A.I H.W-2

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Q1. Using the Bayesian network, show the following

(i) P(Burglary | John Calls) = 0.016

$= \frac{P(J,B)}{P(J)} = P(JBA) + P(JB\neg A)$ — (M)

$P(JB) = P(J|AB)\,P(AB) + P(J|\neg AB)\,P(\neg AB)$

$= P(J|A) \cdot P(AB) + P(J|\neg A)\,P(\neg AB)$

$= (0.90)\,P(AB) + (0.05)\,P(\neg AB)$ — (1)

$P(AB) = P(ABE) + P(AB\neg E)$

$= P(A|BE) \cdot P(B)P(E) + P(A|B\neg E) \cdot P(B)\,P(\neg E)$

$= (0.95)(0.001)(0.002) + (0.95)(0.001)(0.998)$

$P(AB) = 0.00095$ — (2) $\quad \left[\because P(\neg E) = 1 - 0.002 = 0.998\right]$

$P(\neg AB) = P(\neg ABE) + P(\neg AB\neg E)$

$= P(\neg A|BE) \cdot P(BE) + P(\neg A|B\neg E) \cdot P(B\neg E)$

$= (1-0.95) \times (0.001 \times 0.002) + (1-0.95)(0.001)(0.998)$

$= 0.00005$ — (3)

By substituting (2) & (3) in (1), we get

$P(JB) = (0.9) \times (0.00095) + (0.05) \times (0.00005)$

$= 0.00086$ — (4)

$P(J) = P(J|A) \cdot P(A) + P(J|\neg A)\,P(\neg A)$

$= 0.9 \cdot P(A) + 0.05 \cdot P(\neg A)$ — (5)

$$P(A) = P(ABE) + P(A\neg B\neg E) + P(A\neg BE) + P(AB\neg E)$$

$$= P(A|\neg B\neg E)\cdot P(\neg B\neg E) + P(A|BE)\cdot P(BE) + P(A|\neg BE)$$
$$\cdot P(\neg BE) + P(A|B\neg E)\cdot P(B\neg E)$$

$$= 0.001 \times (1-0.001) \times (1-0.002) + 0.95 \times (0.001) \times (0.002)$$
$$+ 0.29 \times 0.999 \times 0.02 + 0.95 \times 0.001 \times 0.998$$

$$= 0.001 + 0.0009 + 0.0006 = 0.00252 \quad (6)$$

By keeping $P(A)$ in (5)

(5) $\Rightarrow$ $$P(J) = 0.9\, P(A) + 0.05\, P(\neg A)$$
$$= 0.9(0.0025) + 0.05(1-0.0025)$$
$$= 0.05212 \quad (7)$$

Substituting (7) & (4) in (M)

$$P(B|J) = P(JB)/P(J)$$
$$= \frac{0.00086}{0.05212} = 0.016 = R.H.S$$

$\therefore$ L.H.S = R.H.S . that is

P(Burglary | JohnCalls) = 0.016

(ii) $P(\text{JohnCalls} \mid \text{Burglary}) = 0.86$

$L.H.S = P(\text{Johncalls} \mid \text{Burglary}) = \frac{P(J,B)}{P(B)}$ —— (M)

From equation (4) in question (i) we have.

$P(J,B) = 0.00086$ —— (1)

and from the given Bayesian network, we have

$P(B) = 0.001$ —— (2)

$\therefore$ By substituting (1) & (2) in (M)

$(M) \Rightarrow \frac{P(J,B)}{P(B)} = \frac{0.00086}{0.001} = 0.86 = R.H.S$

$L.H.S = R.H.S$

$\therefore$ Hence proved

(iii) $P(\text{MaryCalls} \mid \text{Burglary}) = 0.67$

$$= \frac{P(M,B)}{P(B)} \quad \text{—— (M)}$$

$$P(M,B) = P(M \mid A,B)\, P(A,B) + P(M \mid \neg A B)\, P(\neg A B)$$

$$= P(M \mid A) \cdot P(AB) + P(M \mid \neg A)\, P(\neg A B)$$

$$= 0.70\, P(AB) + (0.01)\, P(\neg A B) \quad \text{—— (1)}$$

$$P(A,B) = P(ABE) + P(AB\neg E)$$

$$= P(A \mid B,E) \cdot P(B) P(E) + P(A \mid B \neg E) P(B) P(\neg E)$$

$$= (0.95)(0.001) \times (0.002) + (0.94)(0.001)(0.998)$$

[reasonable approximation]

$$P(A,B) = 0.00094 \quad \text{—— (2)}$$

$$P(\neg A B) = P(\neg A B E) + P(\neg A B \neg E)$$

$$= P(\neg A \mid B,E) \cdot P(B) P(E) + P(\neg A \mid B \neg E) P(B) P(E)$$

$$= (1-0.95)(0.001)(0.002) + (1-0.94)(0.001)(0.998)$$

$$= 0.00006$$

$$(1) \Rightarrow P(M,B) = 0.70\, P(AB) + 0.01\, P(\neg A B)$$

$$= 0.70\,(0.00094) + 0.01\,(0.00006)$$

$$= 0.000665 \quad \text{—— (3)}$$

Substitute (3) in (M)

$$(M) \Rightarrow P(\text{Marycalls} \mid \text{Burglary}) = \frac{P(M,B)}{P(B)} = \frac{0.000665}{0.01}$$

$$= 0.0665 \approx 0.67$$

$\therefore$ Hence proved.

(iv) $P(\text{Burglary} \mid \text{Alarm}) = 0.376$

$$L.H.S = P(B \mid A) = \frac{P(B,A)}{P(A)} \quad \text{——} (M)$$

From equation (2) in question (iii), we have.

$$P(B,A) = 0.00095 \quad \text{——} (1)$$

and from equation (6) in question (i), we have

$$P(A) = 0.00252 \quad \text{——} (2)$$

By Substituting (1) & (2) in eq (M), we get.

$$P(B \mid A) = \frac{P(B,A)}{P(A)} = \frac{0.00095}{0.00252} \simeq 0.376 = R.H.S$$

$$L.H.S = R.H.S$$

$\therefore$ hence proved.

(v) P(Burglary | Alarm, Earthquake) = 0.003

$$P(B|A,E) = \alpha P(B) P(A|B,E)\ P(E) \sum_J P(J|A) \sum_M P(M|A)$$

$$= \alpha P(B) P(A|B,E)\ P(E) \sum_J P(J|A) \{ P(M|A) + P(\neg M|A) \}$$

$$= \alpha P(B) P(A|B,E) P(E)\ \ P(J|A) + P(\neg J|A) \{ (1) \}$$

$$= \alpha P(B) P(A|B,E) P(E) \times 1 \times 1$$

$$= 0.001 \times 0.95 \times 0.002$$

$$= 0.0000019$$

$$P(\neg B|A E) = \alpha P(\neg B) P(A|\neg B, E) P(E) \sum_J P(J|A) \sum_n P(M|A)$$

$$= 0.999 \times 0.29 \times 0.002 \{ 1 \}$$

$$= 0.00057942$$

$$\alpha = \frac{1}{P(B|A,E) + P(\neg B|AE)} = \frac{1}{0.0000019 + 0.0005794}$$

$$= \frac{1}{0.00058} = 1724.13$$

$\therefore$ P(Burglary | Alarm, Earthquake) $= \alpha \cdot P(B, A, E)$

$$= 1724.13 \times 0.0000019$$

$$= 0.003.$$

L.H.S = R.H.S

$\therefore$ Hence proved

(vi) $P(\text{Alarm} \mid \text{JohnCalls}, \neg\text{Earthquake}) = 0.003$

$$L.H.S = P(A \mid J, \neg E) = \frac{P(A, J, \neg E)}{P(J, \neg E)} \quad \text{——} (M)$$

$$P(A, J, \neg E) = P(J \mid A, \neg E) \cdot P(A, \neg E)$$

$$= P(J \mid A) \cdot P(A, \neg E) \quad \text{——} (1)$$

$$P(A, \neg E) = P(A, \neg E, B) + P(A, \neg E, \neg B)$$

$$= P(A \mid B, \neg E)\, P(B, \neg E) + P(A \mid \neg E, \neg B)\, P(\neg B, \neg E)$$

$$= 0.94 \times (0.001)(0.998) + 0.001\, P(\neg B) \times P(\neg E)$$

$$= 0.94 \times 0.001 \times 0.998 + 0.001 \times 0.999 \times 0.998$$

$$P(A, \neg E) = 0.0019352 \quad \text{——} (2)$$

$$P(\neg A, \neg E) = P(\neg A, \neg E, B) + P(\neg A, \neg E, \neg B)$$

$$= P(\neg A \mid \neg E, B)\, P(B)\, P(\neg E) + P(\neg A \mid \neg E, \neg B)\, P(\neg B)\, P(\neg E)$$

$$= 0.06 \times 0.001 \times 0.998 + 0.999 \times 0.999 \times 0.998$$

$$= 0.000059 + 0.996005$$

$$= 0.996064 \quad \text{——} (3)$$

$$P(J, \neg E) = P(J, A, \neg E) + P(J, \neg A, \neg E)$$

$$= P(J \mid A, \neg E)\, P(A, \neg E) + P(J \mid \neg A, \neg E)\, P(\neg A, \neg E)$$

$$= P(J \mid A)\, P(A, \neg E) + P(J \mid \neg A)\, P(\neg A, \neg E)$$

$$= 0.90 \times 0.0019 + 0.05 \times 0.99 = 0.00171 + 0.0495$$

$$= 0.051$$

$\therefore P(J|A)\,P(A\,\neg E)\,/\,P(J,\neg E)$

$$= \frac{0.90 \times 0.0019}{0.051}$$

$$= \frac{0.00171}{0.051}$$

$= 0.033 = R.H.S$

$L.H.S = R.H.S$

$\therefore$ hence proved.

(vii) P(Burglary | JohnCalls, MaryCalls) = 0.284

$L.H.S = P(B \mid J, M) = \alpha \times P(b, j, m)$ —— (M)

$P(b \mid j,m) = \alpha P(b) \sum_a P(j \mid a) P(m \mid a) \sum_e P(a \mid b,e) P(e)$

$= \alpha P(b) \sum_a P(j \mid a) P(m \mid a) \{ P(a \mid b,e) P(e) + P(a \mid b, \neg e) P(\neg e) \}$

$= \alpha P(b) [P(j \mid a) P(m \mid a) \{ P(a \mid b,e) P(e) + P(a \mid b, \neg e) P(\neg e) \}$
$+ P(j \mid \neg a) P(m \mid \neg a) \{ P(\neg a \mid b,e) P(e) + P(\neg a \mid b, \neg e) P(\neg e) \}]$

$= \alpha \times 0.001 \times (0.9 \times 0.7 (0.95 \times 0.002 + 0.94 \times 0.998)$
$+ 0.05 \times 0.01 \times (0.05 \times 0.002 + 0.71 \times 0.998))$

$= \alpha \times 0.00059$ —— (1) $\rightarrow [P(b,j,m) = 0.00059]$

$P(\neg b \mid j,m) = \alpha P(\neg b) \sum_a P(j \mid a) P(m \mid a) \sum_e P(a \mid \neg b, e) P(e)$

$= \alpha P(\neg b) \sum_a P(j \mid a) P(m \mid a) \{ P(a \mid \neg b, e) P(e) + P(a \mid \neg b, \neg e) P(\neg e) \}$

$= \alpha P(\neg b) [P(j \mid a) P(m \mid a) \{ P(a \mid \neg b, e) P(e) + P(a \mid \neg b, \neg e) P(\neg e) \}$
$+ P(j \mid \neg a) P(m \mid \neg a) \{ P(\neg a \mid \neg b, e) P(e) + P(\neg a \mid \neg b, \neg e) P(\neg e) \}]$

$= \alpha \times 0.999 \times (0.9 \times 0.7 (0.29 \times 0.002 + 0.001 \times 0.998)$
$+ 0.05 \times 0.01 \times (0.71 \times 0.002 + 0.999 \times 0.998))$

$= \alpha \times 0.0015$ —— (2) $\rightarrow [P(\neg b, j, m) = 0.0015]$

$$\alpha = \frac{1}{(P(b, j, m) + P(\neg b, j, m))}$$

from (1) and (2)

$$\alpha = \frac{1}{0.00059 + 0.0015}$$

$$\alpha = 478.5$$

(M) $\Rightarrow P(b \mid j, m) = \alpha P(b, j, m)$

$$= 478.5 \times 0.00059$$

$$= 0.28 = R.H.S$$

L.H.S = R.H.S

$\therefore$ hence proved.

Q2. Conditional independence relations

(i) Markov blanket, non-descendants, d-separation

(ii) Markov blanket

(iii) Non-descendants

(iv) d-separation

(v) d-separation