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HW-8

Problem1

n identical balls are thrown into 10 bins, so let $b_1, b_2, b_3, \dots b_{10}$ be number of balls in bin1, bin2, --- ---bin10 respectively. and we can write.

$$b_1 + b_2 + b_3 + \dots b_{10} = n.$$

given condition is that each bin is non empty. that mean $b_i \geq 1$ where $i \in (1, 10)$

let $y_1 = b_1 - 1$, $y_2 = b_2 - 1$, $y_3 = b_3 - 1$ ---- $y_{10} = b_{10} - 1$

$$\Rightarrow y_1 + 1 + y_2 + 1 + y_3 + 1 \dots y_{10} + 1 = n$$

$$\Rightarrow y_1 + y_2 + y_3 + \dots y_{10} = n - 10$$

[If $x_1 + x_2 + \dots x_k = n$ the total no of values for $x_1, x_2 \dots x_k$ is $\binom{n-1+k}{k}$]

$\Rightarrow$ Total no. of configurations with each non empty bin $= \binom{(n-10)-1+10}{10}$

$$= \binom{n-1}{10}$$

Problem 2

No of ways Mr. A and Mr. B do not sit next to each other = Total no. of. ways − No. of ways they sit next to each other.

15!−(A&B)=13

Total no. of. ways 15 people can be seated = $15!$

No. of ways A & B be next to each other = $14! \times 2$.

A & B can be arranged in two ways.

$\therefore$ No of ways A & B do not sit next to each other

$$= 15! - (14! \times 2)$$

$$= 15 \times 14! - (14! \times 2)$$

$$= 14!\,(15-2) = 14! \times 13$$

Problem 3

given $\binom{10}{0}2^0 - 2\binom{10}{1} + 4\binom{10}{2} - 8\binom{10}{3} \cdots + 1024\binom{10}{10}$

$$= \sum_{k=0}^{10} (-1)^k 2^k \binom{10}{k}$$

$$= \sum_{k=0}^{10} (-2)^k \binom{10}{k} = \sum_{k=0}^{10} (-2)^k (1)^{10-k} \binom{10}{k}$$

we know that $\sum_{k=0}^{n} x^k y^{n-k} \binom{n}{k} = (x+y)^n$

$\therefore$ R.H.S $= (-2+1)^{10} = (-1)^{10} = 1$

Problem 4

Given P is a prime number & $1 \le k \le P-1$ $\{ P | \binom{P}{k} \}$

Let $n = \binom{P}{k} = \frac{P!}{(P-k)!\, k!}$ we need to prove $P \mid \binom{P}{k}$

$\Rightarrow n \cdot k! \cdot (P-k)! = P!$ —— (1)

we know that $P! = \cancel{P} \; P \times (P-1)!$ hence P divides $P!$

$\Rightarrow P \mid P!$

$\Rightarrow P \mid n \cdot k! \cdot (P-k)!$ from (1)

but P is prime number. if $k \le P-1$ then $k < P$
so P doesn't divide $k!$ and if $k < P$ the. $(P-k)$ is also less than. P, so $(P-k)!$ is also eliminated because P doesn't divide $(P-k)!$

$\Rightarrow P \mid n$

but $n = \binom{P}{k}$

$\Rightarrow P \mid \binom{P}{k}$ henceproved. //

Problem 5

5 cards are picked from a pack of 52.

so sample space $S = {}^{52}C_5 = \frac{52!}{5!\,47!}$

Event E = 2 cards of 1 kind & 3 cards of another kind.

ways of choosing 2 kinds from 13 = ${}^{13}C_2$.

ways of choosing 2 cards of a kind = ${}^{4}C_2$

ways of choosing 3 cards of a kind = ${}^{4}C_3$.

$\therefore E = {}^{13}C_2 \times {}^{4}C_2 \times {}^{4}C_3$

$$P(E) = \frac{E}{S} = \frac{{}^{13}C_2 \times {}^{4}C_2 \times {}^{4}C_3}{{}^{52}C_5}$$

$$= \frac{\frac{13!}{2!\,11!} \times \frac{4!}{2!\,2!} \times \frac{4!}{3!\,1!}}{\frac{52!}{5! \times 47!}}$$

$$= \frac{\frac{\cancel{13}^{1} \times \cancel{12}^{1}}{\cancel{2}} \times \frac{3 \times \cancel{4}}{\cancel{2}} \times 4}{4\,\frac{\cancel{48} \times 49 \times 50 \times 51 \times \cancel{52}\,4}{5 \times 4 \times 3 \times 2}}$$

$$= \frac{3 \times \cancel{4} \times \cancel{8} \times \cancel{4} \times \cancel{3}^{1} \times \cancel{2}}{\cancel{4} \times 49 \times \cancel{50}_{\cancel{10}\,5} \times \cancel{51}_{17} \times \cancel{4}} = \frac{3}{4,165}$$

Problem 6

In group of 15 people 6 are to be Selected

$\therefore$ Sample Space $S = {}^{15}C_6$

E = Prob (6 doesn't contain both Mr A & Mr B)

= 1 - (6 contain both A & B)

$= 1 - \bar{E}$

$\therefore \bar{E}$ = No of ways 4 (apart from A&B) can be choosen out 13 (15 - A&B)

$$= {}^{13}C_4 = \frac{13!}{4!\,9!} = \frac{\cancel{10}^{5} \times 11 \times \cancel{12} \times 13}{\cancel{4} \times \cancel{3} \times \cancel{2}} = 5 \times 11 \times 13$$

$$P(E) = 1 - P(\bar{E}) = 1 - \frac{\bar{E}}{S}$$

$$= 1 - \frac{{}^{13}C_4}{{}^{15}C_6} = 1 - \frac{5 \times 11 \times 13}{\frac{15 \times 14 \times 13 \times \cancel{12} \times 11 \times 10}{\cancel{6} \times 5 \times 4 \times 3 \times \cancel{2}}}$$

$$= 1 - \frac{\cancel{5} \times \cancel{11} \times \cancel{13} \times \cancel{5} \times \cancel{4} \times \cancel{3}^{\times 1}}{\cancel{15} \times \cancel{14}_{7} \times \cancel{13} \times \cancel{11} \times \cancel{10} \times \cancel{2}}$$

$$= 1 - \frac{1}{7} = 6/7$$