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FOC - HW - 1

Problem1 :- 23, sec 1.2 From Linz

Let G_1 be the grammer of $S \rightarrow aSb | ab | \lambda$

and G_2 be the grammer of $S' \rightarrow aaSbb | aSb | ab | \lambda$

$G_2 \Rightarrow S' \rightarrow a\underbrace{aSb}_{S}b | aSb | ab | \lambda$

$S \rightarrow aSb | aSb | ab | \lambda$ — (1)

Language of G_1 $L(G_1)$ will be $L(G_1) = \{a^n b^n : n \geq 0\}$
↳ (As proved in class)

Language of G_2 $L(G_2)$ will be $L(G_2) = \{a^n b^n : n \geq 0\}$
↳ [as per (1)]

* Two grammars G_1 and G_2 are equivalent if they generate the same language.

As we can see $L(G_1) = L(G_2)$, we can say the grammars G_1 and G_2 are equivalent.

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Problem 2 14 (a), (b), (d), (e), (f) sec 1.2 from linz.

given $\Sigma = \{a, b\}$

a) Grammar for the sets of all strings with exactly two a's

Productions $P \Rightarrow S \to aaA \mid aAa \mid Aaa \mid aAaA \mid AaAa$

$A \to bA \mid \lambda$

$G = (\{S, A\}, \{a, b\}, S, P)$

b) all strings with atleast two a's

Productions $P \Rightarrow S \to aaA \mid aAa \mid Aaa \mid aAaA \mid \cancel{AaAa}\ AS$

$A \to bA \mid aA \mid \lambda$

$G = (\{S, A\}, \{a, b\}, S, P)$

d) all strings with at least three a's

Productions $P \Rightarrow S \to BaBaBa \mid aBaBaB \mid BaaaB$

$B \to Ba \mid Bb \mid \lambda$

$G = (\{S, B\}, \{a, b\}, S, P)$

e) all strings that start with a and end with b

Productions $P \Rightarrow S \to aAb$

$A \to aA \mid bA \mid \lambda$ $\cancel{G = \{}$

$G = (\{S, A\}, \{a, b\}, S, P)$

f) all strings with an even number of b's

Productions $P \Rightarrow S \to aS \mid bA \mid \lambda$

$A \to aA \mid bS$

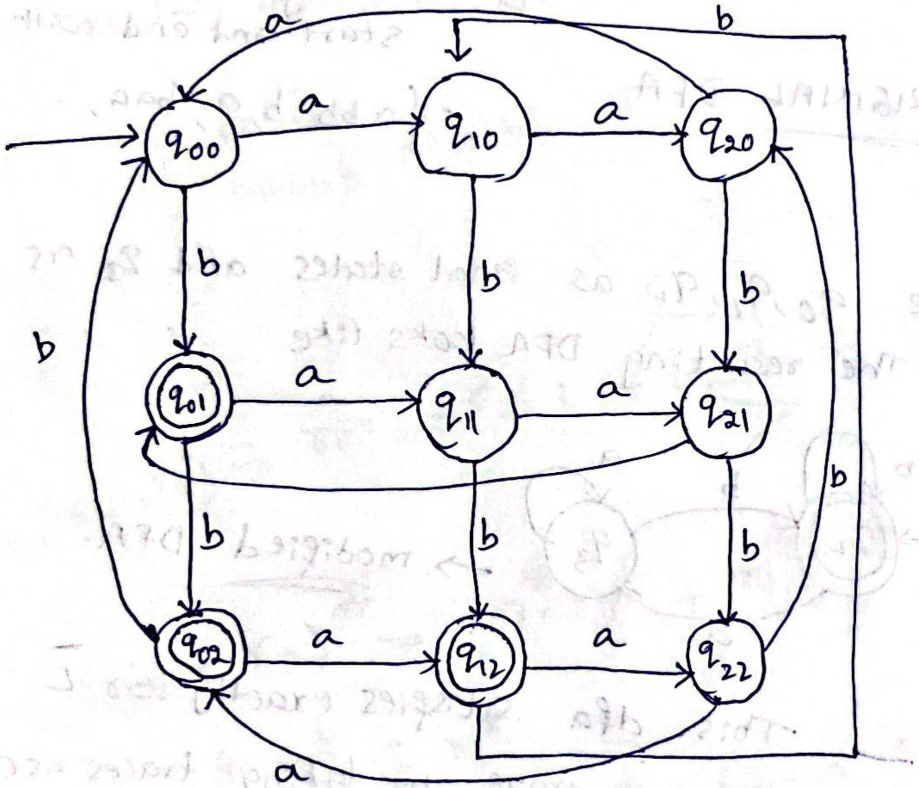
$\therefore G = (\{S, A\}, \{a, b\}, S, P)$

Problem 3

7 d) dfa for $L = \{ w : n_a(w) \bmod 3 < n_b(w) \bmod 3 \}$.

There can be three possible values of mod 3 – 0, 1 and 2

so, create nine states in the DFA each representing remainders of $n_a(w) \bmod 3$ and $n_b(w) \bmod 3$ as a pair.



na	nb
0	1, 2
1	2.

· Initial state is q_0, The state which denotes that $n_a(w) \bmod 3 < n_b(w) \bmod 3$ are states q_{01}, q_{12}, q_{02}.

So the accepting states are q_{01}, q_{02} & q_{12}.

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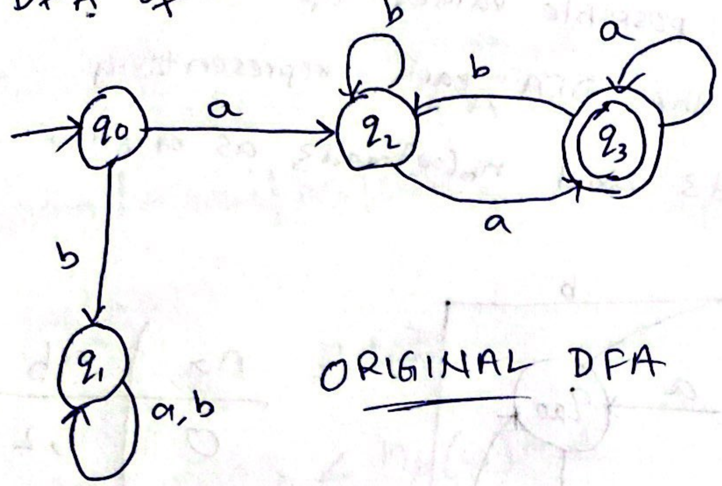
9) Given. $L = \{awa : w \in \{a,b\}^*\}$

DFA of L



ORIGINAL DFA

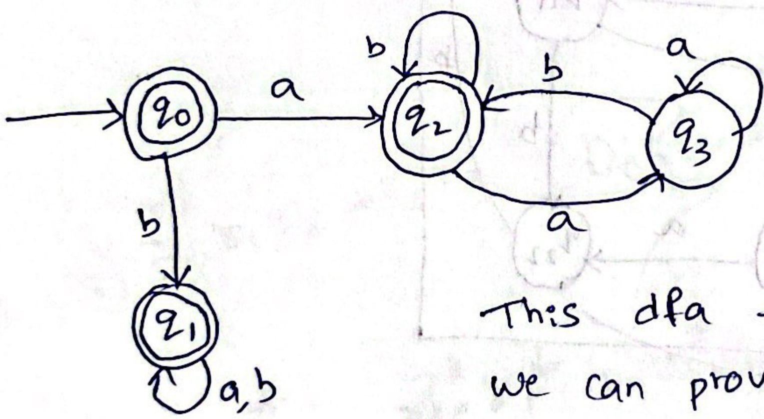
$L = \{awa : w \in \{a,b\}^*\}$

: strings should start and end with 'a'

$\bar{L}$ => strings should not start and end with 'a'

$= \{abb, b, a, baa, \dots\}$

Need to make q_0, q_1, q_2 as final states and q_3 as non-final state. The resulting DFA looks like



→ modified DFA.

This dfa satisfies exactly for $\bar{L}$

we can prove by taking traces as example

Trace 1) abbaa :- if we pass this string in the above dfA of $\bar{L}$, it doesn't end up in final state. It doesn't accepting.

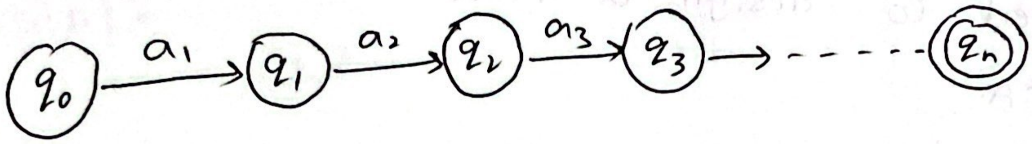
Trace 2) aab :- This string end up in q_2

∴ we can say that the modifications satisfy for $\bar{L}$. and modified DFA accepts $\bar{L}$.

Problem 4: Problem 16, 2.3 from Linz.

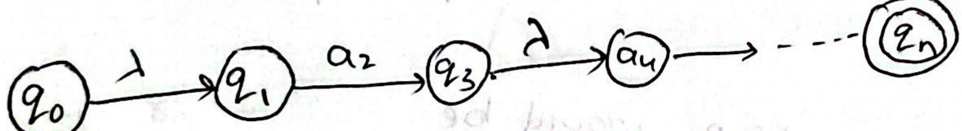
Let L be any regular language & $w = a_1 a_2 a_3 \cdots$

NFA for L be.



$even(w) = a_2 a_4 a_6 a_8 \cdots$

If we replace all the odd transitions with λ transitions then $a_1, a_3, \cdots$ will be λ transitions, the resulting NFA will be like below



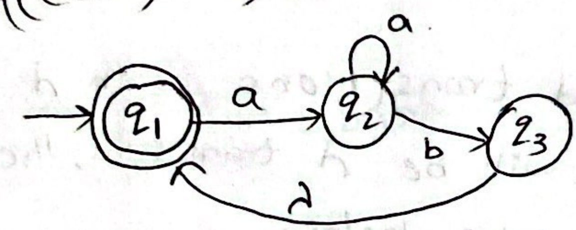
The above NFA accepts even(w). Hence we can say that even(w) is also Regular if L is Regular.

Problem 5 :- 6 (d), (e) from sec 3.2 Linz.

d) $L(((aa^*)^* b)^*)$.

To find a DFA that accepts above Language. We first need to design a NFA and then covert it to DFA.

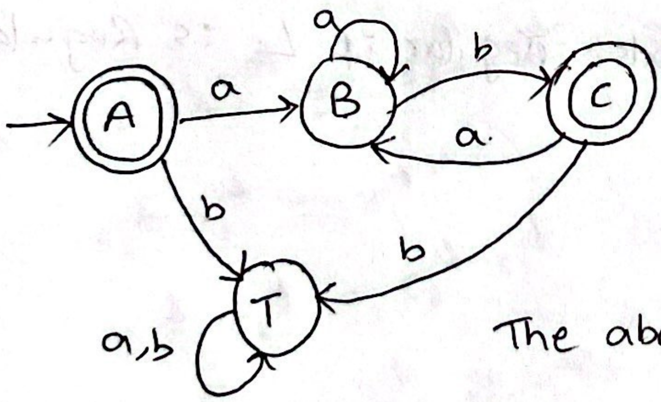
: $(((aa^*)^* b)^*)$



NFA

DFA states	NFA groups	a	b
A	$\{q_1\}$	q_2	Trapstate --
B	$\{q_2\}$	q_2	$\{q_3, q_1\}$
C	$\{q_3, q_1\}$	q_2	Trapstate.

DFA for above NFA would be.



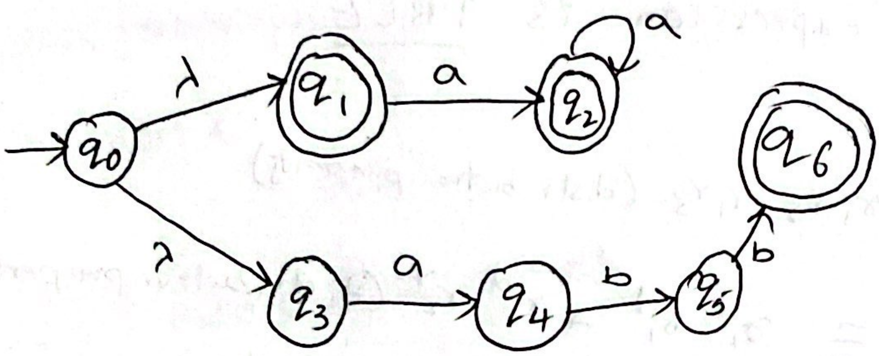
The above. DFA can be checked by two traces.

Trace 1 : The language doesn't consists aabaa, aabaa is not accepted by above DFA

Trace 2 : The language have aab, aab is accepted by above DFA.

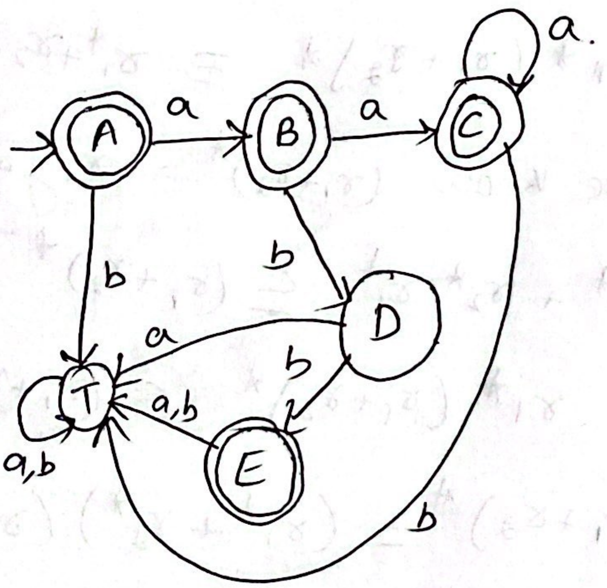
e) $L((aa^*)^* + abb))$

To find a DFA that accepts above Language. we need to design NFA. and then convert it to DFA



NFA

DFA states	NFA state groups	a	b
A	$\{q_0, q_1, q_3\}$	$\{q_2 q_4\}$	ϕ
B	$\{q_2 q_4\}$	q_2	q_5
C	$\{q_2\}$	q_2	ϕ
D	$\{q_5\}$	ϕ	q_6
E	$\{q_6\}$	ϕ	ϕ



DFA

Trace 1 :- Language doesn't have aaabb, aaabb is not accepted by provided DFA.

Trace 2 :- abb. Language has abb, it is also accepted by our DFA//.

Problem 6 : 22 (b), (c), sec 3.1 from Linz.

b) $r_1^*(r_1+r_2)^* \equiv (r_1+r_2)^*$

The ~~st~~ given expression is <u>TRUE</u>.

Proof

$r_1(r_2+r_3) \equiv r_1r_2+r_1r_3$ (distributive property)

$\therefore r_1^*(r_1+r_2)^* \equiv r_1^*r_1^* + r_1^*r_2^*$ (distributive property)

$r_1^*r_1^* \equiv r_1^*$ & $r_1^*r_2^* \equiv r_2^*r_1^*$ (commutative property)

$\Rightarrow r_1^*(r_1+r_2)^* \equiv r_1^*+r_2^*r_1^*$.

But we know $(r_1+r_2)^* = r_1^* + r_2^* + r_1^*r_2^* + r_2^*r_1^* + (r_1^*.r_2^*)^d$

$\because r_1^* + r_2^*r_1^* \subseteq (r_1+r_2)^*$ & $(r_1^*r_2^*)^* \subseteq r_1^*+r_2^*r_1^*$

$\Rightarrow r_1^*(r_1+r_2)^* \subseteq (r_1+r_2)^*$.

$\therefore (r_1+r_2)^* = (r_1^*+r_2^*)(r_1+r_2)^*$.

$(r_1+r_2)^* \subseteq r_1^*(r_1+r_2)^*$.

$\therefore r_1^*(r_1+r_2)^* \subseteq (r_1+r_2)^*$ & $(r_1+r_2)^* \subseteq r_1^*(r_1+r_2)^*$

$\therefore$ The claim is true.

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c) $(r_1 + r_2)^* \equiv (r_1^* r_2^*)^*$

The given expression is TRUE.

Proof

we know that

$$(r_1+r_2)^* = r_1^* + r_2^* + r_1^* r_2^* + r_2^* r_1^* + (r_1^* r_2^*)^*$$

by above statement we can say

$$(r_1+r_2)^* \equiv (r_1^* r_2^*)^*$$ //.

Note:- I have reffered many random websites, textbook, lecture notes, chegg and worked with Vamshi Reddy chirumani and Sankeerth.