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Problem 1 : problem 10, Sec 4.3 from Linz.

Proof :-

Proof by contradiction

Assume $L = \{w \in \{a,b,c\}^* : |w| = 3n_a(w)\}$ is regular.

By Pumping Lemma, there is some integer m with the Property given by PL.

consider $w = a^m b^{2m}$ & $w \in L$, $|w| \geq m$.

so w can be split as xyz where (1) $|xy| \leq m$

(2) $|y| > 0$

where $xy^iz \in L$.

from (1) $xy = a^s$ for some $s \leq m$ & let $y = a^t$ for some $0 < t \leq s$.

But then by PL $xy^2z \in L$.

$\underbrace{xy}_{a^s}\ \underbrace{y}_{a^t}\ \underbrace{z}_{a^{m-s}b^{2m}} \Rightarrow a^{m+t} b^{2m} \in L$

since $t > 0$ $\cancel{m+t+2m}$ $3m+3t \neq m+t+2m$

but clearly $a^{m+t} b^{2m} \notin L$. This is contradiction

$\therefore$ $L = \{w \in \{a,b,c\}^* : |w| = 3n_a(w)\}$ is not regular.

Problem 2 :- 9(d), Sec 5.1 from linz

Sol given $L = \{a^n b^m : 2n \leq m \leq 3n\}$

$G = (\{S\}, \{a,b\}, S, P)$.

where production P are. $S \to aSbb \mid aSbbb \mid \lambda$

$S \in \{S\}$ & $\{a, S, b\} \in (V \cup T)^*$

(i) $abb \in L$, (ii) $abbb \in L$.

Problem 3 :- 12(d), Sec 5.1 from linz.

Sol given $L = \{a^n b^m c^k : k = |n-m|\}$

we have two cases (i) $n \geq m$ (ii) $n \leq m$.

case(i) : $n > m$

$k = n - m$.

$k + m = n$. —(1)

Substitute (1) in $L \Rightarrow L = a^k a^m b^m c^k$.

$\therefore$ context free grammar $G = (\{S, S_1\}, \{a,b,c\}, S, P)$

Productions $\Rightarrow$ $S_1 \to aS_1c \mid S_2 \mid \lambda$

$S_2 \to aS_2b \mid \lambda$.

case(ii) $m > n$

$k = m - n$

$k + n = m$. —(2)

substitute (2) in $L \Rightarrow L = a^n b^k b^n c^k$

Productions $\Rightarrow$ $S_3 \to S_2 S_4$

$S_4 \to bS_4c \mid \lambda$

so. the union of 2 cases is

$S \rightarrow S_1 | S_3$

$S_1 \rightarrow aS_1c | S_2 | \lambda$

$S_2 \rightarrow aS_2b | \lambda$

$S_3 \rightarrow S_2S_4$

$S_4 \rightarrow bS_4c | \lambda$.

Problem 4 : Problem 29, sec 5.1 from Linz.

Sol It is given that productions are of the form :

$A \rightarrow v$ such that $|v| = K > 1$.

This shows there cannot be any λ productions.

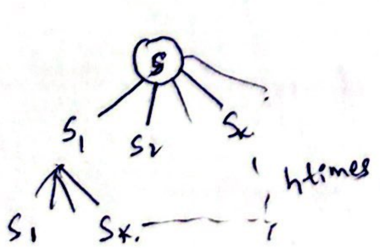
If we draw derivation tree for any $w \in L(G)$ we get a K-ary tree i.e every node has 3 children or no children.

The height of the K-ary tree is minimum if all the leaf nodes are on the same level.

so, if a derivation tree contains h levels & each node gives K children then $|w| = K * K * \text{----} h$ times.

$\Rightarrow |w| = K^h$

$h = \log_h |w|$ — (1)



The height of the K-ary tree is maximum if one of its branch extends continuously.

So let us consider the length of the word is $|w|$ height is h & K is number of branches from each node at each level if we extend just one branch then we get $K-1$ alphabets.

so for h levels, we get $h(K-1)$ alphabets

In the last level, further we don't extend the node.

So we get an extra alphabet.

$\therefore$ total # of alphabets $= (K-1)h+1$, but the total

# of words $= |w|$

$$\Rightarrow |w| = (K-1)h + 1$$

$$\Rightarrow \frac{|w|-1}{K-1} = h. \quad \text{---} (2)$$

from (1) & (2) $\log_K |w| \leq h \leq \frac{|w|-1}{K-1}$ //

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Problem 5 : 9, Sec 6.1 from Linz.

Given production rules P: $S \to AaB \mid aaB$

$A \to \lambda$

$B \to bbA \mid \lambda$

step1: Identifying nullable variables.

$V_0 = \{A, B\}$

$V_1 = \{A, B\}$

Step2: Dropping all λ productions.

(1) dropping $A \to \lambda$

P: $S \to aB \mid aaB$

$B \to bb \mid \lambda$

(2) dropping $B \to \lambda$.

P: $S \to a \mid aa \mid aB \mid aaB$

$B \to bb$ //.

Note:- Referred Textbook, lecture notes and worked with Vamshi Reddy.