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Parameter estimation assignment

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let $(X_1, X_2, .. X_n)$ be a random sample of size n taken from a normal population with parameters mean $= \theta_1$ and variance $= \theta_2$. Find maximum likelihood estimation of these two parameters.

pdf of normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{1}{2}\left(\frac{x-\theta_1}{\sqrt{\theta_2}}\right)^2}$$

where $\theta_2 = \sigma^2$
$\theta_1 = \mu$.

acc. to question ~~θ_1, θ_2~~ $X_1, X_2 \cdots X_n$ are random values from the distribution which makes likelihood f^n as follows:

$$\alpha = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{1}{2}\frac{(x_i-\theta_1)^2}{\theta_2}}$$

Taking log on both sides

$$\log(\alpha) = \log\left(\left(\sqrt{2\pi\theta_2}\right)^{-n} \prod_{i=1}^{n} e^{-\frac{1}{2}\frac{(x_i-\theta_1)^2}{\theta_2}}\right)$$

$$\log(\alpha) = -\frac{n}{2}\log(2\pi\theta_2) + \left(\frac{-1}{2\theta_2}\right)\sum_{i=1}^{n}(x_i-\theta_1)^2$$

Differentiate with 'θ_1'

$$\frac{1}{\alpha}\frac{\partial \alpha}{\partial \theta_1} = \frac{-1}{2\theta_2}\sum_{i=1}^{n} 2(x-\theta_1)(-1)$$

$$\frac{1}{\alpha}\frac{\partial \alpha}{\partial \theta_1} = \frac{1}{2\theta_2}\sum_{i=1}^{n} 2(x_i-\theta_1).$$

Equating $\frac{\partial \alpha}{\partial \theta_1} = 0$.

$$\frac{\partial \alpha}{\partial \theta_1} = \frac{\alpha}{2\theta_2}\sum_{i=1}^{n} 2(x_i-\theta_1) = 0.$$

$$\alpha = 0 \quad \text{or} \quad \frac{1}{2\theta_2}\sum_{i=1}^{n} 2(x_i-\theta_1) = 0$$

$\alpha = 0$ (not possible)

$$\frac{1}{2\theta_2}\sum_{i=1}^{n} 2(x_i-\theta_1) = 0.$$

$$2\sum_{i=1}^{n} x_i = 2\sum_{i=1}^{n}\theta_1$$

$$n\theta_1 = \sum_{i=1}^{n} x_i.$$

$$\Rightarrow \theta_1 = \frac{\sum x_i}{n} \quad \Rightarrow \theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$\theta_1 =$ sample mean.

Differentiate w.r.t. θ_2.

$$\frac{\partial \alpha}{\partial \theta_2}\left(\frac{1}{\alpha}\right) = \frac{-n}{2}\frac{2\pi}{2\pi\theta_2} + \sum_{i=1}^{n}(x_i-\theta_1)^2\left(\frac{1}{2\theta_2^2}\right)$$

put $\frac{\partial \alpha}{\partial \theta_2} = 0$.

$$\frac{-n}{2\theta_2} + \sum_{i=1}^{n}(x_i-\theta_1)^2\frac{1}{\alpha\theta_2^2} = 0$$

$$\sum_{i=1}^{n}(x_i-\theta_1)^2 = n\theta_2.$$

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

$\theta_2 =$ sample variance.

2.) let $X_1, X_2, \ldots X_n$ be a random sample from $B(m,\theta)$ distribution where $\theta \in (0,1)$ is unknown and m is a known positive integer. Compute value of θ using MLE.

pmf of binomial distribution:

$$P(X=K) = {}^{m}C_{K}\,\theta^{K}(1-\theta)^{m-K}.$$

let $x_1, x_2, \ldots x_n$ be random sample from $B(m,\theta)$ distⁿ where for a x_i it represents a number of successes in i^{th} trial.

$$\alpha(\theta) = \prod_{i=1}^{n} {}^{m}C_{x_i}\,\theta^{x_i}(1-\theta)^{m-x_i}$$

Taking log on both sides:

$$\log(\alpha) = \log\left(\prod_{i=1}^{n} {}^{m}C_{x_i}\,\theta^{x_i}(1-\theta)^{m-x_i}\right)$$

$$\log(\alpha) = \sum_{i=1}^{n}\left(\log {}^{m}C_{x_i} + x_i\log\theta + (m-x_i)\log(1-\theta)\right)$$

differentiate w.r.t. θ & equate to 0.

$$\frac{1}{\alpha}\frac{d\alpha}{d\theta} = \frac{1}{\theta}\sum_{i=1}^{n} x_i - \frac{1}{1-\theta}\sum_{i=1}^{n}(m-x_i)$$

$$\frac{\partial \alpha}{\partial \theta} = 0.$$

$$\alpha\left(\frac{1}{\theta}\sum_{i=1}^{n} x_i - \frac{1}{1-\theta}\sum_{i=1}^{n} m-x_i\right) = 0.$$

$\alpha \neq 0$

$$\frac{1}{\theta}\sum_{i=1}^{n} x_i = \frac{1}{1-\theta}\sum_{i=1}^{n} m-x_i$$

$$(1-\theta)\sum_{i=1}^{n} x_i = \theta nm - \theta\sum_{i=1}^{n} x_i$$

$$\theta = \frac{\sum x_i}{nm} \quad \text{where } i=1,2,\ldots n.$$

$$\theta = \frac{1}{m}\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)$$

$$\theta = \frac{\text{sample mean}}{m}$$

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