Home Work 3

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1) Initial state.

The prior probability of getting enough sleep $P(S)$ with no observation is 0.7.

$P(S_0)$
0.7

$Slept_0 \longrightarrow Slept_1$

S_{t-1}	$P(S_t)$
t	0.8
f	0.3

Conditional probability Table P(CPT)

→ The probability of getting enough sleep at night (t) is 0.8 given that the student got enough sleep the previous night, & 0.3 if not

$P(S_t \mid S_{t-1}) = 0.8$

$P(S_t \mid \sim S_{t-1}) = 0.3$

In Observation Model

→ The probability of having red eyes is 0.2, If the student got enough sleep & 0.7 if not

$P(R_t / S_t) = 0.2$, $P(R_t / \sim S_t) = 0.7$

→ The probability of sleeping in class is 0.1, if the student got enough sleep, & 0.3 if not,

$P(SC_t / S_t) = 0.1$, $P(SC_t / \sim S_t) = 0.3$

(Slept t)

R_t SC_t

S_t	$P(R_t)$
t	0.2
f	0.7

S_t	$P(SC_t)$
t	0.1
f	0.3

Conditional Probability Table.

Transitions + observation = slice.

S_{t-1}	$P(S_t)$
t	0.8
f	0.3

$P(S_0)$
0.7

$Slept_0 \longrightarrow Slept_t$

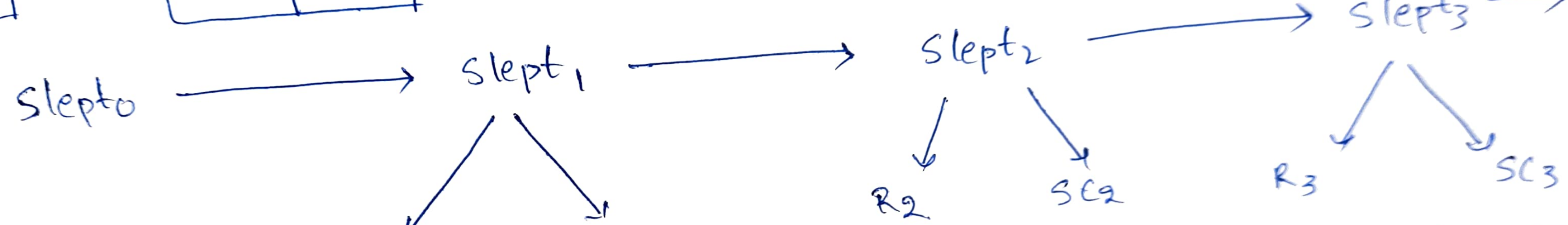
R_1

SC_1

S_t	$P(R_t)$
t	0.2
f	0.7

S_t	$P(SC_t)$
t	0.1
f	0.3

Replicating, Slices "Unrolling"



P(S0)
0.7

S_{t-1}	$P(S_t)$
t	0.8
f	0.3

S_{t-1}	$P(S_t)$
t	0.8
f	0.3

S_{t-1}	$P(S_t)$
t	0.8
f	0.3

S_t	$P(R_t)$
t	0.2
f	0.7

S_t	$P(SC_t)$
t	0.1
f	0.3

S_t	$P(R_t)$
t	0.2
f	0.7

S_t	$P(SC_t)$
t	0.1
f	0.3

Dynamic Bayesian network for our example.

For the hidden Markov Model, the table for $P(E_{t+1} | E_t)$ stays the same for $P(S_t, R_t)$ we assume that S_t & R_t are conditionally independent given E_t:

E_t	r_t, S_t	$r_t, \neg S_t$	$\neg r_t, S_t$	$\neg r_t, \neg S_t$
1	0.02	0.18	0.08	0.72
0	0.21	0.49	0.09	0.21

a) state estimation

We apply the forward algorithm to compute these probabilities

$$P(S_0) = (0.7, 0.3)$$

$$P(S_1) = \sum_{S_0} P(S_1 | S_0) P(S_0)$$

$$= ((0.8, 0.2) 0.7 + (0.3, 0.7) 0.3)$$

$$= (0.65, 0.35)$$

$$P(S_1 | e_1) = \alpha P(e_1 | S_1) P(S_1)$$

$$= \alpha (0.8 \times 0.9, 0.3 \times 0.7) (0.65, 0.35)$$

$$= \alpha (0.72, 0.21) (0.65, 0.35)$$

$$= (0.8643, 0.1357)$$

$$P(S_2 | e_1) = \sum_{S_1} P(S_2 | S_1) P(S_1 | e_1)$$

$$= (0.7321, 0.2679)$$

$P(S_2 | e_{1:2}) = \alpha \, P(e_2 | S_2) \, P(S_2 | e_1)$

$= (0.5010, \ 0.4490)$

$P(S_3 | e_{1:2}) = \sum_{S_2} P(S_3 | S_2) \, P(S_2 | e_{1:2})$

$= (0.5505, \ 0.4495)$

$P(S_3 | e_{1:3}) = \alpha \, P(e_3 | S_3) \, P(S_3 | e_{1:2})$

$= (0.1045, \ 0.8955)$

Similarly to many students during the course of School term, it seems to have a higher likelihood of being sleep deprived

b) $P(e_3 | S_3) = (0.2 \times 0.1, \ 0.7 \times 0.3)$

$= (0.02, \ 0.21)$

$P(e_3 | S_2) = \sum_{S_3} P(e_3 | S_3) \, P(S_3) \, P(S_3 | S_2)$

$= (0.02 \times 0.8 + 0.21 \times 0.2,$
$0.02 \times 0.3 + 0.21 \times 0.7)$

$= (0.0588, \ 0.153)$

$P(e_{2:3} | S_1) = \sum_{S_2} P(e_2 | S_2) \, P(e_3 | S_2) \, P(S_2 | S_1)$

$= (0.0233, \ 0.0556)$

we combines these with forward messages computed previously & normalize.

$$P(S_1 | e_{1:3}) = \alpha P(S_1 | e_1)\, P(e_{2:3} | S_1)$$
$$= (0.7277, 0.2723)$$

$$P(S_2 | e_{1:3}) = \alpha P(S_2 | e_{1:2})\, P(e_3 | S_1)$$

$$= (0.2757, 0.7243)$$

$$P(S_3 | e_{1:3}) = (0.1045, 0.8955)$$

c) The smoothed analysis places the time the student started sleeping poorly one step earlier than the filtered analysis, Integrating future observations indicating lack of sleep at last step.