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Foundations of Computing HW4

References:- Referred to many websites, text book, lecture notes
cheeg & worked with few classmates.

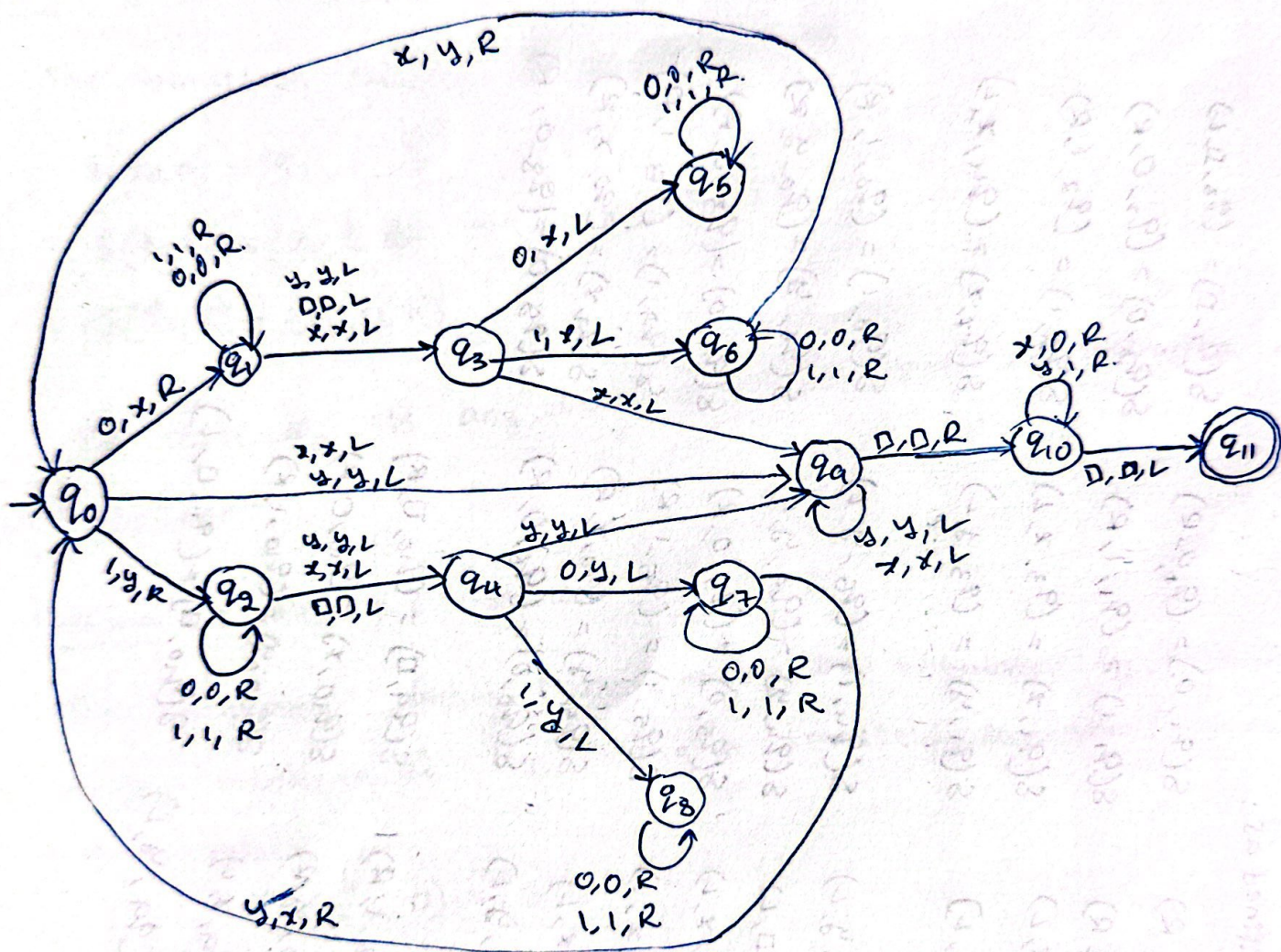
Problem 1: Problem 10, Sec 9.1 from Linz.

Sol Given $f(w) = w^R$, where $w \in \{0,1\}^+$

Let $M = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}\}, \{0,1\}, \{0,1,\square\}, \delta, q_0, \square, \{q_{11}\})$

where δ is defined as:

$\delta(q_0, 0) = (q_1, x, R)$	$\delta(q_1, 0) = (q_2, 0, R)$	$\delta(q_1, \square) = (q_3, \square, L)$
$\delta(q_0, 1) = (q_2, y, R)$	$\delta(q_1, 1) = (q_1, 1, R)$	$\delta(q_1, 0) = (q_2, 0, R)$
$\delta(q_0, x) = (q_0, x, L)$	$\delta(q_1, x) = (q_3, x, L)$	$\delta(q_2, 1) = (q_2, 1, R)$
$\delta(q_0, y) = (q_0, y, L)$	$\delta(q_1, y) = (q_3, y, L)$	$\delta(q_2, x) = (q_4, x, L)$
$\delta(q_2, y) = (q_4, y, L)$	$\delta(q_4, 1) = (q_6, y, L)$	$\delta(q_0, 1) = (q_6, 1, R)$
$\delta(q_2, \square) = (q_4, \square, L)$	$\delta(q_4, y) = (q_9, y, L)$	$\delta(q_6, y) = (q_0, y, R)$
$\delta(q_3, 0) = (q_7, x, L)$	$\delta(q_5, 0) = (q_5, 0, R)$	$\delta(q_7, 0) = (q_8, 0, R)$
$\delta(q_3, 1) = (q_8, x, L)$	$\delta(q_5, 1) = (q_5, 1, R)$	$\delta(q_7, 1) = (q_8, 1, R)$
$\delta(q_3, x) = (q_0, x, L)$	$\delta(q_5, y) = (q_0, x, R)$	$\delta(q_7, x) = (q_0, x, R)$
$\delta(q_4, 0) = (q_5, y, L)$	$\delta(q_6, 0) = (q_6, 0, R)$	$\delta(q_8, 0) = (q_8, 0, R)$
$\delta(q_8, 1) = (q_9, 1, R)$	$\delta(q_9, \square) = (q_{10}, \square, R)$	
$\delta(q_8, x) = (q_0, y, R)$	$\delta(q_{10}, x) = (q_{10}, 0, R)$	
$\delta(q_9, x) = (q_9, x, L)$	$\delta(q_{10}, y) = (q_{10}, 1, R)$	
$\delta(q_9, y) = (q_9, y, L)$	$\delta(q_{10}, \square) = (q_{11}, \square, L)$	



Problem 2 : Problem 8, Sec 9.2 from Linz.

Given macroinstruction:

searchleft(a, q_i, q_j)

$M = (\{q_0, q_i, q_j\}, \{a\}, \{a, b, \square\}, \delta, q_0, \square, \{q_0\})$ is the Turing machine

The transition function δ is defined as

$$\delta(q_0, a) = (q_i, a, L)$$

$$\delta(q_0, b) = (q_0, b, L) \text{ for all } b \in \Gamma - \{a\}$$

$$\delta(q_0, \square) = (q_j, \square, R).$$

The state q_0 is any state where the searchleft instruction may be applied.

Problem 3 : Problem 6, Sec 10.1 from Linz.

The non-erasing Turing Machine M can be simulated by standard Turing machine \hat{M} by adding a transition function (restriction condition) given below.

$$\hat{\delta}(\hat{q}_i, \square) = (\hat{q}_j, \square, L \text{ or } R)$$

Thus the machine \hat{M} on seeing a blank symbol leaves it unchanged and converts it into the input symbol only if it is not a blank symbol.

\therefore No generality is lost by making such restriction on a non erasing Turing Machine.

Problem 4 : Problem 9, Sec 10.1 from Linz

A Turing machine that always writes a symbol other than the one it reads is defined by the transition function given below

$$\delta(q_i, a) = (q_j, b, L \text{ or } R) \text{ where } a, b \text{ are different.}$$

A standard Turing machine can make the following transition,

$$\delta(q_i, a) = (q_j, a, L \text{ or } R).$$

That is, it can rewrite the old symbol before moving the read-write head left or right.

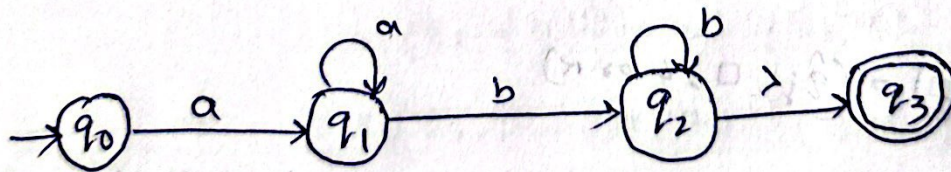
The above discussed Turing machine cannot make such a transition.

Therefore such a Turing machine is less powerful when compared to the standard Turing machine.

Problem 5 : Problem 12, Sec 10.1 from Linz

Transition function for Turing machine for $L(aa^*bb^*)$

$\delta(q_i, \{a, b\}) = (q_j, c, R)$ i.e. move if current symbol is neither a nor b NFA for language.



Turing machine :- $Q = \{q_0, q_1, q_2, q_3, q_f\}$ $\Sigma = \{a, b\}$ $\Gamma = \{a, b, \square\}$
 $F = \{q_f\}$

$$\delta \rightarrow \{(q_0, a) = (q_1, a, R) \\ (q_1, a) = (q_1, a, R)$$

$$(q_1, b) = (q_2, b, R)$$

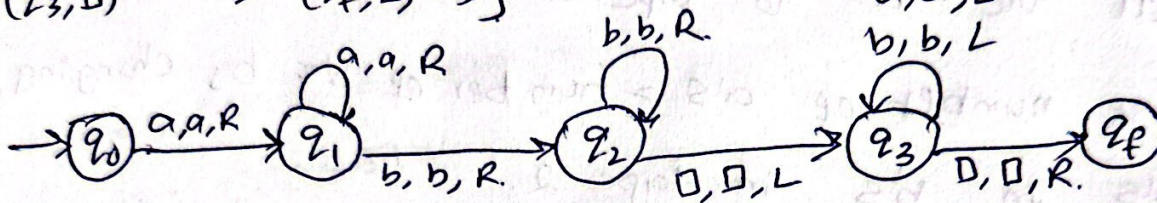
$$(q_2, b) = (q_2, b, R)$$

$$(q_3, \square) = (q_3, \square, L)$$

$$(q_3, b) = (q_3, b, L)$$

$$(q_3, a) = (q_3, a, L)$$

$$(q_3, \square) = (q_4, \square, R)$$



To implement with the transactions like previous exercise.

$$S = \{(q_0, \{b, \square\}) = (q_1, a, R)\}$$

$$(q_1, \{b, \square\}) = (q_1, a, R)$$

$$(q_1, \{a, \square\}) = (q_2, b, R)$$

$$(q_2, \{a, \square\}) = (q_2, b, R)$$

$$(q_2, \{a, b\}) = (q_3, \square, L)$$

$$(q_3, \{a, \square\}) = (q_3, b, L)$$

$$(q_3, \{b, \square\}) = (q_3, a, L)$$

$$(q_3, \{a, b\}) = (q_4, \square, R)$$

Problem 6's Problem 11e, Sec 10.2 from Linz.

We can implement $L = \{n_a(w) = n_b(w) = n_c(w)\}$ using two-tape Turing machine by implementing tape's (as they accept half or stay along with move left & right.

We have 3 steps in implementing this

1. Copy all the a's to tape 2.
2. check if number of a's = number of b's by changing all the a's to b's in tape 2.
3. check if $n_b(w) = n_c(w)$ like the way it is explained

Program for Turing machines:-

$$Q = \{q_0, q_1, q_2, q_f\}, \Sigma = \{a, b, c\}, \Gamma = \{a, b, c, \square\}, F = \{q_f\}$$

$$\delta = \{(q_0, a, \square) \rightarrow (q_0, a, a, R, L)$$

$$(q_0, b, \square) \rightarrow (q_1, b, \square, S, R)$$

$$(q_1, b, a) \rightarrow (q_1, b, a, R, R)$$

$$(q_1, c, \square) \rightarrow (q_2, c, \square, S, L)$$

$$(q_2, c, b) \rightarrow (q_3, c, b, R, L)$$

$$(q_3, \square, \square) \rightarrow (q_f, \square, \square, S, S)\}$$