Barituziga Banuna CHEME 7770 Prelim 1 P2

Sunday, May 10, 2020 8:13 PM

Problem 2

A) $$\frac{d\tilde{X}}{d\tilde{t}} = \frac{\tilde{\alpha}_x + \tilde{\beta}_x S}{1+S+(\tilde{Z}/\tilde{Z}_x)^{n_{zx}}} - \tilde{\delta}_x \tilde{X}$$

$$\frac{d\tilde{Z}}{d\tilde{t}} = \frac{\tilde{\alpha}_z}{1+(\tilde{X}/\tilde{X}_z)^{n_{xz}}} - \tilde{\delta}_z \tilde{Z}$$

B) $\delta_z = \frac{\tilde{\delta}_z}{\tilde{\delta}_x}$ $\quad t = \tilde{t}\tilde{\delta}_x$ $\quad \alpha_x = \frac{\tilde{\alpha}_x}{\tilde{\alpha}_z}$ $\quad \beta_x = \frac{\tilde{\beta}_x}{\tilde{\alpha}_z}$ $\quad z_x = \frac{\tilde{z}_x \tilde{\delta}_x}{\tilde{\alpha}_z}$ $\quad x_z = \frac{\tilde{x}_z \tilde{\delta}_x}{\tilde{\alpha}_z}$

$X = \frac{\tilde{X}\tilde{\delta}_x}{\tilde{\alpha}_z}$ $\quad Z = \frac{\tilde{Z}\tilde{\delta}_x}{\tilde{\alpha}_z}$

$\Rightarrow$ $\tilde{\delta}_z = \delta_z \tilde{\delta}_x$ $\quad \tilde{t} = t/\tilde{\delta}_x$ $\quad \tilde{\alpha}_x = \alpha_x \tilde{\alpha}_z$ $\quad \tilde{\beta}_x = \beta_x \tilde{\alpha}_z$ $\quad \tilde{z}_x = \frac{z_x \tilde{\alpha}_z}{\tilde{\delta}_x}$ $\quad \tilde{x}_z = \frac{x_z \tilde{\alpha}_z}{\tilde{\delta}_x}$

$\tilde{X} = \frac{X\tilde{\alpha}_z}{\tilde{\delta}_x}$ $\quad \tilde{Z} = \frac{Z\tilde{\alpha}_z}{\tilde{\delta}_x}$

$$\frac{d X \cancel{\tilde{\alpha}_z} / \cancel{\tilde{\delta}_x}}{dt/\cancel{\tilde{\delta}_x}} = \frac{\alpha_x \cancel{\tilde{\alpha}_z} + \beta_x \cancel{\tilde{\alpha}_z} S}{1+S\left(\frac{Z\cancel{\tilde{\alpha}_z}}{\cancel{\tilde{\delta}_x}} \div \frac{z_x \cancel{\tilde{\alpha}_z}}{\cancel{\tilde{\delta}_x}}\right)^{n_{zx}}} - \cancel{\tilde{\delta}_x}\frac{X\cancel{\tilde{\alpha}_z}}{\cancel{\tilde{\delta}_x}}$$

$$\Rightarrow \frac{dX}{dt} = \frac{\alpha_x + \beta_x S}{1+S(Z/z_x)^{n_{zx}}} - X$$

$$\frac{d Z \cancel{\tilde{\alpha}_z} / \cancel{\tilde{\delta}_x}}{dt/\cancel{\tilde{\delta}_x}} = \frac{\cancel{\tilde{\alpha}_z}}{1+S\left(\frac{X\cancel{\tilde{\alpha}_z}}{\cancel{\tilde{\delta}_x}} \div \frac{x_z \cancel{\tilde{\alpha}_z}}{\cancel{\tilde{\delta}_x}}\right)^{n_{xz}}} - \delta_z \cancel{\tilde{\delta}_x} \frac{Z\cancel{\tilde{\alpha}_z}}{\cancel{\tilde{\delta}_x}}$$

$$\Rightarrow \frac{dZ}{dt} = \frac{1}{1+S(X/x_z)^{n_{xz}}} - \delta_z Z$$

The small error is in the time non-dimensionalization. They wrote $t = \tilde{t}\,\delta_x$ rather than the correct $t = \tilde{t}\,\tilde{\delta}_x$

• They wrote $t = \tilde{t}\, \delta_x$ rather than the correct $t = \tilde{t}\, \tilde{\delta}_x$

The other small error is in the $\frac{d\tilde{z}}{d\tilde{t}}$ definition

• z is written rather than the correct $\tilde{z}$

c) See Matlab file

d) Results are reproducable. See Matlab file

e) See Matlab code

Below Hopf they are incoherent

Above saddle they are coherent

f) This seems like it could be possible since in e we have coherent oscillations for the saddle bifurcation