Fractal animation introduction

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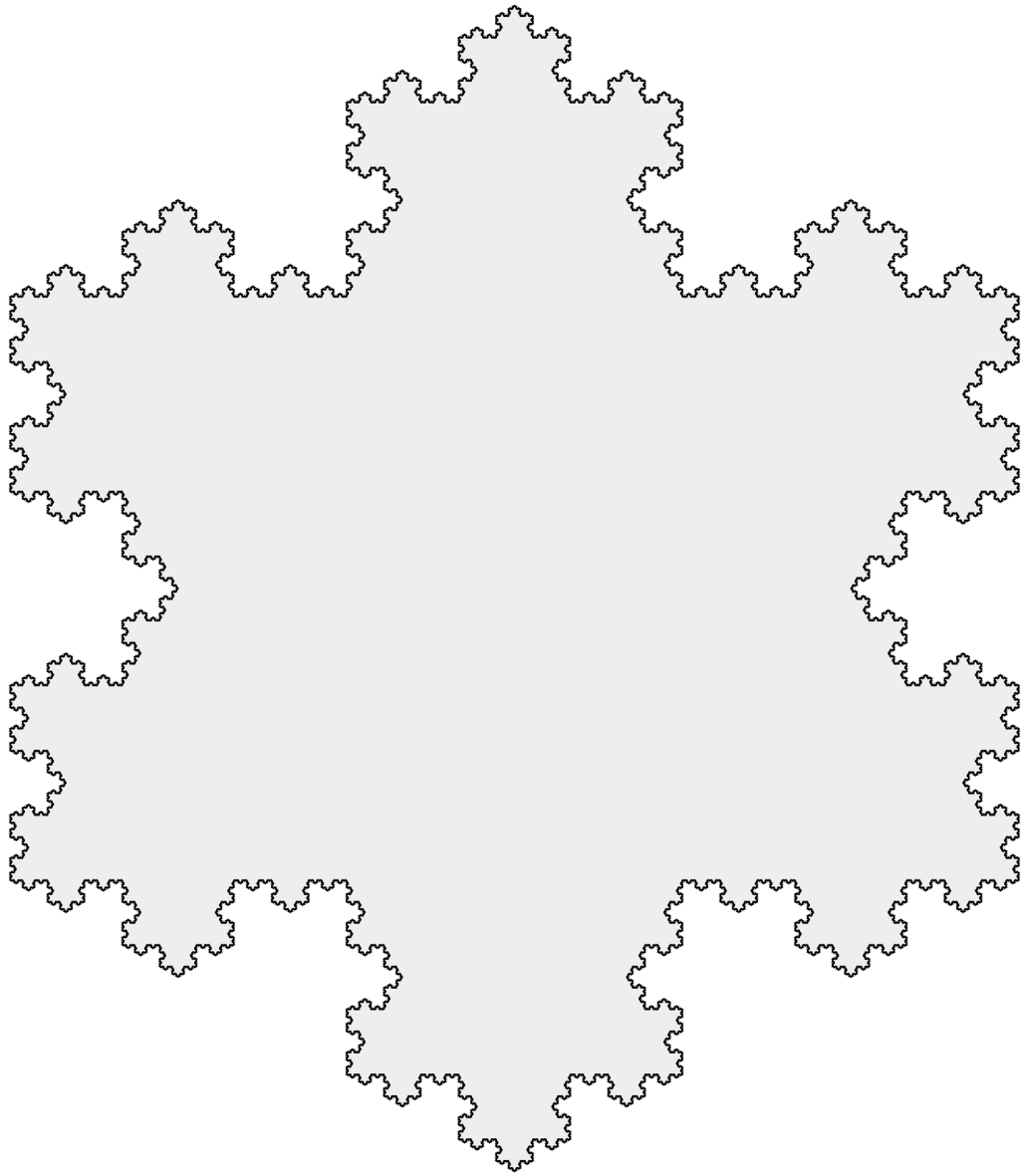
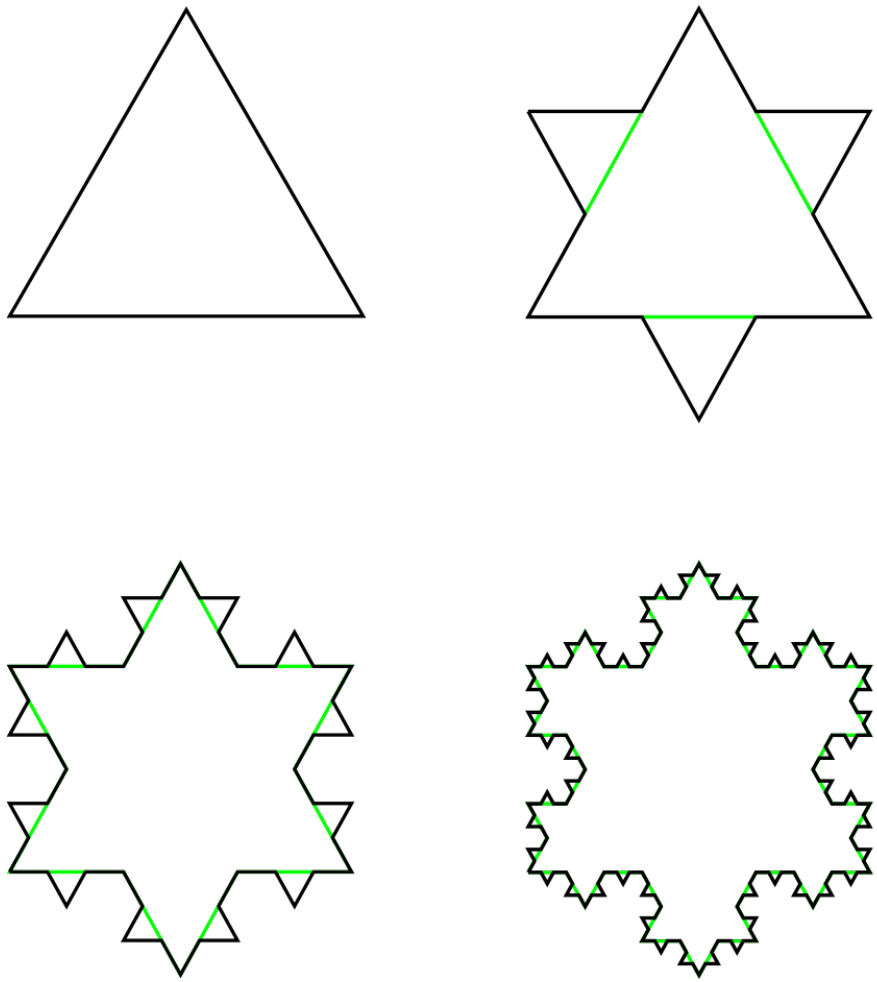
What are fractals?

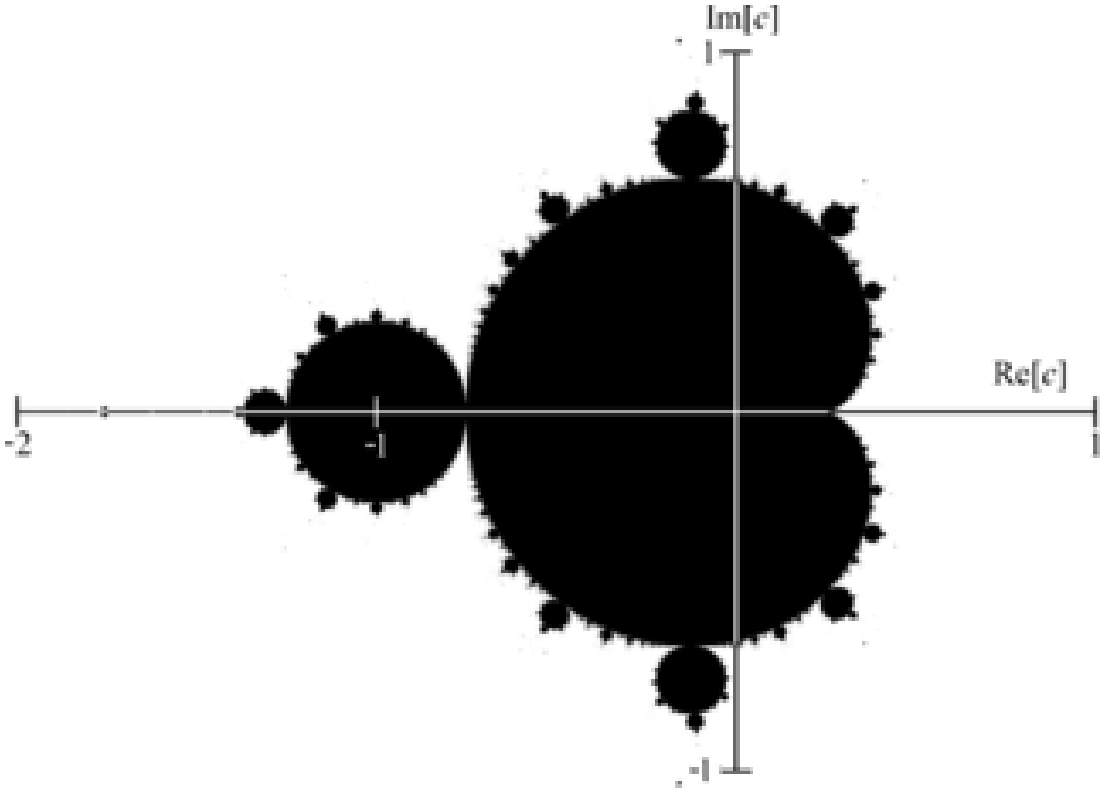


Koch snowflake



Pythagora's tree demo

Mandelbrot set



Mandelbrot set

Complex numbers and complex plane

$$c = ai + b$$

$$i^2 = -1$$

The *modulus* of a complex number is its euclidiean distance to 0:

$$c = ai + b$$

$$|c| = \sqrt{a^2 + b^2}$$

```
In [1]: c = complex(1, 2)

In [2]: c
Out[2]: (1+2j)

In [3]: c + complex(0,3)
Out[3]: (1+5j)

In [4]: c * c
Out[4]: (-3+4j)
```

Mandelbrot set

Function to iterate

The formula that generates everything is the following one:

$$z_0 = c$$
$$z_{n+1} = z_n^2 + c$$

For each pixel (x, y) of the screen, we compute the corresponding complex number $c_{x,y}$.

Now we compute a fixed number of terms of the sequence above starting with $z_0 = c_{x,y}$.

$$\begin{aligned} z_0 &= c_{x,y} \\ z_1 &= c_{x,y}^2 + c_{x,y} \\ z_2 &= z_1^2 + c_{x,y} = \left(c_{x,y}^2 + c_{x,y}\right)^2 + c_{x,y} \\ &\dots \\ z_{100} &= z_{99}^2 + c_{x,y} \end{aligned}$$

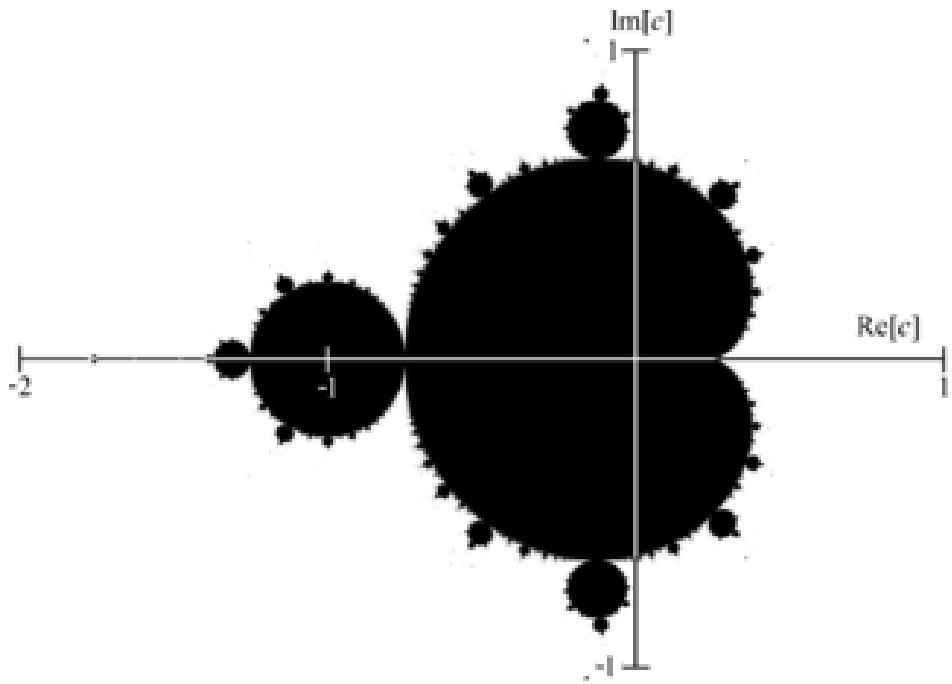
In Python:

```
mandelbrot_function = lambda z, c: z ** 2 + c
```

Mandelbrot set

From the formula to the drawing

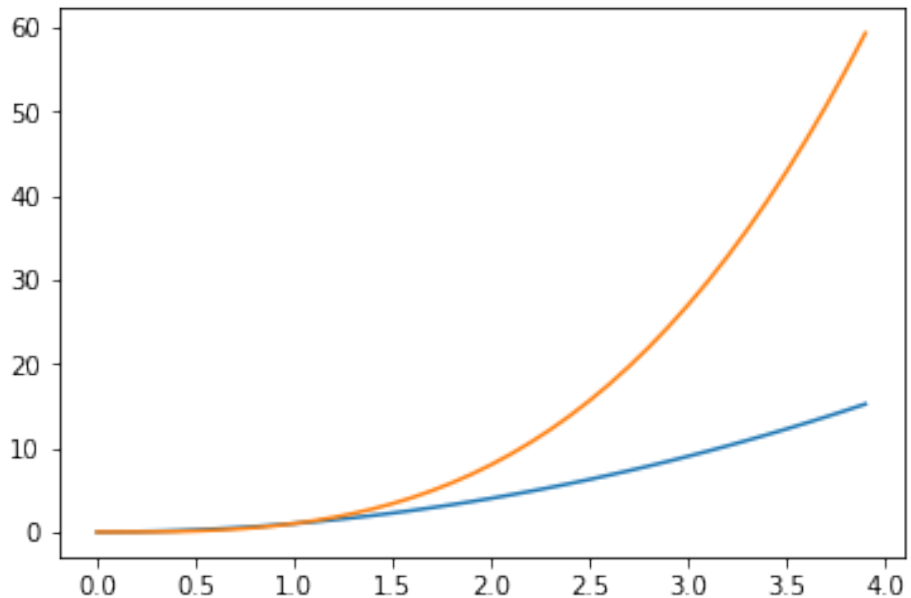
While we are iterating the formula for a pixel (x, y), if we find a step i for which $|z_i| > 2$ then we draw the (x, y) pixel white, otherwise we draw it black.



Mandelbrot set zoom animation

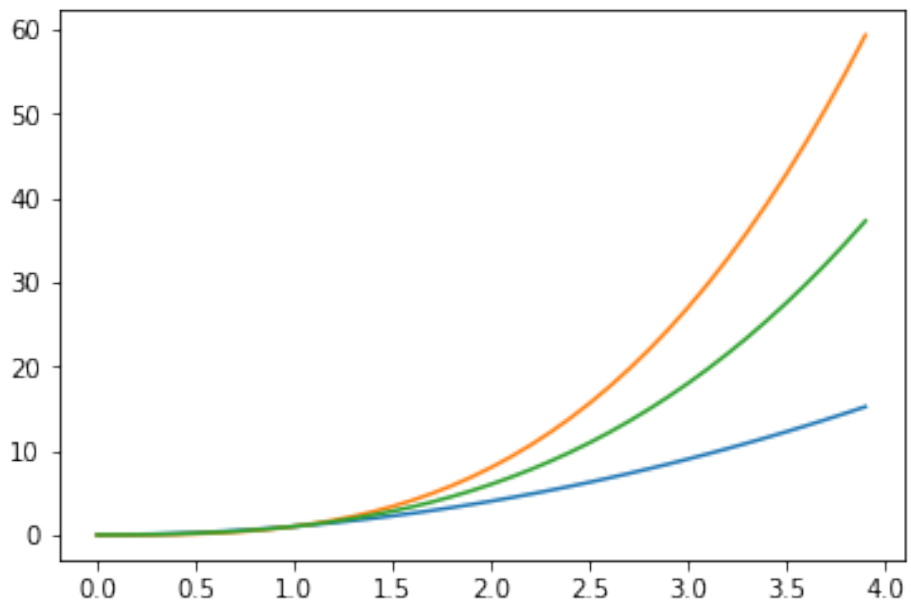
Function interpolation with Python

```
f_square = lambda x: x ** 2
f_cube   = lambda x: x ** 3
```



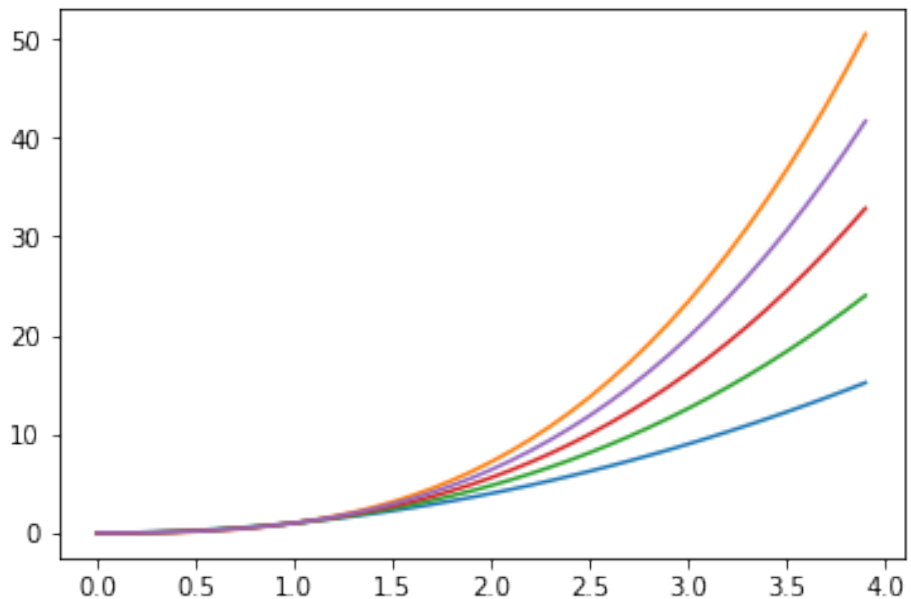
Function interpolation with Python

```
f_square = lambda x: x ** 2
f_cube   = lambda x: x ** 3
f_inter  = lambda x: (f_square(x) + f_cube(x)) / 2
```



Function interpolation with Python

```
n_int   = 5
int_fns = []
for i in range(n_int):
    new_fn = lambda x, i = i, n_int = n_int: (
        (n_int - i) * f_square(x) +
        i * f_cube(x)
    ) / n_int
    int_fns.append(new_fn)
```



Mandelbrot set formula animations