

Representation of common geometric relations with point geometry

NO.	Geometric relations	Expressions in point geometry	Expressions in vector geometry
1	A, B, C are collinear and $\overrightarrow{CA} = (t-1)\overrightarrow{CD}$. If $t = 1/2$, C is the midpoint of AB.	$C = tA + (1-t)B$	$\overrightarrow{OC} = t\overrightarrow{OA} + (1-t)\overrightarrow{OB}$
2	ABCD is a parallelogram	$D = A - B + C$	$\overrightarrow{CD} = \overrightarrow{BA}$
3	ABCD is a trapezoid with AB//CD and $CD = t \cdot BA$	$D = t(A - B) + C, t \neq 1$	$\overrightarrow{CD} = t\overrightarrow{BA}$
4	G is the centroid of $\triangle ABC$.	$G = \frac{A+B+C}{3}$	$\overrightarrow{OG} = \frac{\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}}{3}$
5	ABC is a triangle with orthocenter H and circumcenter D	$H = A + B + C - 2D$	$\overrightarrow{OH} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} - 2\overrightarrow{OD}$
6	$AB \perp CD$	$(A - B)(C - D) = 0$	$\overrightarrow{BA} \cdot \overrightarrow{DC} = 0$
7	A is on the perpendicular bisector of BC.	$(A - \frac{B+C}{2})(B - C) = 0$	$(\overrightarrow{OA} - \frac{\overrightarrow{OB} + \overrightarrow{OC}}{2}) \cdot \overrightarrow{CB} = 0$
8	$\angle BAC = 90^\circ$	$(A - B)(A - C) = 0$	$\overrightarrow{BA} \cdot \overrightarrow{CA} = 0$
9	H is the orthocenter of $\triangle ABC$.	$(A - H)(B - C) = 0$ and $(B - H)(C - A) = 0$	$\overrightarrow{HA} \cdot \overrightarrow{CB} = 0$ and $\overrightarrow{HB} \cdot \overrightarrow{CA} = 0$
10	O is the circumcenter of $\triangle ABC$.	$(O - A)^2 - (O - B)^2 = 0$ and $(O - A)^2 - (O - C)^2 = 0$	$\overrightarrow{OA}^2 = \overrightarrow{OB}^2$ and $\overrightarrow{OA}^2 = \overrightarrow{OC}^2$
11	Projection Theorem: BAC is a triangle with $BA \perp AC$ and $AD \perp BC$ where D is on BC.	$(A - D)^2 - (B - D)(D - C) = 0$	$\overrightarrow{DA}^2 - \overrightarrow{DB} \cdot \overrightarrow{DC} = 0$
		$(A - B)^2 - (B - D)(B - C) = 0$	$\overrightarrow{BA}^2 - \overrightarrow{DB} \cdot \overrightarrow{CB} = 0$
12	Circle-Power Theorem: ABCD is a cyclic quadrilateral, AC intersects BD at P.	$(P - A)(P - B) - (P - C)(P - D) = 0$	$\overrightarrow{PA} \cdot \overrightarrow{PB} - \overrightarrow{PC} \cdot \overrightarrow{PD} = 0$
13	$\sqrt{k_1}AB = \sqrt{k_2}CD$	$k_1(A - B)^2 + k_2(C - D)^2 = 0$	$k_1\overrightarrow{BA}^2 - k_2\overrightarrow{DC}^2 = 0$
14	$AB^2 = k_1CD^2 + k_2EF^2$	$(A - B)^2 - k_1(C - D)^2 - k_2(E - F)^2 = 0$	$\overrightarrow{AB}^2 - k_1\overrightarrow{CD}^2 - k_2\overrightarrow{EF}^2 = 0$
15	$CD \parallel EF$ and $k_1AB^2 = k_2CD \cdot EF$	$k_1(A - B)^2 - k_2(C - D)(E - F) = 0$	$k_1\overrightarrow{AB}^2 - k_2\overrightarrow{CD} \cdot \overrightarrow{EF} = 0$
16	A, B, C are collinear. D, E, F are collinear. And $k_1AB \cdot BC = k_2DE \cdot EF$	$k_1(A - B)(B - C) - k_2(D - E)(E - F) = 0$	$k_1\overrightarrow{AB} \cdot \overrightarrow{BC} = k_2\overrightarrow{DE} \cdot \overrightarrow{EF}$
17	$AB \parallel CD$. E, F, G are collinear. And $k_1AB \cdot CD = k_2EF \cdot FG$	$k_1(A - B)(C - D) - k_2(E - F)(F - G) = 0$	$k_1\overrightarrow{AB} \cdot \overrightarrow{CD} = k_2\overrightarrow{EF} \cdot \overrightarrow{FG}$
18	$AB \parallel CD$, $EF \parallel GH$ and $k_1AB \cdot CD = k_2EF \cdot GH$	$k_1(A - B)(C - D) - k_2(E - F)(G - H) = 0$	$k_1\overrightarrow{AB} \cdot \overrightarrow{CD} = k_2\overrightarrow{EF} \cdot \overrightarrow{GH}$