Assignment 8

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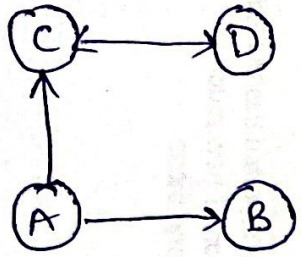
Q1



a) the adjacent matrices for the above graph are

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad M^T = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} h(A) \\ h(B) \\ h(C) \\ h(D) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a(A) \\ a(B) \\ a(C) \\ a(D) \end{bmatrix}$$

$$\begin{bmatrix} a(A) \\ a(B) \\ a(C) \\ a(D) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} h(A) \\ h(B) \\ h(C) \\ h(D) \end{bmatrix}$$

Co-efficient Matrices

$$* \quad M M^T = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+1+0 & 0 & 0 & 0+0+1+0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0+0+0+1 & 0 \\ 0+0+1+0 & 0 & 0 & 0+0+1+0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

* $M^T M = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1+0+0+0 & 1+0+0+0 & 0 \\ 0 & 1+0+0+0 & 1+0+0+1 & 0+0+0+0 \\ 0 & 0 & 0 & 0+0+1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Iteration step hub weights

$\vec{h} = M M^T \vec{h}$ $\quad MM^T = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

Hub non-normalized

Iteration No:	1	2	3	4
A	1	3	5.333	6.001
B	1	0	0	0
C	1	1	0.667	0.286
D	1	2	3.333	3.715

Hub (Normalized)

Iteration NO	1	2	3	4
A	1	2	2.286	2.4
B	1	0	0	0
C	1	0.667	0.286	0.114
D	1	1.333	1.429	1.486

Authority Weights

$\vec{a} = M^T M \vec{a}$ $\quad M^T M = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Authority (non-normalized)

Iteration No:	1	2	3	4
A	1	0	0	0
B	1	2	3.333	3.725
C	1	3	5.333	6.007
D	1	1	0.667	0.286

Authority (normalized)

Iteration No:	1	2	3	4
A	1	0	0	0
B	1	1.333	1.429	1.486
C	1	2	2.286	2.4
D	1	0.667	0.286	0.114

b) Ranking Score of A = h(A) + a(A)
= 2.4 + 0

$\boxed{R(A) = 2.4}$

∴ Ranking order is C, A, D, B

Ranking Score of B = h(B) + a(B) = 0 + 1.486

$\boxed{R(B) = 1.486}$

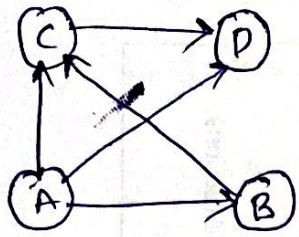
Ranking Score of C = h(C) + a(C) = 0.114 + 2.4

$\boxed{R(C) = 2.514}$

Ranking score of D = h(D) + a(D) = 1.486 + 0.114

$\boxed{R(D) = 1.6}$

Q2



Total number of nodes N = 4.

M(V)

Inlink neighbours to A = ϕ

Inlink neighbours to B = {A}

Inlink neighbours to C = {A, B}

Inlink neighbours to D = {A, C}

C(V)

Out degree of node A = 3

Out degree of node B = 1

Out degree of node C = 1

Out degree of node D = 0

Let initial page ranks for each node be 1 i.e PR

PR(A) = PR(B) = PR(C) = PR(D) = 1. $\alpha = 0.15$.

Iteration 1: $PR(A) = \alpha\left(\frac{1}{N}\right) + (1-\alpha)\sum_{i=1}^{n}\frac{PR(t_i)}{C(t_i)}$

$= 0.15\left(\frac{1}{4}\right) + (1-0.15) \times 0 = 0.0375.$

$PR(B) = \alpha\left(\frac{1}{N}\right) + (1-\alpha)\sum_{i=1}^{n}\left(\frac{PR(t_i)}{C(t_i)}\right) = \frac{0.15}{4} + (1-0.15)\left(\frac{1}{3}\right)$

$= 0.0375 + 0.2833 = 0.3208.$

$PR(C) = \frac{0.15}{4} + (1-0.15)\left[\frac{1}{3}+1\right] = 0.0375 + 1.3333$

$= 1.7083$

$PR(D) = \frac{0.15}{4} + (1-0.15)\left(\frac{PR(A)}{C(A)} + \frac{PR(C)}{C(C)}\right)$

$= \frac{0.15}{4} + 0.85\left(\frac{1}{3}+\frac{1}{1}\right) = 1.7083$

2nd iteration

$PR(A) = \frac{0.15}{4} + (1-0.15)0 = 0.0375$ //

$PR(B) = \frac{0.15}{4} + (1-0.15)\left(\frac{PR(A)}{C(A)}\right) = 0.0375 + 0.85\left[\frac{0.0375}{3}\right]$

$= 0.0481$ //

$PR(C) = \frac{0.15}{4} + (1-0.15)\left[\frac{PR(A)}{C(A)} + \frac{PR(B)}{C(B)}\right]$

$= 0.0375 + 0.85\left[\frac{0.0375}{3} + 0.3208\right]$

$= 0.0375 + 0.85[0.3333]$

$= 0.3208$ //

$PR(D) = \frac{0.15}{4} + (1-0.15)\left[\frac{PR(A)}{C(A)} + \frac{PR(C)}{C(C)}\right]$

$= 0.0375 + 0.85\left[\frac{0.0375}{3} + 1.7083\right]$

$PR(D) = 1.5002$ //