2. For each of the following matrices, find all real numbers x such that it is invertible.

$$A = \begin{pmatrix} x & 0 & 0 & 0 \\ 2 & x & 2 & 0 \\ -2 & -3 & 1 & -2 \\ 1 & 0 & 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & x & 4 \\ 1 & -2 & 4 \\ -2 & -1 & x \end{pmatrix}$$

$$|A| = |A|^T = \begin{vmatrix} x & 2 & -2 & 1 \\ 0 & x & -3 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{vmatrix} = x \begin{vmatrix} x & -3 & 0 \\ 2 & 1 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$|A| = x(x+6)$$

$$\therefore x \neq 0, \; x \neq -6$$

$$|B| = \begin{vmatrix} 2 & x & 4 \\ 1 & -2 & 4 \\ -2 & -1 & x \end{vmatrix}$$

$$= 2\begin{vmatrix} -2 & 4 \\ -1 & x \end{vmatrix} - x\begin{vmatrix} 1 & 4 \\ -2 & x \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ -2 & -1 \end{vmatrix}$$

$$= 2(-2x+4) - x(x+8) + 4(-1-4)$$

$$= -4x + 8 - x^2 - 8x - 20$$

$$= -x^2 - 12x - 12$$

$$= -(x^2 + 12x + 12)$$

$$= -6 \pm 2\sqrt{6}$$

$$\therefore x \neq -6 \pm 2\sqrt{6}$$

3. $|A| = 2$

5. $\left|(2A^{-1})^T\right|$

6. $\left|(2(-A)^T)^{-1}\right|$

5) $= |(2A^{-1})|^T$

$n \times n$

$= (2A^{-1}) = 2^4 |A^{-1}| = \frac{16}{|A|} = 8$

6) $\left|(2(-A)^T)^{-1}\right| = \frac{1}{|2(-A)^T|} = \frac{1}{2^4(-A)}$

$= 16(-1)^4 (A) = \frac{1}{32}$

4. Compute $|A| + |B|$ if $A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 3 \\ 1 & 4 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 3 \\ 0 & 1 & -1 \end{pmatrix}$.

$$|A| = \left|\begin{matrix} 2 & 1 & 3 \\ 1 & 0 & 3 \\ 1 & 4 & 7 \end{matrix}\right| \begin{matrix} 2 & 1 \\ 1 & 0 \\ 1 & 4 \end{matrix}$$

$= (0 + 3 + 12) - (0 + 24 + 7) = 15 - 31 = -16$

$$|B| = \left|\begin{matrix} 2 & 1 & 3 \\ 1 & 0 & 3 \\ 0 & 1 & -1 \end{matrix}\right| \begin{matrix} 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{matrix}$$

$= (0 + 0 + 3) - (0 + 6 + (-1)) = 3 - 5 = -2$

$\therefore |A| + |B| = -18$

1. Determine which of the following equations are linear.

a. $x + 2y + z = 6$

b. $2x - 6y + z = 1$

c. $-\sqrt{2}x + 6^{-\frac{2}{3}}y = 4 - 3z$

d. $3x_1 + 2x_2 + 4x_3 + 5x_4 = 1$

2. For each of the following systems of linear equations, find its coefficient and augmented matrices.

1. $\begin{cases} 2x_1 - x_2 = 6 \\ 4x_1 + x_2 = 3 \end{cases}$

2. $\begin{cases} -x_1 + 2x_2 + 3x_3 = 4 \\ 3x_1 + 2x_2 - 3x_3 = 5 \\ 2x_1 + 3x_2 - x_3 = 1 \end{cases}$

d) is a linear equation

2. $\left(\begin{array}{ccc|c} -1 & 2 & 3 & 4 \\ 3 & 2 & -3 & 5 \\ 2 & 3 & -1 & 1 \end{array}\right)$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ -1 & 1 & 0 & -2 \\ 0 & -1 & 1 & 0 \end{array}\right) \quad \left(\begin{array}{cc|c} 2 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 3 & -1 \end{array}\right)$$

3.

$A = \begin{cases} x + 2y + 3z = 1 \\ -x + y + 0 = -2 \\ 0 + -y + z = 0 \end{cases}$ $\quad B = \begin{cases} 2x + y = 1 \\ -y = -2 \\ 3y = -1 \end{cases}$

4. For each of the following linear systems, write it into the form $A\vec{X} = \vec{b}$ and express $\vec{b}$ as a linear combination of the column vectors of A.

a. $\begin{cases} -x_1 + x_2 + 2x_3 = 3 \\ 2x_1 + 6x_2 - 5x_3 = 2 \\ -3x_1 + 7x_2 - 5x_3 = -1 \end{cases}$

$a = \begin{vmatrix} -1 & 1 & 2 \\ 2 & 6 & -5 \\ -3 & 7 & -5 \end{vmatrix}$ $\quad \vec{b} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ $\quad c_1 = \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$ $\quad c_2 = \begin{pmatrix} 1 \\ 6 \\ 7 \end{pmatrix}$ $\quad c_3 = \begin{pmatrix} 2 \\ -5 \\ -5 \end{pmatrix}$

$\vec{b} = c_1x_1 + c_2x_2 + c_3x_3$