Sindhuja Yerramalla

Syrrmlla@memphis.edu

U00839259.

HW7

Problem 1

There are 8 odd numbered positions and 7 even numbered positions

Condition A: There will be only 2 choices for filling all the 8 odd numbered positions with same digit [either '0' or '1'].
(Task A)

Condition B: The remaining 7 positions should be filled with '1' or '0' in any order. The total possible ways are 2^7 [2 choices, 7 places]
(Task B)

$\boxed{\text{product principle} = A \times B}$ $= 2 \times 2^7$

$= 2 \times 2^7 = 2^8 = 256$ possible ways.

Problem 2

TA: we can select 2 men out of 10 men in ${}^{10}C_2$ possible ways

TB: we can select 4 women out of 8 women in ${}^{8}C_4$ ways.

product principle $= A \times B = {}^{10}C_2 \times {}^{8}C_4$

The total no of ways 6 people committee can forms. $= \dfrac{10!}{8!\,2!} \cdot \dfrac{8!}{4!\,4!}$ ways. //

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Problem 3

The word "ABRACADABRA" has 11 letters in which there are 5 A's, 2 B's, 2 C's so we divide by $5! \times 2! \times 2!$ [repeated letters] for because we want distinct words.

$\therefore$ Total no of distinct $= \dfrac{11!}{5!\,2!\,2!}$ //

Problem 4

given, mapping $f: \{1,2,3,4,5,6\} \rightarrow \{1,2,3,4,5,6,7,8,9,10\}$

Conditions are $f(1) = f(6)$ and $f(3) > f(5)$

Task 1: $f(1)$ ~~has~~ can be mapped to any number from 1 to 10
$\therefore$ it has 10 ways.

Task 2: $f(2)$ has also 10 ways.

Task 3: $f(3)$ has also 10 ways.

Task 4: $f(4)$ has also 10 ways

Task 5: $f(5)$ should be equal to $f(3)$. There is ~~no~~ only 1 choice.

Task 6: $f(6)$ should be equal to $f(1)$. There is only 1 choice.

$\therefore$ The number functions $= 10 \times 10 \times 10 \times 10 \times 1 \times 1$ [product Principle]

$= 10{,}000$ ways.

Problem 5

i) The namber of non-negative integer solutions for the given equation $x_1+x_2+x_3+x_4+x_5=30$ is. $^{(30+5-1)}C_{(5-1)}$

[as per the counting problem discussed in the class]

$\therefore\ ^{(30+5-1)}C_{(5-1)} = {}^{34}C_4 = \frac{34!}{30!\,4!}$ ways.

ii) given condition $x_i >= 2$. [used $^{n+k-1}C_{k-1}$ formula]

let $y_1 = x_1-2$, $y_2 = x_2-2$, $y_3 = x_3-2$, $y_4 = x_4-2$, $y_5 = x_5-2$.

where $y_i >= 0$ for $i=1,2,3,4,5$

$\therefore$ we can write $y_1+2+y_2+2+y_3+2+y_4+2+y_5+2=30$

$\Rightarrow y_1+y_2+y_3+y_4+y_5 = 30-10$

$\Rightarrow y_1+y_2+y_3+y_4+y_5 = 20$

$\therefore$ The total namber ways for the condition $x_i >= 2$ is

$^{(20+5-1)}C_{(5-1)} = {}^{24}C_4$

$= \frac{24!}{20!\,4!}$

Note: All answers are concluded in generalised form. Not solving at the end because of complicated calculations.

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