

HW9Problem 1

given $n=6$, x is the random variable that is the number of heads is the outcome, x has range $\{0, 1, 2, 3, 4, 5, 6\}$.

probability of success $(P) = 1/2$

$$\Rightarrow q = 1 - P = 1 - 1/2 = 1/2.$$

$$\text{mean} = E(x) = np$$

$$= 6 \times 1/2 = 3 //$$

$$\text{Variance} = \text{Var}(x) = E((x - E(x))^2) = npq$$

$$= 6 \times \frac{1}{2} \times \frac{1}{2} = 1.5 //$$

Problem 2 :-

of ways of arranging n balls in m bins.

$$= \binom{n-1+m}{m}$$

Given bin 1 receives 2 balls & bin m receives 3 balls.

\therefore probab. # for arranging $(n-5)$ balls into $m-2$ bins $= \binom{n-5-1+m-2}{m-2}$

$$= \binom{n+m-8}{m-2}$$

$$\text{Sample space} = \binom{n-1+m}{m}$$

$$\text{Probability} = \frac{\binom{n+m-8}{m-2}}{\binom{n+m-1}{m}}$$

Problem 3 :-

Given Urn A contains 4 red & 7 green
Urn B have 5 red & 10 green

$$\text{Probability of green from A} = \frac{7}{11} = P(E_{AG})$$

$$\text{Probability of getting red from A} = \frac{4}{11} = P(E_{AR})$$

$$\text{Probability of green after green chosen from A \& put it in B} = \frac{11}{16} = P(E_G/E_{AG})$$

Probability of choosing green after Red.
is chosen from A & put it in B = $\frac{10}{16} = P(E_G/E_{AR})$

Probability of choosing green $P(E)$

$$P(E) = P(E_G/E_{AG})P(E_{AG}) + P(E_G/E_{AR})P(E_{AR})$$

$$= \frac{7}{11} \times \frac{11}{16} + \frac{10}{16} \times \frac{4}{11}$$

$$= \frac{77 + 40}{176} = \frac{117}{176} //$$

Problem 4 :-

Given experiment of rolling 10 pair dice

(i) Sample Space size.

Dice \rightarrow	D_1	D_2	D_3	D_4	...	D_{10}
outcome \rightarrow	6	6	6	6	...	6

$$S = 6^{10} //$$

2. Event A = exactly $5^4 3 = \frac{(10)}{(5)} \left(\frac{1}{6}\right)^5 \left(\frac{5}{6}\right)^5$

Event B = exactly $5^3 = \frac{(10)}{(5)} \left(\frac{1}{6}\right)^5 \left(\frac{5}{6}\right)^5$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(A \cap B) = \frac{\# \text{ of ways of rearranging } 5^4 3}{6^{10}}$

$\# \text{ ways of rearranging } 5^4 3 = \frac{10!}{5! 5!}$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = 252$$

$$P(A \cap B) = \frac{252}{6^{10}}$$

$$P(B) = \frac{252}{6^{10}} \times 5^5$$

$$P(A|B) = 1/5^5$$

$$P(A) = \frac{252}{6^{10}} \times 5^5$$

$$P(A/B) \neq P(A)$$

$\therefore A$ & B are not independent

Problem 5

Given a standard pack of 52 cards

5 cards are chosen

size of sample space = $52C_5$

choosing no aces = choosing 5 cards from 48 cards

$$\therefore e. \quad 52 - 4 \text{ Aces} = 48$$

of ways of choosing 5 from 48 = $48C_5$

$$\begin{aligned} \text{Probability that there is no ace} &= \frac{48C_5}{52C_5} \\ &= \frac{35673}{541456} \end{aligned}$$