Name - Sindhuja Yerramalla
Email - syrrmlla@memphis.edu

HW 5

Problem 1

Symmetric: Relation R on A is symmetric if for every $(x,y) \in R$ we have $(y,x) \in R$.

Transitive: if ~~wh~~ whenever xRy and yRz then xRz.

Given $A = \{1, 2, 4, 7\}$

let R be relation on A, where

$$R = \{(1,1), (1,2), (1,4), (2,7), (2,4), (4,7)\}$$

case (i) $(1,2) \in R$ & $(2,4) \in R \Rightarrow (1,4) \in R$

Similarly $(2,4) \in R$ & $(4,7) \in R \Rightarrow (2,7) \in R$.

$\therefore$ R is transitive

Case (ii) $(1,2) \in R$ but $(2,1) \notin R$.

$\therefore$ R is not symmetric.

Problem 2:

Given $f: R \to [-2\ 2)$ & $f(x) = \sin(x) + \cos(x)$

* If f is injective. $f(a) = f(b) \Rightarrow a = b$.

let us consider $0, 2\pi \in R$.

$f(0) = \sin(0) + \cos(0) = 0 + 1 = 1$

$f(2\pi) = \sin(2\pi) + \cos(2\pi)$

$= 0 + 1 = 1$

$f(0) = f(2\pi)$

but $0 \neq 2\pi$.

$\therefore$ the given function $f(x)$ is not injective —(1)

* $f: A \to B$ is a surjective if for every $b \in B$ there is some $a \in A$ such that $f(a) = b$

Consider 2, $2 \in [-2, 2]$

Assume $x \in \mathbb{R} \Rightarrow f(x) = 2$

$\Rightarrow \sin(x) + \cos(x) = 2 = 1 + 1$

WKT. Range of $\sin(x)$, $\cos(x)$ is $-1 \leq \sin(x), \cos(x) \leq 1 \; \forall x \in \mathbb{R}$

$\therefore$ The maximum value for $\sin(x)$, $\cos(x)$ should be 1 each

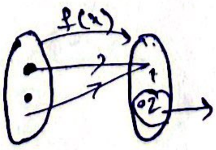
$\Rightarrow \sin(x) = 1, \quad \cos(x) = 1$

$\Rightarrow \sin^2(x) + \cos^2(x) = 1^2 + 1^2 = 1 + 1 = 2$

But $\sin^2(x) + \cos^2(x) = 1 \quad \forall x \in \mathbb{R}$

$\Rightarrow 2 = 1$

This is a contradiction, There is no $x \in \mathbb{R}$ such that $f(x) = 2$.



no incomming arrow

$\therefore$ $f(x)$ is not surjective —(2)

* A function is said to be bijective if it is both injective and Surjective

from (1) & (2) $f(x)$ is not injective neither Surjective

$\therefore$ $f(x)$ is not bijective by the definition

$\Rightarrow$ The given function $f(x)$ is not injective, not Surjective and not bijective.

Problem 3

$f: R\setminus\{1\} \to R\setminus\{2\}$ & $f(x) = \frac{2x-7}{x-1}$

$$f(x) = \frac{2x-7}{x-1}$$

$$= \frac{2x-2}{x-1} - \frac{5}{x-1}$$

$$= 2\left[\frac{x-1}{x-1}\right] - \frac{5}{x-1}$$

$$= 2 - \frac{5}{x-1}$$

$\forall\, x \in R\setminus\{1\}$ there exists $y \in R\setminus\{2\}$ [injective]

& $\forall\, y \in R\setminus\{2\}$ there exists $x \in R\setminus\{1\}$ [Surjective]

where $y = f(x)$

$\therefore$ the function is bijective.

(4)

let $f(x) = y$

$\Rightarrow x = f^{-1}(y)$

$\Rightarrow \cancel{x} = f^{-1}(\cancel{y})$

$$y = f(x) = \frac{2x-7}{x-1}$$

$$y(x-1) = 2x-7$$

$$yx - y = 2x - 7$$

$$7 - y = 2x - yx.$$

$$7 - y = x(2-y)$$

$$\frac{7-y}{2-y} = x.$$

$$\Rightarrow x = \frac{7-y}{2-y}$$

$$f^{-1}(y) = \frac{7-y}{2-y}$$

$$\boxed{\Rightarrow f^{-1}(x) = \frac{7-x}{2-x}} \quad \forall\ x \in R \setminus \{2\}$$

Problem 4

$f: A \to B$ and $g: B \to C$

i) Suppose $g \circ f$ is injective.

let $a, b \in A$.

Suppose $f(a) = f(b)$ then $g \circ f(a) = g(f(a))$
$= g(f(b))$
$= g \circ f(b)$

But since $g \circ f$ in injective $\Rightarrow a = b$

$\therefore f(a) = f(b) \Rightarrow a = b$.

$\therefore f$ is injective //.

ii) let $z \in C$

Since $g \circ f$ is surjective, there exist $x \in A$,

Such that $g \circ f(x) = g(f(x)) = z$

$\therefore$ if we let $y = f(x) \in B$, then.

$\boxed{g(y) = z.}$

$\therefore g$ is surjective //.

Scanned with CamScanner