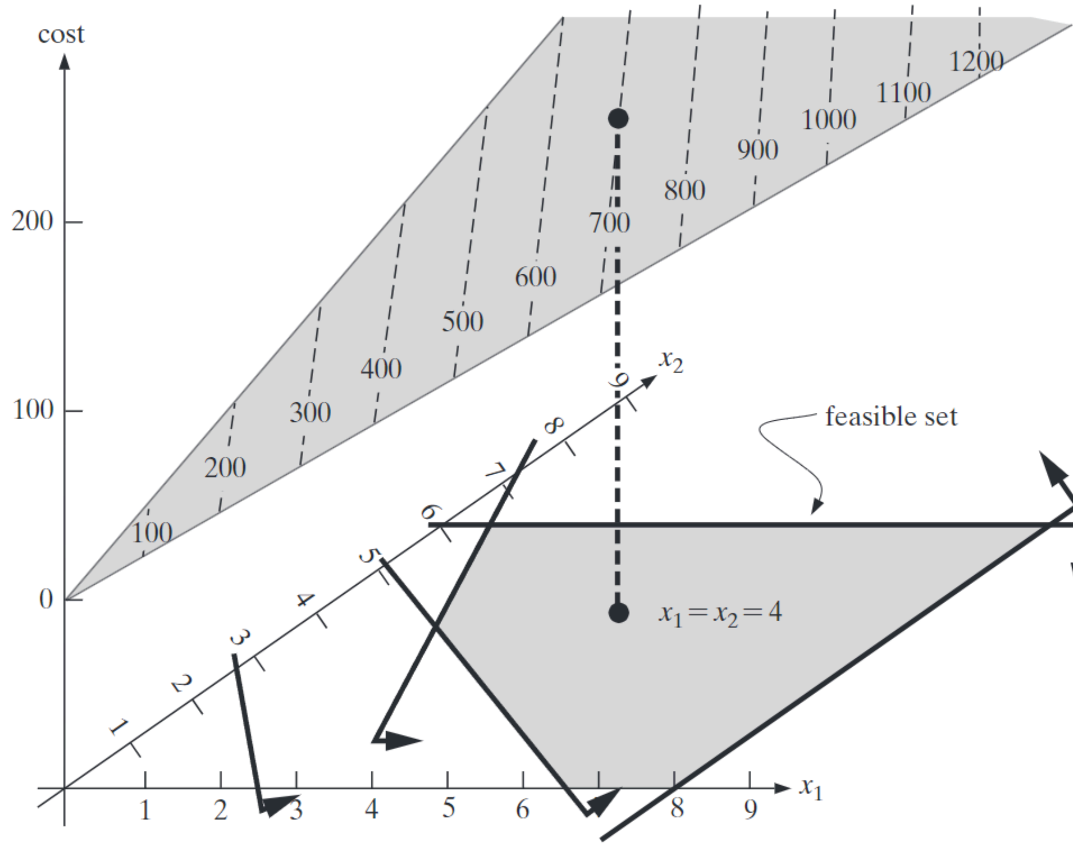
$$4x_1 + 2x_2 \geq 15$$
$$2x_1 + 3x_2 \geq 5$$

Feasible Set



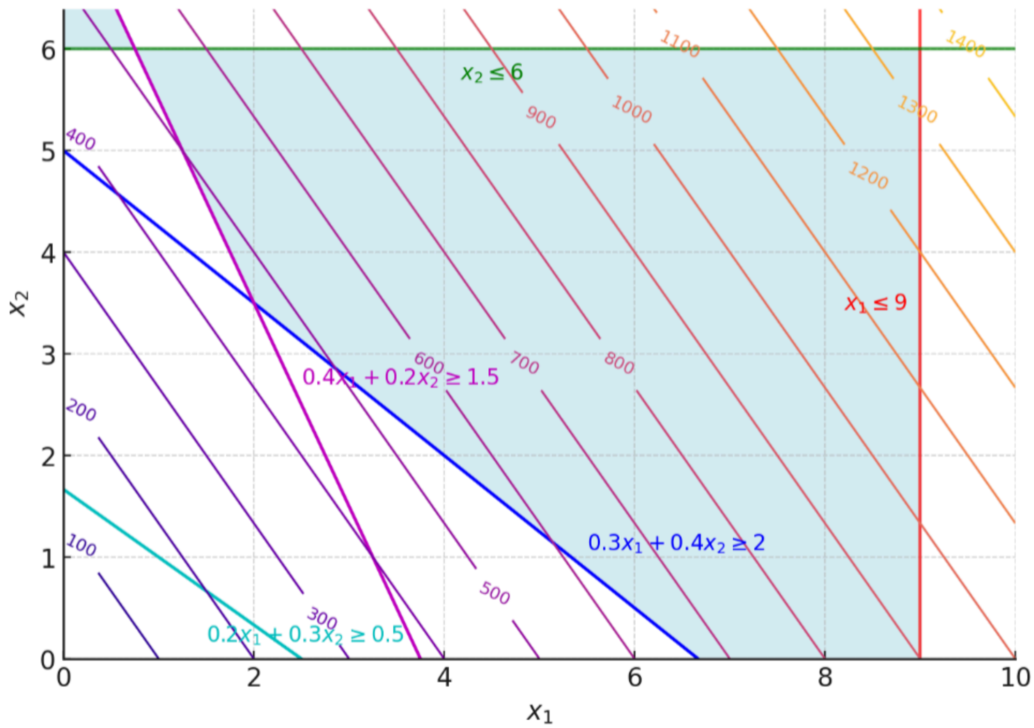
[Fig.4] 2 Dimensional Feasible Set

If we intro the cost axis to show that data, we got:



[Fig.5] 3 Dimensional Feasible Set

Then we can mapping z-axis on x-y plane:



[Fig.6] mapped data

This figure is something like equipotential surface in physics or contour line in geometry. Thus, we could get the answer easily, the target point is: The intersection point of the functions $0.3x_1 + 0.4x_2 = 2$ and $0.4x_1 + 0.2x_2 = 1.5$.