Simulation and control of three degree of freedom robot based on Comau smart5 NJ165

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1st Section

Basic investigation of the robot and degrees of freedom

Today, industrial robots play an important role in performing complex operations at high speeds. This category of robots must have the ability to perform precise and sensitive operations continuously and repeatedly. However, robot manufacturers make controllers available to users so that they can improve its capabilities and computing power. On the other hand, the scientific community requires conducting numerous researches and improvements on systems in a short period of time. In this project, work will be done on the Comau Smart5 NJ165 robot, whose basic information is given in Figure 2: Basic information of the Comau Smart5 robot.

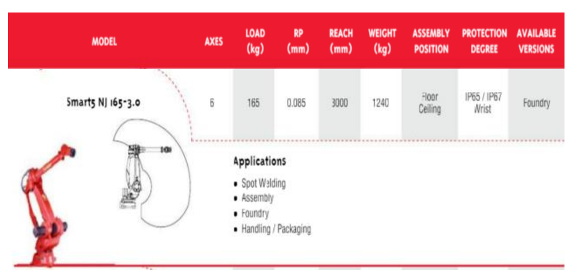


Figure 1 : Comau Smart5

Below is an image of the mentioned robot which has 6 degrees of freedom (DOF).

Robot specifications

```
Mass properties of Part0
    Configuration: Default
    Coordinate system: -- default --

Density = 7860.00 kilograms per cubic meter

Mass = 741.86 kilograms

Volume = 0.09 cubic meters

Surface area = 1.72 square meters

Center of mass: ( meters )
    X = 0.02
    Y = 0.13
    Z = 0.00

Principal axes of inertia and principal moments of inertia: ( kilograms * square n
Taken at the center of mass.
    Ix = ( 1.00,  0.01,  0.00)      Px = 24.46
    Iy = ( 0.00,  0.00, -1.00)      Py = 29.48
    Iz = (-0.01,  1.00,  0.00)      Pz = 43.14

Moments of inertia: ( kilograms * square meters )
Taken at the center of mass and aligned with the output coordinate system. (Usi
    Lxx = 24.47          Lxy = 0.16          Lxz = 0.00
    Lyx = 0.16           Lyy = 43.14         Lyz = 0.00
    Lzx = 0.00           Lzy = 0.00          Lzz = 29.48

Moments of inertia: ( kilograms * square meters )
Taken at the output coordinate system. (Using positive tensor notation.)
    Ixx = 37.46          Ixy = 2.16          Ixz = 0.00
    Iyx = 2.16           Iyy = 43.45         Iyz = 0.00
    Izx = 0.00           Izy = 0.00          Izz = 42.78
```

2nd Section

Denavit-Hartenberg parameters

In engineering sciences, Denavit-Hartenberg parameters or DH parameters include four parameters that are determined by a special law and by deploying them the reference frames of different links of a robot can be related to each other.

To determine the DH parameters of the mentioned robot, we first must determine the coordinates of each joint of the robot.

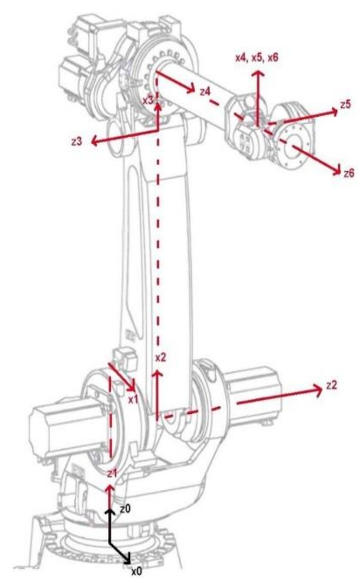


Figure3 : coordinates of each joint

Next, we complete the DH table using the modified method.

Before completing the table, it is necessary to know how to calculate and the relationship between each of the used variables.

- ✓ d_i indicates the distance between x_{i-1} and x_i in the direction of the z_i axis.
- ✓ θ_i indicates the angle between x_{i-1} and x_i around z_i axis.
- ✓ a_i indicates the distance between z_i and z_{i+1} in the direction of x_i axis.
- ✓ α_i indicates the angle between z_i and z_{i+1} around x_i axis.

i	α_{i-1}	a_{i-1} (cm)	d_i (cm)	θ_i
1	0	0	0.83	θ_1
2	-90	0.4	0	$-90+\theta_2$
3	180	1.175	0	θ_3
4	90	0.25	1.444	θ_4
5	90	0	0	θ_5
6	-90	0	0.23	θ_6

3rd Section

In this section, with the help of the values of Table 1: Denavit-Hartenberg parameters and placing them according to the matrix below, the T matrices of the robot can be obtained. As shown below, matrix $R_{3\times3}$ will indicate the rotational position of the final operator and matrix $T_{3\times1}$ will indicate the position of the final operator.

The position and periodic status of the end effector

$$T_1^0 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 12.45 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} S_2 & -C_2 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ C_2 & -S_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} C_3 & -S_3 & 0 & 17.625 \\ -S_3 & -C_3 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} C_4 & -S_4 & 0 & 3.75 \\ 0 & 0 & -1 & -21.66 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^4 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^5 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_6 & -C_6 & 0 & 0.23 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then we calculate the position of the end effector by successively multiplying the transformation matrices of the position of each joint relative to the zero frame.

$$T_3^0 = \begin{bmatrix} C_1S_{2-3} & -C_1C_{2-3} & -S_1 & 1.175C_1S_2 + 0.4C_1 \\ S_1S_{2-3} & -S_1C_{2-3} & -C_1 & 1.175S_1S_2 + 0.4S_1 \\ C_{2-3} & S_{2-3} & 0 & 1.175C_2 + 12.45 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = \begin{bmatrix} C_1C_4S_{2-3} - S_1S_4 & C_1C_4S_{2-3} - S_1C_4 & C_1C_{2-3} & 0.25C_1S_{2-3} + 1.444C_1C_{2-3} + 1.175C_1S_2 + 0.4 \\ S_1C_4S_{2-3} - C_1S_4 & -S_1S_4S_{2-3} - C_1C_4 & S_1C_{2-3} & 0.25S_1S_{2-3} + 1.444S_1C_{2-3} + 1.175S_1S_2 + 0.4 \\ C_4S_{2-3} & -S_4C_{2-3} & -S_{2-3} & 0.25C_{2-3} + 1.444S_{2-3} + 1.175C_2 + 0.4 + 12.4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = \begin{bmatrix} C_1S_2 & C_1C_2 & -S_1 & 0.4C_1 \\ S_1S_2 & C_2S_1 & C_1 & 0.4S_1 \\ C_2 & -S_2 & 0 & 12.45 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^0 = \begin{bmatrix} -C_5S_1S_4 + C_1S_5C_{2-3} + C_1C_4C_5S_{2-3} & -C_1C_4S_5S_{2-3} + S_1S_4S_5 + C_1C_5C_{2-3} & S_1C_4 - C_1S_4C_{2-3} & 0.25C_1S_{2-3} - 1.444C_1C_{2-3} + 1.175C_1S_2 + 0.4C_1 \\ -S_1C_4S_{2-3}C_5 - C_1S_4C_5 + S_1S_5S_{2-3} & -S_1C_4S_5S_{2-3} + C_1S_4S_5 + S_1C_5C_{2-3} & S_1S_4S_{2-3} - C_1C_4 & 0.25S_1S_{2-3} - 1.444S_1C_{2-3} + 1.175S_1S_2 + 0.4S_1 \\ C_4C_{2-3}C_5 - S_5S_{2-3} & -C_4S_5C_{2-3} - C_5S_{2-3} & S_4S_{2-3} & 0.25C_{2-3} - 1.444S_{2-3} + 1.175C_2 + 12.45 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^0 = \begin{bmatrix} -S_1S_4C_5C_6 + C_1S_5C_6C_{2-3} + C_1C_4C_5C_6S_{2-3} - S_1C_4S_6 + C_1S_4S_5C_{2-3} & S_1S_4C_5S_6 - C_1S_5S_6C_{2-3} - C_1C_4C_5C_6S_{2-3} - S_1C_4C_6 + C_1S_4C_6C_{2-3} & -C_1C_4S_5S_{2-3} + S_1S_4S_5 + C_1C_5C_{2-3} & 0.25C_1S_{2-3} - 1.444C_1C_{2-3} + 1.175C_1S_2 + 0.4C_1 \\ -S_2C_4C_6C_5C_{2-3} - C_1S_4C_5C_6 + S_1S_5C_6C_{2-3} - S_1S_4S_6S_{2-3} - C_1S_4S_6 & S_1S_4S_6C_5S_{2-3} + C_1S_4C_5S_6 - S_1S_5S_6C_{2-3} - S_1S_4C_6C_{2-3} + C_1S_4C_6 & -S_1C_4S_5S_{2-3} + C_1S_4S_5 + S_1C_5C_{2-3} & 0.25S\_1S\_\{2-3\} - 1.444S\_1C\_\{2-3\} + 1.175S\_1S\_2 + \\ C_4C_5C_6C_{2-3} - S_5C_6S_{2-3} - S_4S_6C_2C_{2-3} & -C_4C_5S_6C_{2-3} + S_5S_6S_{2-3} - S_4C_6C_2C_{2-3} & -C_4S_5C_{2-3} - C_5S_{2-3} & 0.25C\_\{2-3\} - 1.444S\_\{2-3\} + 1.175C\_2 + 12.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4th Section

Euler's angles

```
alpha=

angle(-(abs(sin(t2 - t3)*cos(t5) + cos(t2)*cos(t3)*cos(t4)*sin(t5) + cos(t4)*sin(t2)*sin(t3)*sin(t5))*(cos(t1((
```

```
beta=
```

```
atan2(((cos(t6)*(sin(t5)*(cos(t2 - pi/2)*cos(t3) + sin(t2 - pi/2)*sin(t3)) - cos(t5)*(cos(t4)*(cos(t2 - pi/2)*sin(t3) - sin(t2 - pi/2)*cos(t3)) + (sin(t4)*(cos(t2 - pi/2)*sin(t3) - sin(t2 - pi/2)*cos(t3))))^2 + (sin(t6)*(sin(t5)*(cos(t2 - pi/2)*cos(t3) + sin(t2 - pi/2)*sin(t3)) - cos(t5)*(cos(t4)*(cos(t2 - pi/2)*sin(t3) - sin(t2 - pi/2)*cos(t3)) - sin(t4)*(cos(t2 - pi/2)*sin(t3) - sin(t2 - pi/2)*cos(t3))))^2)^(1/2), cos(t5)*(cos(t2 - pi/2)*cos(t3) + sin(t2 - pi/2)*sin(t3)) + sin(t5)*(cos(t4)*(cos(t2 - pi/2)*sin(t3) - sin(t2 - pi/2)*cos(t3((
```

```
gama =

angle(-(abs(sin(t2 - t3)*cos(t5(( (
```

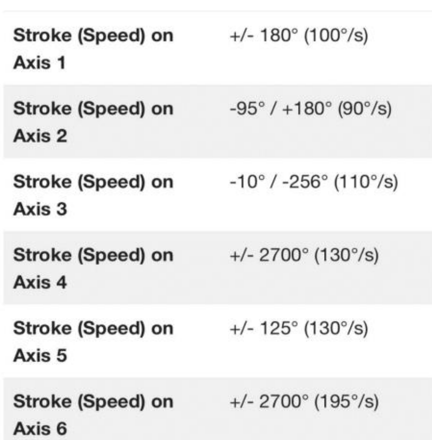
5^{th} Section

Robot's workspace

The workspace is available in Solidworks. Considering that the last three joints of their axes pass through the same point, it does not affect the movement of the end effector and only determines its orientation.

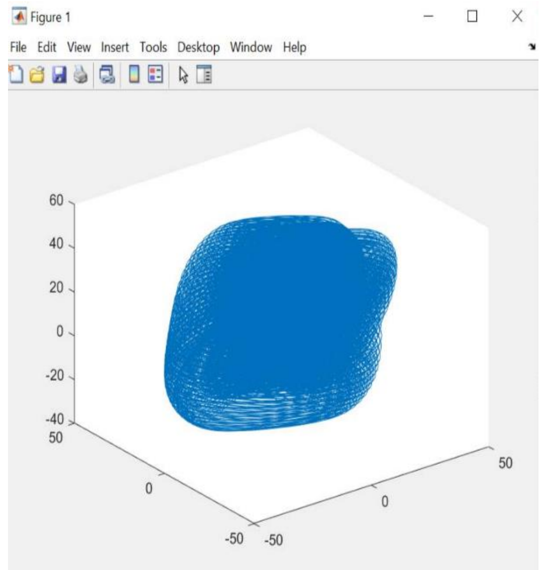
So, to get the working space, it is sufficient to move the first three joints and get the super points. Using the information in the datasheet, we have: So, to get the working space, we only need to move the first three joints and get the hyper points. Using the information in the datasheet, we

have:



Stroke (Speed) on Axis 1	+/- 180° (100°/s)
Stroke (Speed) on Axis 2	-95° / +180° (90°/s)
Stroke (Speed) on Axis 3	-10° / -256° (110°/s)
Stroke (Speed) on Axis 4	+/- 2700° (130°/s)
Stroke (Speed) on Axis 5	+/- 125° (130°/s)
Stroke (Speed) on Axis 6	+/- 2700° (195°/s)

Below is the workspace drawn with the help of MATLAB software:

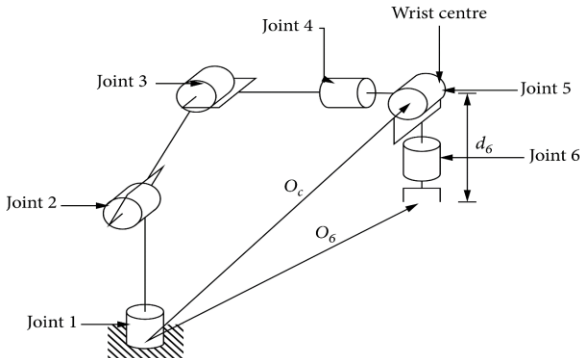


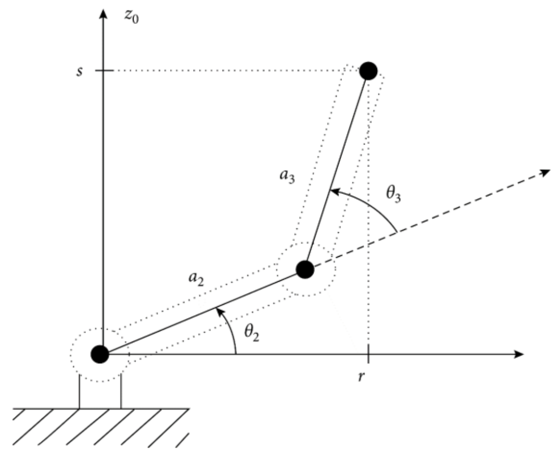
Robot movement animation

6^{th} Section

Inverse kinematics is finding the ϴ values of the robot arm for a given position P and direction O. To find inverse kinematics, inverse direction and inverse position are required.

Inverse kinematics





According to the obtained matrix, the position will always be as follows:

$$P = \begin{pmatrix} 0 \\ 0 \\ 0.23 \end{pmatrix}$$

$$d6 = 0.23$$

So we have:

$$O = O_c + d_6 R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad O_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

Based on the previous relationship, we can obtain the angle of the first three chapters:

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6(-C_1C_4S_5C_{2-3} + S_1S_4S_5 + C_1C_5C_{2-3}) \\ o_y - d_6(-S_1C_4S_5S_{2-3} + C_1S_4C_5 + S_1C_3C_{2-3}) \\ o_z - d_6(-C_4C_5S_6S_{2-3} + S_5S_6S_{2-3} + S_4C_6C_{2-3}) \end{bmatrix}$$

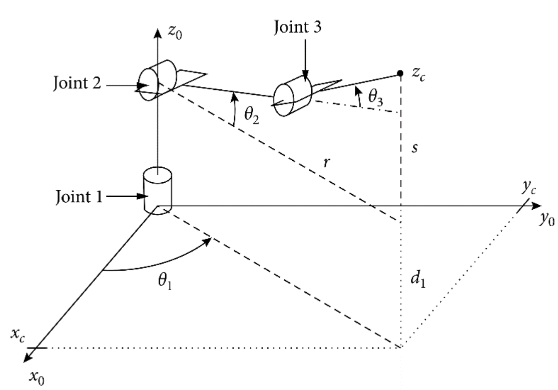
$$P_0^6 = \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix} = \begin{bmatrix} 3.75C_1S_{2-3} - 21.66C_1C_{2-3} + 17.625C_1C_2 + 6C_1 \\ 3.75C_1S_1S_{2-3} + 21.66S_1C_{2-3} + 17.625S_1S_2 + 6S_1 \\ 3.75C_{2-3} - 21.66S_{2-3} + 17.625C_2 + 12.68 \end{bmatrix}$$

We also know:

$$R = R_3^0 R_6^3 \quad , \quad R_6^3 = R_3^{0-1} R$$

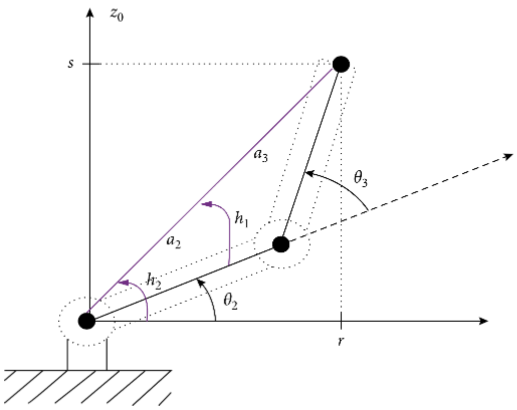
$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} 3.75C_1S_{2-3} - 21.66C_1C_{2-3} + 17.625C_1C_2 + 6C_1 + 0.23C_1C_4S_5C_{2-3} - 0.23S_1S_4S_5 - 0.23C_1C_5C_{2-3} \\ 3.75C_1S_1S_{2-3} + 21.66S_1C_{2-3} + 17.625S_1S_2 + 6S_1 + 0.23S_1C_4S_5S_{2-3} - 0.23C_1S_4C_5 - 0.23S_1C_5C_{2-3} \\ 3.75C_{2-3} - 21.66S_{2-3} + 17.625C_2 + 12.68 + 0.23C_1C_4C_5S_6S_{2-3} - 0.23S_5S_6S_{2-3} + 0.23S_4C_6C_{2-3} \end{bmatrix}$$

This gives a triangular projection from which the value of angle Θ1 is obtained:

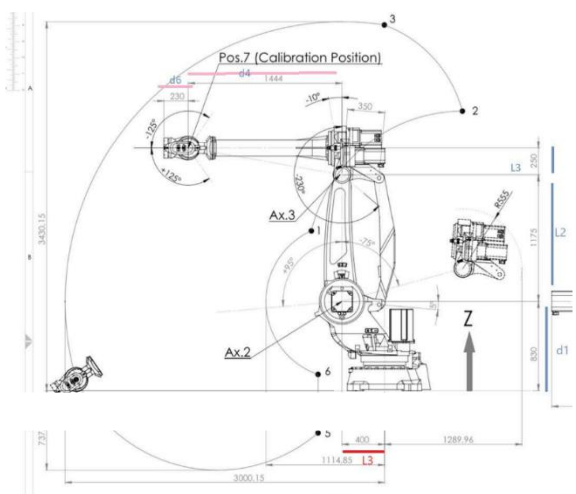


$$\theta_1 = atan2(x_c, y_c) \quad , \quad \theta_1 = atan2(x_c, y_c) + \pi$$

$$L_2 = a_2$$



The cosine law can be used as follows to obtain the joint angle 3:



$$cos\,\theta_3 = \frac{r^2 + s^2 - a_2^2 - a_3^2}{2a_2a_3}$$

$$cos\,\theta_3 = \frac{(x_c^2 + y_c^2) + (2c - d_1)^2 - a_2^2 - a_3^2}{2a}$$

$$\xRightarrow{a_2 = L_2} cos\,\theta_3 = \frac{x_c^2 + y_c^2 + (2c - d_1)^2 - L_2^2 - a_3^2}{2L_2a_3}$$

$$\sin\theta_3 = \sqrt{1 - \cos\theta_3^2}$$

$$\tan\theta_3 = \sqrt{\frac{1 - \cos\theta_3^2}{cos\,\theta_3}} = \sqrt{\frac{1 - D^2}{D^2}}$$

$$\theta_3 = atan2\left(D, \pm\sqrt{1 - D^2}\right)$$

According to the shape of the robot, the value of L = 1.117.

To calculate Θ2 we must first find the value of h2 h1.

$$\theta_2 = h_1 - h_2$$

$$h_1 = atan2(r, s) \;=\; atan2(\sqrt{x_c^2 + y_c^2}, 2c - d_1)$$

$$h_2 = atan2(a_2 + a_3\cos\theta_3\,, a_3\sin\theta_3)$$

7th Section

The Jacobian matrix indicates the relationship between the speed of the joints and the speed of the end effector in the robot. In other words, if we have the movement speed of the joints of a robot, we can find out the speed of the end effector.

To calculate the Jacobian matrix, it is necessary to use one of the following two relationships according to the type of joints and complete one column of the matrix with relevant information for each joint.

Jacobian matrix

$$J = \begin{bmatrix} -0.45S_1 - 1.175S_1S_2 - 0.25S_1S_{2-3} - 1.44S_1C_{2-3} & C_1(0.25C_{2-3} - 1.44S_{2-3} + 1.175C_2) & 0 & 0 & 0 & 0 \\ 0.4C_1 + 1.175C_1S_2 + 1.44C_1C_{2-3} + 0.25C_1S_{2-3} & S_1(0.25C_{2-3} - 1.44S_{2-3} + 1.175C_2) & -1.175S_2 - 0.25S_{2-3} - 1.44C_{2-3} & 0 & 0 & 0 \\ 0 & -1.44S_1C_2S_3 - 0.25S_1C_2C_3 & 0.25S_2C_3 + 1.44S_2S_3 & 0 & 0 & 0 \\ 0 & 0 & S_1 & C_1C_{2-3} & -C_4S_1 + C_1S_4S_{2-3} & S_1S_4S_5 - C_1C_5C_{2-3} - C_1C_4S_5S_{2-3} \\ 0 & 0 & -C_1 & S_1C_{2-3} & C_1C_4 + S_1S_4S_{2-3} & -C_1S_4S_5 - S_1C_5C_{2-3} + S_1C_4S_5S_{2-3} \\ 1 & 0 & 0 & -S_{2-3} & S_4C_{2-3} & C_5S_{2-3} + C_4S_5C_{2-3} \end{bmatrix}$$

The Jacobian matrix can be calculated analytically.

8th Section

The singularity of the rob

```
clc
clear all
syms t1 t2 t3 t4 t5 t6 J11;
J11 = [-0.4*sin(t1)-1.175*sin(t1)*sin(t2)-0.25*sin(t1)*sin(t2-t3)-1.44*sin(t1)*cos(t2-t3)  cos(t1)*(0.25*cos(t2-t3)-1.44*sin(t2-t3)+1.175*cos(t2))  0;
    0.4*cos(t1)+1.175*cos(t1)*sin(t2)+1.44*cos(t1)*cos(t2-t3)+0.25*cos(t1)*sin(t2-t3)  sin(t1)*(0.25*cos(t2-t3)-1.44*sin(t2-t3)+1.175*cos(t2))  -1.175*sin(t2)-0.2
    0 -1.44*sin(t1)*cos(t2)*sin(t3)-0.25*sin(t1)*cos(t2)*cos(t3)  0.25*sin(t2)*cos(t3)+1.44*sin(t2)*sin(t3)];
J22 = [cos(t1)*cos(t2-t3)  -cos(t4)*sin(t1)+cos(t1)*sin(t4)*sin(t2-t3)  sin(t1)*sin(t4)*sin(t5)-cos(t1)*cos(t5)*cos(t2-t3)-cos(t1)*cos(t4)*sin(t5)*sin(t2-t3);
    sin(t1)*cos(t2-t3)  cos(t1)*cos(t4)+sin(t1)*sin(t4)*sin(t2-t3)  -cos(t1)*sin(t4)*sin(t5)-sin(t1)*sin(t5)*cos(t2-t3)+sin(t1)*cos(t4)*sin(t5)*sin(t2-t3);
    -sin(t2-t3)  sin(t4)*cos(t2-t3)  cos(t5)*sin(t2-t3)+cos(t4)*sin(t5)*cos(t2-t3)];
theta1 = [];
theta2 = [];
theta3 = [];
cnt = 1;
for i = 0:0.01:2*pi
    for j = 0:0.01:2*pi
        for k = 0:0.01:2*pi
            J111=subs(J11,[t1,t2,t3],[i,j,k]);
            det11 = det(J111);
            if det11<=(10^(-6))
                theta1(cnt) = i;
                theta2(cnt) = j;
                theta3(cnt) = k;
                cnt = cnt + 1;
                fprintf('%d %d %d %d\n', i, j, k);
            end
        end
    end
end
```

9th Section

Robot dynamics

D, C and G matrices

To calculate the matrix G, we must derive the potential force relative to each joint variable.

$$p = m_1 l_1 g + m_2 l_2 g \sin(-\theta_2) + (m_3 + m_4) g l_3 \sin(-\theta_2 + \theta_3) + (m_5 + m_6) g l_5 \sin(-\theta_2 + \theta_3 - \theta_5)$$

Next, we calculate the G matrix.

$$G = \begin{bmatrix} 0 \\ -m_2 l_2 g \cos(\theta_2) - (m_3 + m_4) g l_3 \cos(-\theta_2 + \theta_3) - (m_5 + m_6) g l_5 \cos(-\theta_2 + \theta_3 - \theta_5) \\ (m_3 + m_4) g l_3 \cos(-\theta_2 + \theta_3) + (m_5 + m_6) g l_5 \cos(-\theta_2 + \theta_3 - \theta_5) \\ 0 \\ -(m_5 + m_6) g l_5 \cos(-\theta_2 + \theta_3 - \theta_5) \\ 0 \end{bmatrix}$$