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A.I. MIDTERM EXAM.

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13.3

a) If $P(a|b,c) = P(b|a,c)$, then $P(a|c) = P(b|c)$

Sol From the definition of conditional probability.

$$P(A, B|C) = \frac{P(A,B,C)}{P(C)} \quad \text{——} \quad (1)$$

and by the product rule, we know

$$P(b,c)\, P(a|b,c) = P(a,c)\, P(b|a,c)$$

[$\because P(a|b,c) = P(b|a,c)$ from question.]

Therefore. $P(a,c) = P(b,c)$,,

$\therefore$ when two sides are equal by product rule. it is divided by $P(c)$

$$\frac{P(a,c)}{P(c)} = \frac{P(b,c)}{P(c)}$$

then $\boxed{P(a|c) = P(b|c)}$ [like eqtn (1)]

The given statement is True.

b) If $P(a|b,c) = P(a)$, then $P(b|c) = P(b)$

Sol :- We have $P(a|b,c) = P(a)$

From the above derivation it clearly states that 'a' is independent of 'b' & 'c'.

The result of event 'A' does not effect the occurence of event 'B' & event 'C'.

by counter example :- a & b, the result of 2 independent coin flips & $C = b$

if a & b events give the results of two independent coin flips, then a and b might not have same results and they are not related and for the third event 'c' which is also an independent flip has nothing to do with a & b, In that case we can't say the relation. So,

This statement is False.

c) If $P(a|b) = P(a)$, then $P(a|b,c) = P(a|c)$

Sol From the statement $P(a|b) = P(a)$ implies that 'a' is independent of 'b'. It does not imply that 'a' is conditionally independent of 'b' given 'c'.

Counter Example:- 'a' and 'b' record the results of two independent coin flips and 'c' equals XOR of a and b

$\therefore$ The given statement is false.

14.1

a) Draw the Bayesian network corresponding to this setup and define the necessary CPTs.

Sol when a coin is drawn randomly from a bag containing 3 coins of (a, b, c) with outcome. Heads (or) Tails with the random variable. 'c' denoting coins (a, b, c)

The network has 'c' at the root and x_1, x_2 and x_3 as children.

The Conditional probability Table for C is

C	P(C)
a	1/3
b	1/3
c	1/3

The conditional probability table for X_i given C are the same, and equal to:

C	X_1	$P(C)$
a	heads	0.2
b	heads	0.6
c	heads	0.8

b) Calculate which coin was most likely to have been drawn from the bag if the observed flips come out heads twice and tails once.

Sol The coin most likely to have been drawn from the bag given this sequence. is the value of C with greatest posterior probability $P(C|2H,1T)$; Now,

$$P(C|2H,1T) = P(2H,1T|C)\,P(C) \,/\, P(2(H),1T)$$

$$\propto P(2H,1T|C)\,P(C)$$

$$\propto P(2H,1T|C)$$

∴ Hence we can observe that the constant of proportionality $1/P(2H,1T)$ is independent of C. and in the last we observe that $P(C)$ is also independent of the value of C Since it is, by hypothesis equal to $1/3$

→ C is independent of $P(2H, 1T)$

'C' is independent of $x_1, x_2\ x_3$

For $C=a$ (If the random variable 'C' is coin 'a').

$P(H|C=a) = 0.2 \quad P(T|C=a) = 0.8$

$P(x_1=T, x_2=H, x_3=H \mid C=a) = P(x_1=T \mid C=a) \times P(x_3=H \mid C=a)$
$P(x_3=H \mid C=a)$

$= 0.8 \times 0.2 \times 0.2.$

$= 0.032.$

→ From CPT we can get same probability for any ordering of 2H & 1T

∴ For '3' orderings $P(2H, 1T \mid C=a)$

$= 3 \times 0.032.$

$= 0.096.$

Similarly for $C=b$

$P(H|C=b) = 0.6 \quad P(T|C=b) = 0.4$

$P(x_1=T, x_2=H, x_3=H, C=b) = P(x_1=T \mid C=b)\, P(x_2=H \mid C=b)$
$P(x_3=H \mid C=b)$

$= 0.4 \times 0.6 \times 0.6$

$= 0.144$

From CPT we can get Some probability for any ordering of 2H & 1T

For '3' orderings $P(2H, 1T \mid c=b) = 3 \times 0.144$
$= 0.432.$

→ Similarly for $c = c$

$P(H \mid C = c) = 0.8$ $\quad P(T \mid C = c) = 0.2$

$$P(X_1 = T, X_2 = H, X_3 = H \mid C = c) = P(X_1 = T \mid C = c)\, P(X_2 = H \mid C = c)\, P(X_3 = H \mid C = c)$$

$$= 0.2 \times 0.8 \times 0.8$$

$$= 0.128$$

→ From conditional probability Table we can get Same probability for any ordering of 2H & 1T.

For $P(2H, 1T \mid C = c) = 3 \times 0.128$
$= 0.384.$

From probability of coin, B has more chances than coin A & C.