Let $X(t)$ for $t \in (0, T]$ denote the value of the asset at time t. For an Asian Option, the payoff of the option is given by:

$$Payoff = (\overline{X}(T) - K)^+$$

where $\overline{X}(T)$ denote the average stock value over the period $[0, T]$. Assuming a risk - free interest rate r, the value of the option is given by:

$$v = E[e^{-rT}(\overline{X}(T) - K)^+].$$

Usually the price of the underlying asset is modeled as a Geometric Brownian Motion (GBM), a continuous time, continuous state stochastic process. In particular, the average value of the stock is given by computing an integral. This cannot be done! Instead, we estimate the average by computing the Rimann sum for interval length δt. This introduces a bias, in particular, it is not accurate.

Geometric Brownian motion: Suppose the stochastic process $S(t)$ is a GBM. Then we get:

$$S(t) = S(0)e^{X(t)}, \quad t \geq 0,$$

where $X(t)$ is a brownian motion with drift:

$$X(t) = \sigma B(t) + \mu(t)$$

μ is called the drift, and σ is the variance. Note that, property of Brownian motions gives for two time stamps t_{i-1}, t_i:

$$\frac{S(t_i)}{S_{t_{i-1}}} = exp\{\mu(t_i - t_{i-1}) + \sigma\sqrt{t_i - t_{i-1}}Z_i, \tag{1}$$

where Z_i is a standard normal variable. We use equation 1, to produce successive values of the asset price for computing the Riemann sum.