

### Representations of geometric relations in point geometry and vector geometry

| NO. | Geometric relations  | Expressions in point geometry                                  | Expressions in vector geometry   |
|-----|--|--|--|
| 1   | $A, B, C$ are collinear and $t\overrightarrow{CA} = (t-1)\overrightarrow{CD}$ . If $t = 1/2$ , $C$ is the midpoint of $AB$ . | $C = tA + (1-t)B$  | $\overrightarrow{OC} = t\overrightarrow{OA} + (1-t)\overrightarrow{OB}$  |
| 2   | $ABCD$ is a parallelogram  | $D = A - B + C$  | $\overrightarrow{CD} = \overrightarrow{BA}$  |
| 3   | $ABCD$ is a trapezoid with $AB \parallel CD$ and $CD = t \cdot BA$   | $D = t(A - B) + C, t \neq 1$                                   | $\overrightarrow{CD} = t\overrightarrow{BA}$   |
| 4   | $G$ is the centroid of $\triangle ABC$ .   | $G = \frac{A+B+C}{3}$  | $\overrightarrow{OG} = \frac{\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}}{3}$              |
| 5   | $ABC$ is a triangle with orthocenter $H$ and circumcenter $D$  | $H = A + B + C - 2D$   | $\overrightarrow{OH} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} - 2\overrightarrow{OD}$ |
| 6   | $AB \perp CD$  | $(A - B)(C - D) = 0$   | $\overrightarrow{BA} \cdot \overrightarrow{DC} = 0$  |
| 7   | $A$ is on the perpendicular bisector of $BC$ .   | $(A - \frac{B+C}{2})(B - C) = 0$                               | $(\overrightarrow{OA} - \frac{\overrightarrow{OB} + \overrightarrow{OC}}{2}) \cdot \overrightarrow{CB} = 0$    |
| 8   | $\angle BAC = 90^\circ$  | $(A - B)(A - C) = 0$   | $\overrightarrow{BA} \cdot \overrightarrow{CA} = 0$  |
| 9   | $H$ is the orthocenter of $\triangle ABC$ .  | $(A - H)(B - C) = 0$ and<br>$(B - H)(C - A) = 0$               | $\overrightarrow{HA} \cdot \overrightarrow{CB} = 0$ and<br>$\overrightarrow{HB} \cdot \overrightarrow{CA} = 0$ |
| 10  | $O$ is the circumcenter of $\triangle ABC$ .   | $(O - A)^2 - (O - B)^2 = 0$ and<br>$(O - A)^2 - (O - C)^2 = 0$ | $\overrightarrow{OA}^2 = \overrightarrow{OB}^2$ and<br>$\overrightarrow{OA}^2 = \overrightarrow{OC}^2$         |
| 11  | Projection Theorem:<br>$BAC$ is a triangle with $BA \perp AC$ and $AD \perp BC$ where $D$ lies on $BC$ .                     | $(A - D)^2 - (B - D)(D - C) = 0$                               | $\overrightarrow{DA}^2 - \overrightarrow{DB} \cdot \overrightarrow{CD} = 0$                                    |
|     |  | $(A - B)^2 - (B - D)(B - C) = 0$                               | $\overrightarrow{BA}^2 - \overrightarrow{DB} \cdot \overrightarrow{CB} = 0$                                    |
| 12  | Circle-Power Theorem:<br>$ABCD$ is a cyclic quadrilateral, $AC$ intersects $BD$ at $P$ .                                     | $(P - A)(P - B) - (P - C)(P - D) = 0$                          | $\overrightarrow{AP} \cdot \overrightarrow{BP} - \overrightarrow{CP} \cdot \overrightarrow{DP} = 0$            |
| 13  | $\sqrt{k_1}AB = \sqrt{k_2}CD$  | $k_1(A - B)^2 - k_2(C - D)^2 = 0$                              | $k_1\overrightarrow{BA}^2 - k_2\overrightarrow{DC}^2 = 0$  |
| 14  | $AB^2 = k_1CD^2 + k_2EF^2$   | $(A - B)^2 - k_1(C - D)^2 - k_2(E - F)^2 = 0$                  | $\overrightarrow{AB}^2 - k_1\overrightarrow{CD}^2 - k_2\overrightarrow{EF}^2 = 0$                              |
| 15  | $CD \parallel EF$ and $k_1AB^2 = k_2CD \cdot EF$   | $k_1(A - B)^2 - k_2(C - D)(E - F) = 0$                         | $k_1\overrightarrow{AB}^2 - k_2\overrightarrow{DC} \cdot \overrightarrow{FE} = 0$                              |
|     | $C, D, E$ are collinear and $k_1AB^2 = k_2CD \cdot DE$   | $k_1(A - B)^2 - k_2(C - D)(D - E) = 0$                         | $k_1\overrightarrow{AB}^2 - k_2\overrightarrow{CD} \cdot \overrightarrow{DE} = 0$                              |
| 16  | $A, B, C$ are collinear. $D, E, F$ are collinear. And $k_1AB \cdot BC = k_2DE \cdot EF$                                      | $k_1(A - B)(B - C) - k_2(D - E)(E - F) = 0$                    | $k_1\overrightarrow{AB} \cdot \overrightarrow{BC} = k_2\overrightarrow{DE} \cdot \overrightarrow{EF}$          |
| 17  | $AB \parallel CD$ . $E, F, G$ are collinear. And $k_1AB \cdot CD = k_2EF \cdot FG$   | $k_1(A - B)(C - D) - k_2(E - F)(F - G) = 0$                    | $k_1\overrightarrow{AB} \cdot \overrightarrow{CD} = k_2\overrightarrow{EF} \cdot \overrightarrow{FG}$          |
| 18  | $AB \parallel CD$ , $EF \parallel GH$ and $k_1AB \cdot CD = k_2EF \cdot GH$  | $k_1(A - B)(C - D) - k_2(E - F)(G - H) = 0$                    | $k_1\overrightarrow{AB} \cdot \overrightarrow{CD} = k_2\overrightarrow{EF} \cdot \overrightarrow{GH}$          |