Modeller

MSB 105 - Assignment 4

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```
suppressPackageStartupMessages({
library(tidyverse)
library(lubridate)
library(modelr)
library(broom)
library(lmtest)
library(sandwich)
library(viridis)
})
```

Henter csv. filen:

```
pm2 <- read_csv("data/pm2.csv", show_col_types = FALSE)
```

```
## New names:
## * `` -> ...1
```

Muterer:

```
pm2 <- pm2 %>%
  mutate(
    fnr = str_sub(knr, 1,2),
    aar_f = str_sub(aar)
  )
```

```
head(pm2)
```

```
## # A tibble: 6 x 19
##    ...1 knr     aar knavn    pm2 Menn_ya_p Kvinner_ya_p Total_ya_p inc_k1 inc_k5
##   <dbl> <chr> <dbl> <chr>  <dbl>     <dbl>        <dbl>      <dbl>  <dbl>  <dbl>
## 1     1 0101   2008 Halden 13427      59.7         56.8       58.3   24.5   13.6
## 2     2 0101   2009 Halden 13095      59.8         57.0       58.4   24.4   14.1
## 3     3 0101   2010 Halden 13832      59.6         57.1       58.3   23.9   13.7
## 4     4 0101   2011 Halden 14915      59.8         57.2       58.5   24     14
## 5     5 0101   2012 Halden 15473      59.5         57.0       58.2   23.9   14
## 6     6 0101   2013 Halden 15461      59.0         56.7       57.9   24.1   13.4
## # ... with 9 more variables: uni_k_mf <dbl>, uni_k_m <dbl>, uni_k_f <dbl>,
## #   uni_l_mf <dbl>, uni_l_m <dbl>, uni_l_f <dbl>, Trade_p <dbl>, fnr <chr>,
## #   aar_f <chr>
```

parse_factor funksjonen:

```
pm2 %>%
  mutate(
    fnr = parse_factor(fnr, levels = fnr),
    aar_f = parse_factor(aar_f, levels = aar_f)
  )
```

```
## # A tibble: 2,140 x 19
##     ...1 knr     aar knavn    pm2 Menn_ya_p Kvinner_ya_p Total_ya_p inc_k1 inc_k5
##    <dbl> <chr> <dbl> <chr>  <dbl>     <dbl>        <dbl>      <dbl>  <dbl>  <dbl>
##  1     1 0101   2008 Halden 13427      59.7         56.8       58.3   24.5   13.6
##  2     2 0101   2009 Halden 13095      59.8         57.0       58.4   24.4   14.1
##  3     3 0101   2010 Halden 13832      59.6         57.1       58.3   23.9   13.7
##  4     4 0101   2011 Halden 14915      59.8         57.2       58.5   24     14
##  5     5 0101   2012 Halden 15473      59.5         57.0       58.2   23.9   14
##  6     6 0101   2013 Halden 15461      59.0         56.7       57.9   24.1   13.4
##  7     7 0101   2014 Halden 17164      58.8         56.7       57.7   23.9   13.5
##  8     8 0101   2015 Halden 17427      58.7         56.8       57.8   24     13.7
##  9     9 0101   2016 Halden 18941      58.7         56.6       57.7   24     13.8
## 10    10 0101   2017 Halden 20143      58.9         56.9       57.9   23.7   14
## # ... with 2,130 more rows, and 9 more variables: uni_k_mf <dbl>,
## #   uni_k_m <dbl>, uni_k_f <dbl>, uni_l_mf <dbl>, uni_l_m <dbl>, uni_l_f <dbl>,
## #   Trade_p <dbl>, fnr <fct>, aar_f <fct>
```

muterer:

```
pm2 <- pm2 %>%
  mutate(
    Trade_pc_100K = Trade_p/100000
  )
```

```
head(pm2, n = 4)
```

```
## # A tibble: 4 x 20
##    ...1 knr     aar knavn    pm2 Menn_ya_p Kvinner_ya_p Total_ya_p inc_k1 inc_k5
##   <dbl> <chr> <dbl> <chr>  <dbl>     <dbl>        <dbl>      <dbl>  <dbl>  <dbl>
## 1     1 0101   2008 Halden 13427      59.7         56.8       58.3   24.5   13.6
## 2     2 0101   2009 Halden 13095      59.8         57.0       58.4   24.4   14.1
## 3     3 0101   2010 Halden 13832      59.6         57.1       58.3   23.9   13.7
## 4     4 0101   2011 Halden 14915      59.8         57.2       58.5   24     14
## # ... with 10 more variables: uni_k_mf <dbl>, uni_k_m <dbl>, uni_k_f <dbl>,
## #   uni_l_mf <dbl>, uni_l_m <dbl>, uni_l_f <dbl>, Trade_p <dbl>, fnr <chr>,
## #   aar_f <chr>, Trade_pc_100K <dbl>
```

Modell

```
mod1 <- 'pm2 ~ aar_f + Total_ya_p + inc_k1 + inc_k5 + uni_k_mf + uni_l_mf + Trade_pc_100K'
```

i.

```
lm1 <- lm(mod1, data = pm2, subset = complete.cases(pm2))
```

```
summary(lm1)
```

```
##
## Call:
## lm(formula = mod1, data = pm2, subset = complete.cases(pm2))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8516.6 -1472.1   -29.9  1467.3 15736.3
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -20400.74    2663.02  -7.661 2.79e-14 ***
## aar_f2009        104.15     244.77   0.426 0.670512
## aar_f2010        908.13     245.16   3.704 0.000217 ***
## aar_f2011       1663.93     245.86   6.768 1.68e-11 ***
## aar_f2012       2240.48     247.10   9.067  < 2e-16 ***
## aar_f2013       2869.30     248.31  11.555  < 2e-16 ***
## aar_f2014       2863.22     250.54  11.428  < 2e-16 ***
## aar_f2015       3525.22     253.08  13.929  < 2e-16 ***
## aar_f2016       4274.99     255.81  16.711  < 2e-16 ***
## aar_f2017       5146.33     258.50  19.909  < 2e-16 ***
## Total_ya_p       582.44      38.94  14.957  < 2e-16 ***
## inc_k1          -376.99      30.29 -12.445  < 2e-16 ***
## inc_k5           194.35      22.87   8.498  < 2e-16 ***
## uni_k_mf         -82.02      29.42  -2.788 0.005357 **
## uni_l_mf        1206.86      42.22  28.585  < 2e-16 ***
## Trade_pc_100K    871.99     218.42   3.992 6.77e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2531 on 2124 degrees of freedom
## Multiple R-squared:  0.8346, Adjusted R-squared:  0.8334
## F-statistic: 714.3 on 15 and 2124 DF,  p-value: < 2.2e-16
```

ii. Legger til residualer:

```
pm2 %>%
  add_residuals(lm1)
```

```
## # A tibble: 2,140 x 21
##     ...1 knr     aar knavn    pm2 Menn_ya_p Kvinner_ya_p Total_ya_p inc_k1 inc_k5
##    <dbl> <chr> <dbl> <chr>  <dbl>     <dbl>        <dbl>      <dbl>  <dbl>  <dbl>
##  1     1 0101   2008 Halden 13427      59.7         56.8       58.3   24.5   13.6
##  2     2 0101   2009 Halden 13095      59.8         57.0       58.4   24.4   14.1
##  3     3 0101   2010 Halden 13832      59.6         57.1       58.3   23.9   13.7
##  4     4 0101   2011 Halden 14915      59.8         57.2       58.5   24     14
##  5     5 0101   2012 Halden 15473      59.5         57.0       58.2   23.9   14
##  6     6 0101   2013 Halden 15461      59.0         56.7       57.9   24.1   13.4
##  7     7 0101   2014 Halden 17164      58.8         56.7       57.7   23.9   13.5
```

```
##  8     8 0101   2015 Halden 17427      58.7         56.8       57.8   24     13.7
##  9     9 0101   2016 Halden 18941      58.7         56.6       57.7   24     13.8
## 10    10 0101   2017 Halden 20143      58.9         56.9       57.9   23.7   14
## # ... with 2,130 more rows, and 11 more variables: uni_k_mf <dbl>,
## #   uni_k_m <dbl>, uni_k_f <dbl>, uni_l_mf <dbl>, uni_l_m <dbl>, uni_l_f <dbl>,
## #   Trade_p <dbl>, fnr <chr>, aar_f <chr>, Trade_pc_100K <dbl>, resid <dbl>
```

```
head(pm2, n = 4)
```

```
## # A tibble: 4 x 20
##    ...1 knr     aar knavn    pm2 Menn_ya_p Kvinner_ya_p Total_ya_p inc_k1 inc_k5
##   <dbl> <chr> <dbl> <chr>  <dbl>     <dbl>        <dbl>      <dbl>  <dbl>  <dbl>
## 1     1 0101   2008 Halden 13427      59.7         56.8       58.3   24.5   13.6
## 2     2 0101   2009 Halden 13095      59.8         57.0       58.4   24.4   14.1
## 3     3 0101   2010 Halden 13832      59.6         57.1       58.3   23.9   13.7
## 4     4 0101   2011 Halden 14915      59.8         57.2       58.5   24     14
## # ... with 10 more variables: uni_k_mf <dbl>, uni_k_m <dbl>, uni_k_f <dbl>,
## #   uni_l_mf <dbl>, uni_l_m <dbl>, uni_l_f <dbl>, Trade_p <dbl>, fnr <chr>,
## #   aar_f <chr>, Trade_pc_100K <dbl>
```

i.

Man leser ut gjennomsnittlig kvadratmeterpris for en enebolig ($pm2$) for de forskjellige årene. Vi ser at $pm2$ stiger jevnt og trutt.

ii.

Vi vil si at fortegnene er som forventet. Dersom vi har tolket modellen riktig, så vil $pm2$ være mindre for dem nederste kvintilen (inc_k1) enn for den øverste (inc_k5). Det samme gjelder for de med kort og lang utdanning.

Dette er nok fordi den rikere delen av befolkninge, og de med høyere utdanning, sannsynligvis har mer attraktive eneboliger som gjør at $pm2$ er høyere.

Heteroskedastisitet

i.

```
bptest(lm1)
```

```
##
##  studentized Breusch-Pagan test
##
## data:  lm1
## BP = 352.89, df = 15, p-value < 2.2e-16
```

ii.

Veldig høy p-verdi. Da kan H_0 forkastes og vi kan med sterke bevis si at det foreligger Heteroskedastisitet.

iii.

```
coeftest(lm1)
```

```
##
## t test of coefficients:
##
##                 Estimate Std. Error  t value  Pr(>|t|)
## (Intercept)   -20400.742   2663.022  -7.6607 2.790e-14 ***
## aar_f2009        104.150    244.767   0.4255 0.6705118
## aar_f2010        908.129    245.156   3.7043 0.0002174 ***
## aar_f2011       1663.926    245.857   6.7679 1.685e-11 ***
## aar_f2012       2240.475    247.095   9.0672 < 2.2e-16 ***
## aar_f2013       2869.297    248.315  11.5551 < 2.2e-16 ***
## aar_f2014       2863.224    250.537  11.4283 < 2.2e-16 ***
## aar_f2015       3525.223    253.083  13.9291 < 2.2e-16 ***
## aar_f2016       4274.990    255.812  16.7114 < 2.2e-16 ***
## aar_f2017       5146.326    258.498  19.9086 < 2.2e-16 ***
## Total_ya_p       582.436     38.941  14.9568 < 2.2e-16 ***
## inc_k1          -376.989     30.291 -12.4455 < 2.2e-16 ***
## inc_k5           194.354     22.871   8.4979 < 2.2e-16 ***
## uni_k_mf         -82.023     29.424  -2.7876 0.0053574 **
## uni_l_mf        1206.857     42.219  28.5853 < 2.2e-16 ***
## Trade_pc_100K    871.993    218.422   3.9922 6.768e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
vcovHC(lm1)
```

```
##               (Intercept)    aar_f2009   aar_f2010   aar_f2011   aar_f2012
## (Intercept)    9297989.37 -26519.17426 -34751.3931 -64358.9799 -88195.7750
## aar_f2009       -26519.17  42579.51052  22306.6988  22379.0191  22461.1963
## aar_f2010       -34751.39  22306.69876  41857.2132  22643.0594  22816.5776
## aar_f2011       -64358.98  22379.01911  22643.0594  45210.7304  23406.9880
## aar_f2012       -88195.78  22461.19628  22816.5776  23406.9880  47055.4187
## aar_f2013       -93332.22  22562.49160  23016.0483  23690.1311  24270.5328
## aar_f2014      -128032.51  22647.20878  23232.1454  24076.5421  24791.9383
## aar_f2015      -177893.27  22637.74268  23267.9132  24237.7165  25055.0255
## aar_f2016      -229170.12  22623.80635  23323.0788  24446.1520  25385.7301
## aar_f2017      -231919.09  22624.44448  23352.3686  24515.4258  25408.7607
## Total_ya_p     -134378.95     89.41919    277.8154    681.8928   1112.5721
## inc_k1          -48847.48    -46.78668   -117.7882    188.8338    193.4766
## inc_k5          -26724.41    110.78484    126.8286    397.1950    455.5137
## uni_k_mf        -23624.40   -129.42390   -212.3787   -468.5265   -572.7298
## uni_l_mf         79213.28    -45.36231   -237.3954   -324.3915   -491.9711
## Trade_pc_100K   145568.84    497.16540   1261.8579    987.3383    936.1196
##                  aar_f2013    aar_f2014    aar_f2015    aar_f2016    aar_f2017
## (Intercept)   -93332.21682 -128032.5143 -177893.2733 -229170.1243 -231919.0869
## aar_f2009      22562.49160   22647.2088   22637.7427   22623.8064   22624.4445
## aar_f2010      23016.04825   23232.1454   23267.9132   23323.0788   23352.3686
## aar_f2011      23690.13111   24076.5421   24237.7165   24446.1520   24515.4258
## aar_f2012      24270.53282   24791.9383   25055.0255   25385.7301   25408.7607
```

```
## aar_f2013      49220.90256   25428.8815   25755.4473   26135.5595   26169.5465
## aar_f2014      25428.88146   53475.4422   27156.8674   27482.0673   27045.3309
## aar_f2015      25755.44730   27156.8674   63394.1122   28309.5656   27655.2812
## aar_f2016      26135.55952   27482.0673   28309.5656   75087.4602   28071.1160
## aar_f2017      26169.54649   27045.3309   27655.2812   28071.1160   89424.5717
## Total_ya_p      1311.74280    1662.7240    2349.7551    3130.9906    3266.6554
## inc_k1           -23.25608     237.9932     438.1822     706.9105     723.9683
## inc_k5           419.80206     750.9501     927.6337    1166.2786    1178.1709
## uni_k_mf        -695.90501    -198.2867     136.4018    -110.1222    -816.2879
## uni_l_mf        -632.27758   -2195.0185   -3034.7846   -2540.7427   -1110.7783
## Trade_pc_100K   2510.69810    2684.4013    2764.2300     282.6406    1862.4720
##                  Total_ya_p       inc_k1      inc_k5     uni_k_mf    uni_l_mf
## (Intercept)   -134378.94615 -48847.47803 -26724.4053 -23624.40438 79213.27980
## aar_f2009          89.41919    -46.78668    110.7848   -129.42390   -45.36231
## aar_f2010         277.81538   -117.78822    126.8286   -212.37867  -237.39541
## aar_f2011         681.89276    188.83384    397.1950   -468.52650  -324.39148
## aar_f2012        1112.57212    193.47663    455.5137   -572.72977  -491.97106
## aar_f2013        1311.74280    -23.25608    419.8021   -695.90501  -632.27758
## aar_f2014        1662.72401    237.99318    750.9501   -198.28673 -2195.01848
## aar_f2015        2349.75511    438.18220    927.6337    136.40176 -3034.78456
## aar_f2016        3130.99055    706.91052   1166.2786   -110.12216 -2540.74265
## aar_f2017        3266.65535    723.96826   1178.1709   -816.28793 -1110.77830
## Total_ya_p       2167.75020    426.37025    133.2185     51.21924  -614.02732
## inc_k1            426.37025    801.89764    496.4444    158.26504  -500.25996
## inc_k5            133.21845    496.44438    547.3448    104.53767  -690.28424
## uni_k_mf           51.21924    158.26504    104.5377   1515.96690 -2398.54359
## uni_l_mf         -614.02732   -500.25996   -690.2842  -2398.54359  5463.68941
## Trade_pc_100K   -1619.34164  -2293.03278   -115.1786  -2608.77275   651.94105
##                Trade_pc_100K
## (Intercept)     145568.8365
## aar_f2009          497.1654
## aar_f2010         1261.8579
## aar_f2011          987.3383
## aar_f2012          936.1196
## aar_f2013         2510.6981
## aar_f2014         2684.4013
## aar_f2015         2764.2300
## aar_f2016          282.6406
## aar_f2017         1862.4720
## Total_ya_p       -1619.3416
## inc_k1           -2293.0328
## inc_k5            -115.1786
## uni_k_mf         -2608.7728
## uni_l_mf           651.9410
## Trade_pc_100K    60897.1826
```

iv.

```
pm2 <- pm2 %>%
  add_residuals(lm1)
```

v.

```
pm2 <- pm2 %>%
  mutate(aar_d = make_date(aar))
```

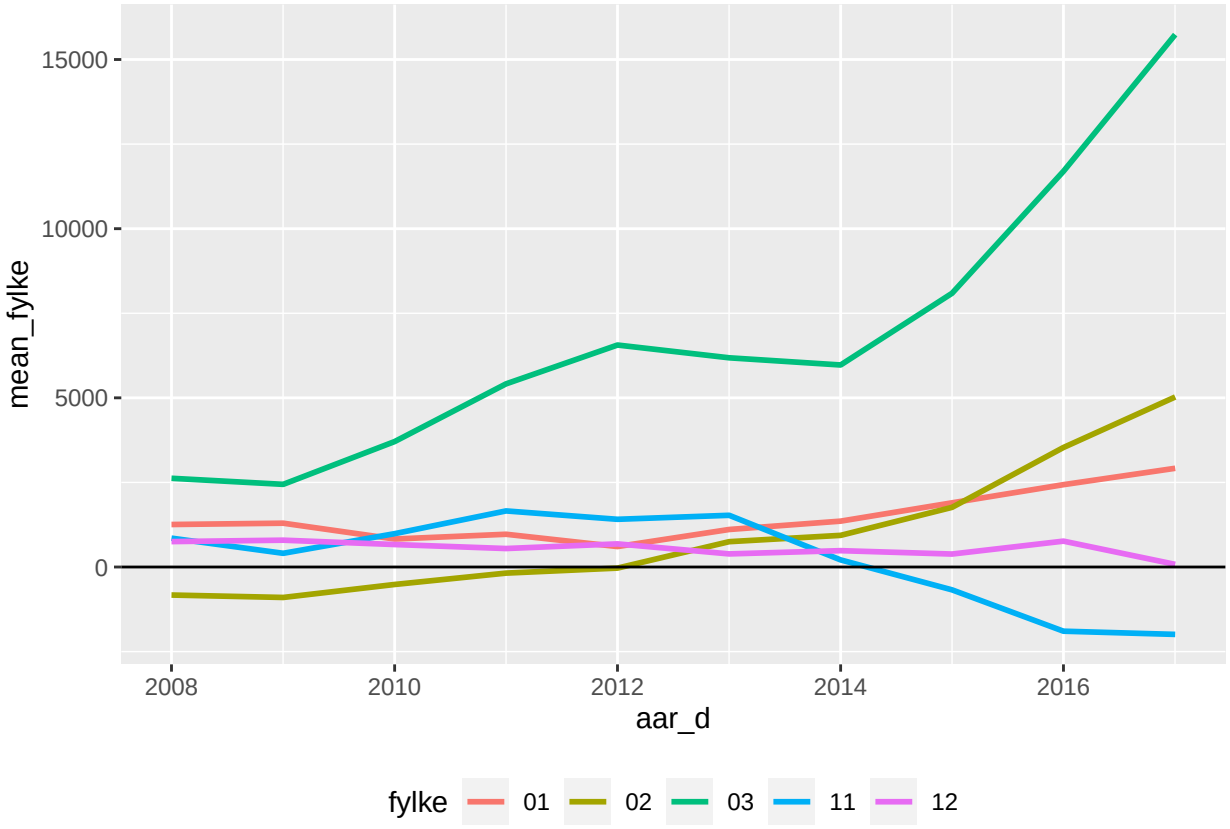
vi.

```
pm2 <- pm2 %>%
  mutate(fylke = substr(knr, start = 1, stop = 2))
```

vii -x.

```
pm2 %>%
  filter(fylke %in% c("01", "02", "03", "11", "12")) %>%
  unnest(c(fylke)) %>%
  group_by(fylke, aar_d) %>%
  summarize(mean_fylke = mean(resid)
            ) %>%
  ggplot(aes(x = aar_d, y = mean_fylke, colour = fylke)) +
  geom_line(lwd=1) +
  theme(legend.position = "bottom")+
  geom_hline(yintercept = 0, colour = "black")
```

```
## `summarise()` has grouped output by 'fylke'. You can override using the `.groups` argument.
```



Dummy fylke og år

i & ii.

```
mod2 <- 'pm2 ~ aar_f*fnr + Total_ya_p + inc_k1 + inc_k5 + uni_k_mf + uni_l_mf + Trade_pc_100K'

lm2 <- lm(mod2, data = pm2)

summary(lm2)
```

```
##
## Call:
## lm(formula = mod2, data = pm2)
##
## Residuals:
##    Min     1Q Median     3Q    Max
##  -8546  -1191     32   1198   8328
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)     -21200.688   2521.645  -8.407  < 2e-16 ***
## aar_f2009           94.009    744.240   0.126 0.899496
## aar_f2010          417.129    744.379   0.560 0.575290
```

```
## aar_f2011         1280.914    744.731   1.720 0.085597 .
## aar_f2012         1455.525    745.679   1.952 0.051088 .
## aar_f2013         2479.533    746.367   3.322 0.000910 ***
## aar_f2014         2795.831    747.254   3.741 0.000188 ***
## aar_f2015         3987.973    748.109   5.331 1.09e-07 ***
## aar_f2016         5264.965    749.169   7.028 2.89e-12 ***
## aar_f2017         6618.572    749.430   8.831  < 2e-16 ***
## fnr02            -1482.789    702.970  -2.109 0.035045 *
## fnr03             3248.234   2190.443   1.483 0.138260
## fnr04            -1049.219    774.264  -1.355 0.175537
## fnr05            -1937.388    758.293  -2.555 0.010696 *
## fnr06            -2172.731    772.094  -2.814 0.004941 **
## fnr07             -737.995   1080.348  -0.683 0.494620
## fnr08            -3213.279    878.620  -3.657 0.000262 ***
## fnr09            -1219.813    913.691  -1.335 0.182020
## fnr10             -281.375    852.265  -0.330 0.741323
## fnr11             -565.360    771.927  -0.732 0.464012
## fnr12             -903.071    742.464  -1.216 0.224012
## fnr14            -3339.829   1182.013  -2.826 0.004768 **
## fnr15            -3619.198    715.832  -5.056 4.69e-07 ***
## fnr16            -1093.217    759.677  -1.439 0.150296
## fnr17            -2005.965    917.216  -2.187 0.028860 *
## fnr18            -1567.503    774.530  -2.024 0.043126 *
## fnr19            -2856.881   1326.142  -2.154 0.031341 *
## fnr20            -2656.315   1180.088  -2.251 0.024500 *
## Total_ya_p         511.787     36.100  14.177  < 2e-16 ***
## inc_k1            -243.050     27.007  -9.000  < 2e-16 ***
## inc_k5             251.645     22.916  10.981  < 2e-16 ***
## uni_k_mf           178.253     28.157   6.331 3.02e-10 ***
## uni_l_mf           732.442     42.235  17.342  < 2e-16 ***
## Trade_pc_100K     1067.760    190.885   5.594 2.54e-08 ***
## aar_f2009:fnr02    -40.505    978.026  -0.041 0.966969
## aar_f2010:fnr02    792.694    978.020   0.811 0.417747
## aar_f2011:fnr02    992.480    978.070   1.015 0.310359
## aar_f2012:fnr02   1565.161    978.102   1.600 0.109716
## aar_f2013:fnr02   1953.373    978.298   1.997 0.045996 *
## aar_f2014:fnr02   2019.269    978.649   2.063 0.039214 *
## aar_f2015:fnr02   2401.120    979.036   2.453 0.014273 *
## aar_f2016:fnr02   3656.344    979.067   3.735 0.000193 ***
## aar_f2017:fnr02   4707.776    979.374   4.807 1.65e-06 ***
## aar_f2009:fnr03     84.133   3068.211   0.027 0.978127
## aar_f2010:fnr03   2004.378   3068.354   0.653 0.513677
## aar_f2011:fnr03   3891.025   3068.768   1.268 0.204970
## aar_f2012:fnr03   5674.403   3069.281   1.849 0.064642 .
## aar_f2013:fnr03   5108.375   3070.149   1.664 0.096297 .
## aar_f2014:fnr03   4938.603   3071.105   1.608 0.107979
## aar_f2015:fnr03   6985.367   3073.112   2.273 0.023131 *
## aar_f2016:fnr03  10264.572   3074.072   3.339 0.000856 ***
## aar_f2017:fnr03  13986.613   3075.071   4.548 5.74e-06 ***
## aar_f2009:fnr04   -330.219   1089.318  -0.303 0.761813
## aar_f2010:fnr04   -191.813   1089.355  -0.176 0.860250
## aar_f2011:fnr04   -775.700   1089.399  -0.712 0.476523
## aar_f2012:fnr04   -808.528   1089.510  -0.742 0.458115
## aar_f2013:fnr04  -1206.685   1089.615  -1.107 0.268240
```

```
## aar_f2014:fnr04  -1456.367   1089.708  -1.336 0.181550
## aar_f2015:fnr04  -1912.336   1089.754  -1.755 0.079446 .
## aar_f2016:fnr04  -2459.017   1089.893  -2.256 0.024169 *
## aar_f2017:fnr04  -3549.658   1089.920  -3.257 0.001146 **
## aar_f2009:fnr05    416.862   1069.758   0.390 0.696816
## aar_f2010:fnr05    655.342   1069.794   0.613 0.540221
## aar_f2011:fnr05    183.865   1069.834   0.172 0.863563
## aar_f2012:fnr05    820.104   1070.017   0.766 0.443507
## aar_f2013:fnr05   -198.536   1070.094  -0.186 0.852832
## aar_f2014:fnr05   -254.055   1070.253  -0.237 0.812388
## aar_f2015:fnr05  -1326.089   1070.254  -1.239 0.215480
## aar_f2016:fnr05  -2117.228   1070.338  -1.978 0.048059 *
## aar_f2017:fnr05  -2397.820   1070.176  -2.241 0.025165 *
## aar_f2009:fnr06   -163.759   1089.292  -0.150 0.880516
## aar_f2010:fnr06    189.332   1089.409   0.174 0.862046
## aar_f2011:fnr06     33.963   1089.394   0.031 0.975132
## aar_f2012:fnr06    800.976   1089.455   0.735 0.462302
## aar_f2013:fnr06    410.281   1089.375   0.377 0.706497
## aar_f2014:fnr06    571.152   1089.474   0.524 0.600167
## aar_f2015:fnr06     22.631   1089.626   0.021 0.983431
## aar_f2016:fnr06   -598.671   1089.701  -0.549 0.582801
## aar_f2017:fnr06     60.036   1089.704   0.055 0.956069
## aar_f2009:fnr07    134.353   1525.051   0.088 0.929808
## aar_f2010:fnr07    728.914   1525.112   0.478 0.632745
## aar_f2011:fnr07    275.017   1525.266   0.180 0.856930
## aar_f2012:fnr07   1047.940   1525.235   0.687 0.492122
## aar_f2013:fnr07    890.998   1525.236   0.584 0.559173
## aar_f2014:fnr07    582.123   1525.332   0.382 0.702772
## aar_f2015:fnr07    990.944   1525.354   0.650 0.515996
## aar_f2016:fnr07    447.813   1525.278   0.294 0.769099
## aar_f2017:fnr07    960.018   1525.236   0.629 0.529146
## aar_f2009:fnr08    329.317   1240.237   0.266 0.790631
## aar_f2010:fnr08   1281.636   1240.345   1.033 0.301597
## aar_f2011:fnr08    646.495   1240.336   0.521 0.602269
## aar_f2012:fnr08   1090.416   1240.413   0.879 0.379470
## aar_f2013:fnr08    575.599   1240.249   0.464 0.642628
## aar_f2014:fnr08    689.084   1240.251   0.556 0.578548
## aar_f2015:fnr08   -776.910   1240.290  -0.626 0.531130
## aar_f2016:fnr08  -1716.491   1240.468  -1.384 0.166595
## aar_f2017:fnr08  -2045.538   1240.415  -1.649 0.099294 .
## aar_f2009:fnr09    686.715   1288.922   0.533 0.594245
## aar_f2010:fnr09    986.486   1288.914   0.765 0.444149
## aar_f2011:fnr09    599.582   1288.944   0.465 0.641860
## aar_f2012:fnr09   1071.846   1289.011   0.832 0.405779
## aar_f2013:fnr09     64.585   1289.204   0.050 0.960050
## aar_f2014:fnr09   -186.541   1289.179  -0.145 0.884965
## aar_f2015:fnr09  -1242.730   1289.232  -0.964 0.335201
## aar_f2016:fnr09  -1987.219   1289.181  -1.541 0.123368
## aar_f2017:fnr09  -3223.036   1289.344  -2.500 0.012510 *
## aar_f2009:fnr10    231.288   1199.909   0.193 0.847172
## aar_f2010:fnr10    924.121   1199.916   0.770 0.441302
## aar_f2011:fnr10    168.648   1199.944   0.141 0.888243
## aar_f2012:fnr10    321.458   1200.216   0.268 0.788856
## aar_f2013:fnr10   -515.180   1200.200  -0.429 0.667793
```

```
## aar_f2014:fnr10   -674.319   1200.339  -0.562 0.574335
## aar_f2015:fnr10  -1492.749   1200.502  -1.243 0.213856
## aar_f2016:fnr10  -3090.918   1200.777  -2.574 0.010124 *
## aar_f2017:fnr10  -3807.142   1200.767  -3.171 0.001545 **
## aar_f2009:fnr11   -414.412   1069.772  -0.387 0.698515
## aar_f2010:fnr11    642.468   1069.866   0.601 0.548235
## aar_f2011:fnr11   1243.418   1070.024   1.162 0.245359
## aar_f2012:fnr11   1467.212   1070.665   1.370 0.170728
## aar_f2013:fnr11   1179.371   1071.062   1.101 0.270979
## aar_f2014:fnr11   -183.391   1071.523  -0.171 0.864124
## aar_f2015:fnr11  -1489.385   1072.451  -1.389 0.165063
## aar_f2016:fnr11  -3274.743   1072.946  -3.052 0.002303 **
## aar_f2017:fnr11  -3863.610   1073.185  -3.600 0.000326 ***
## aar_f2009:fnr12     21.853   1036.805   0.021 0.983186
## aar_f2010:fnr12    381.898   1036.801   0.368 0.712658
## aar_f2011:fnr12    165.379   1036.901   0.159 0.873297
## aar_f2012:fnr12    669.171   1037.128   0.645 0.518864
## aar_f2013:fnr12    -69.430   1037.183  -0.067 0.946636
## aar_f2014:fnr12   -147.825   1037.277  -0.143 0.886690
## aar_f2015:fnr12   -711.755   1037.476  -0.686 0.492767
## aar_f2016:fnr12   -901.775   1037.688  -0.869 0.384941
## aar_f2017:fnr12  -2046.447   1038.104  -1.971 0.048828 *
## aar_f2009:fnr14   -220.698   1663.985  -0.133 0.894498
## aar_f2010:fnr14    536.844   1663.957   0.323 0.747009
## aar_f2011:fnr14   1984.847   1664.012   1.193 0.233090
## aar_f2012:fnr14   1739.551   1664.177   1.045 0.296018
## aar_f2013:fnr14    208.353   1664.208   0.125 0.900381
## aar_f2014:fnr14    253.302   1664.812   0.152 0.879084
## aar_f2015:fnr14  -1695.187   1665.139  -1.018 0.308783
## aar_f2016:fnr14  -1552.417   1665.259  -0.932 0.351330
## aar_f2017:fnr14  -2074.192   1665.271  -1.246 0.213077
## aar_f2009:fnr15    205.720    998.429   0.206 0.836779
## aar_f2010:fnr15    548.008    998.671   0.549 0.583249
## aar_f2011:fnr15    463.880    998.884   0.464 0.642414
## aar_f2012:fnr15    463.860    999.265   0.464 0.642556
## aar_f2013:fnr15      7.994    999.213   0.008 0.993617
## aar_f2014:fnr15   -481.056    999.093  -0.481 0.630220
## aar_f2015:fnr15   -587.449    999.385  -0.588 0.556727
## aar_f2016:fnr15  -1872.887    999.582  -1.874 0.061126 .
## aar_f2017:fnr15  -2799.827    999.681  -2.801 0.005149 **
## aar_f2009:fnr16   -346.631   1069.772  -0.324 0.745955
## aar_f2010:fnr16   -237.962   1069.934  -0.222 0.824020
## aar_f2011:fnr16   -497.945   1069.952  -0.465 0.641705
## aar_f2012:fnr16    380.682   1070.437   0.356 0.722154
## aar_f2013:fnr16   -347.235   1070.757  -0.324 0.745754
## aar_f2014:fnr16   -229.362   1070.812  -0.214 0.830418
## aar_f2015:fnr16   -139.973   1070.880  -0.131 0.896019
## aar_f2016:fnr16  -1074.143   1070.970  -1.003 0.316004
## aar_f2017:fnr16  -2278.453   1070.923  -2.128 0.033499 *
## aar_f2009:fnr17   -288.412   1288.940  -0.224 0.822969
## aar_f2010:fnr17   -422.338   1289.001  -0.328 0.743214
## aar_f2011:fnr17    257.671   1289.086   0.200 0.841590
## aar_f2012:fnr17    637.493   1289.624   0.494 0.621133
## aar_f2013:fnr17    203.405   1289.762   0.158 0.874704
```

```
## aar_f2014:fnr17    -61.073   1289.824  -0.047 0.962239
## aar_f2015:fnr17   -867.834   1289.740  -0.673 0.501107
## aar_f2016:fnr17  -1612.215   1290.487  -1.249 0.211703
## aar_f2017:fnr17  -2761.733   1290.527  -2.140 0.032479 *
## aar_f2009:fnr18   -148.285   1089.412  -0.136 0.891744
## aar_f2010:fnr18    402.939   1089.510   0.370 0.711545
## aar_f2011:fnr18    252.454   1089.674   0.232 0.816812
## aar_f2012:fnr18    482.679   1089.761   0.443 0.657871
## aar_f2013:fnr18    201.272   1090.026   0.185 0.853524
## aar_f2014:fnr18   -393.115   1090.258  -0.361 0.718459
## aar_f2015:fnr18   -439.127   1090.372  -0.403 0.687190
## aar_f2016:fnr18  -1361.291   1090.771  -1.248 0.212178
## aar_f2017:fnr18  -2661.041   1090.689  -2.440 0.014785 *
## aar_f2009:fnr19    453.061   1872.733   0.242 0.808864
## aar_f2010:fnr19    982.125   1872.779   0.524 0.600045
## aar_f2011:fnr19   -669.729   1872.850  -0.358 0.720682
## aar_f2012:fnr19    727.671   1872.902   0.389 0.697670
## aar_f2013:fnr19    278.261   1873.128   0.149 0.881921
## aar_f2014:fnr19   1688.165   1873.121   0.901 0.367563
## aar_f2015:fnr19    369.085   1873.412   0.197 0.843839
## aar_f2016:fnr19    906.286   1873.612   0.484 0.628646
## aar_f2017:fnr19   -716.410   1873.886  -0.382 0.702272
## aar_f2009:fnr20   -927.061   1664.164  -0.557 0.577542
## aar_f2010:fnr20   -547.207   1664.063  -0.329 0.742313
## aar_f2011:fnr20   -542.321   1664.293  -0.326 0.744568
## aar_f2012:fnr20   -378.342   1664.741  -0.227 0.820240
## aar_f2013:fnr20  -1110.163   1664.836  -0.667 0.504960
## aar_f2014:fnr20  -1563.827   1665.176  -0.939 0.347778
## aar_f2015:fnr20  -3266.760   1665.444  -1.961 0.049964 *
## aar_f2016:fnr20  -3169.910   1665.821  -1.903 0.057200 .
## aar_f2017:fnr20  -3922.387   1665.464  -2.355 0.018615 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2105 on 1944 degrees of freedom
## Multiple R-squared:  0.8953, Adjusted R-squared:  0.8848
## F-statistic: 85.21 on 195 and 1944 DF,  p-value: < 2.2e-16
```

iii.

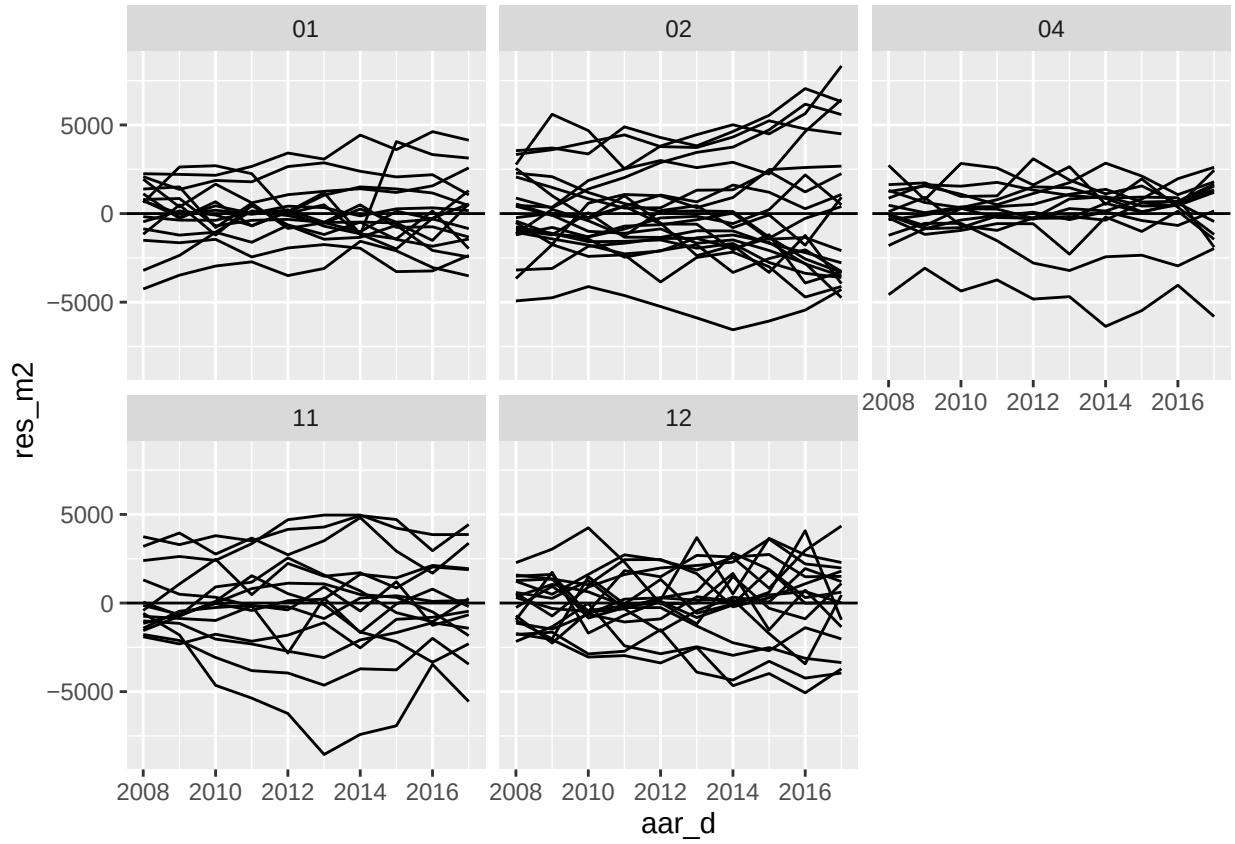
```
pm2 <- pm2 %>%
  mutate(res_m2 = resid(lm2))
```

iv.

Delplott:

```
pm2 %>% filter(fnr %in% c("01", "02", "04", "11", "12")) %>%
ggplot(mapping = aes(x = aar_d, y = res_m2)) +
geom_line(aes(group = knavn)) +
scale_size_manual(values = c(seq(2.0, 0.5, by = -0.1))) +
```

```
geom_hline(yintercept = 0) +
theme(legend.position = 'bottom') +
  facet_wrap(~fylke)
```



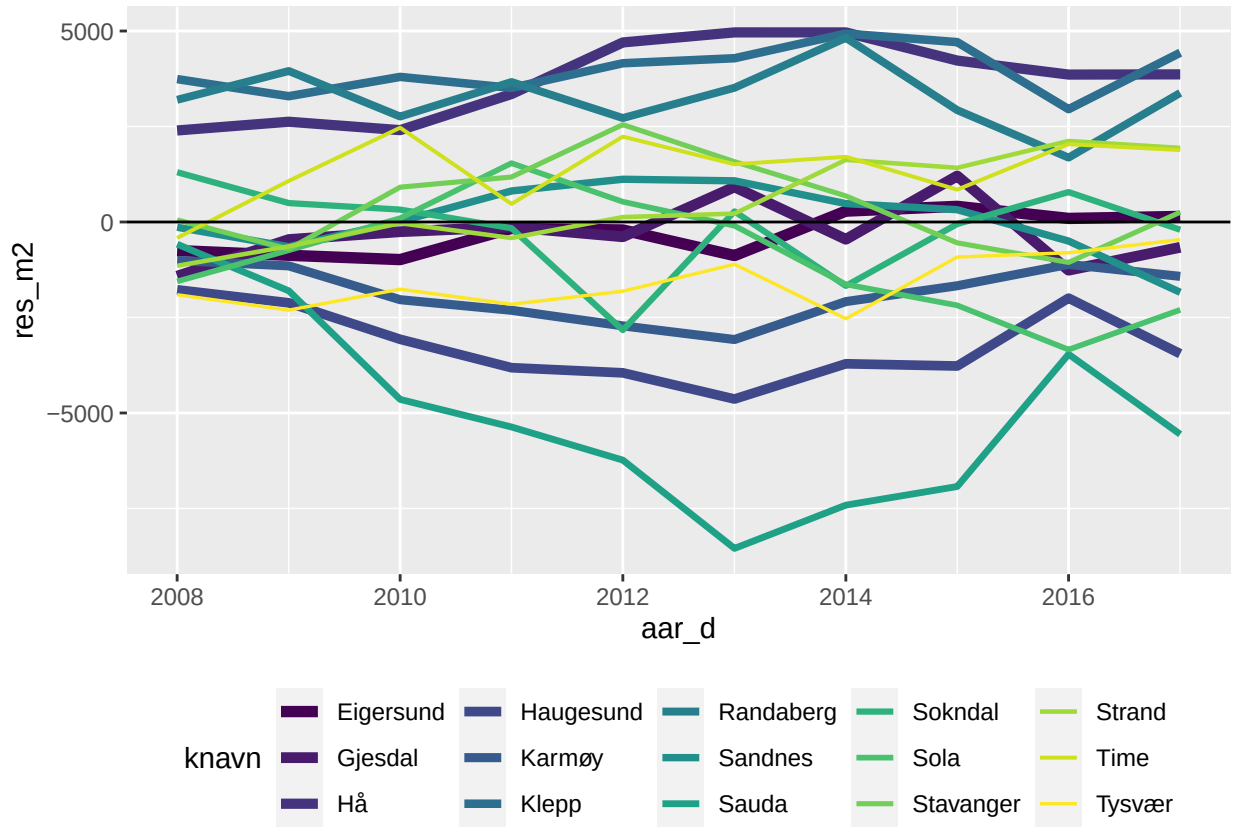
i & ii.

Kvaliteten på modellen er ikke helt optimal da den mangler noen variabler. Dette kan ha noe med heteroskedatisitet i modell at det er stor variasjon. Det er store residualer, spesielt i Rogaland.

Ut i fra grafene så ser man at variasjonen er stor. Dette indikerer et heteroskedastisitetsproblem, og dermed er det grunn til at det er utelatte viktige variabler (brudd på TS.3/TS'.3)

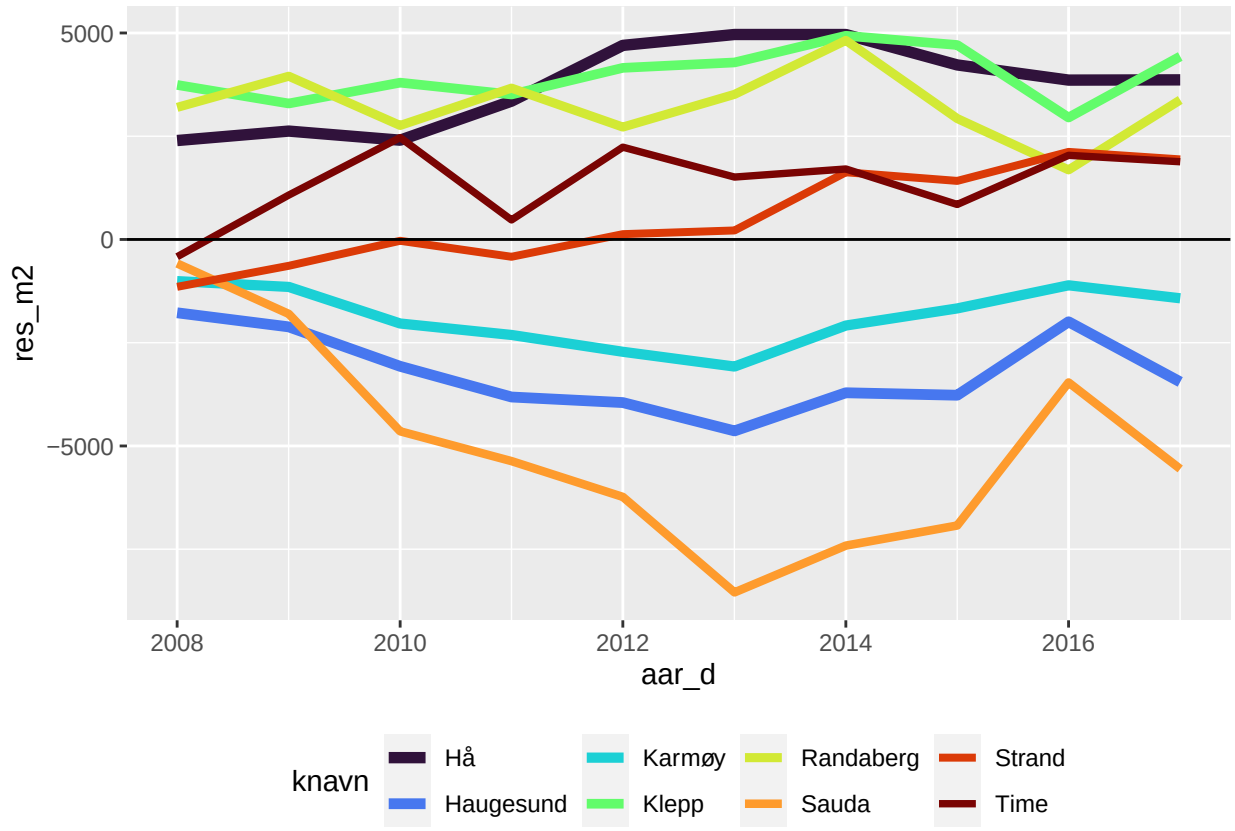
iii.

```
pm2 %>% filter(fnr %in% c("11")) %>%
ggplot(mapping = aes(x = aar_d, y = res_m2)) +
scale_color_viridis(discrete = TRUE, option = "D") +
geom_line(aes(group = knavn, colour = knavn, size =knavn)) +
scale_size_manual(values = c(seq(2.0, 0.5, by = -0.1))) +
geom_hline(yintercept = 0) +
theme(legend.position = 'bottom')
```



i.

```
pm2 %>% filter(knr %in% c("1119", "1120", "1127", "1121", "1130", "1135", "1106", "1149")) %>%
ggplot(mapping = aes(x = aar_d, y = res_m2)) +
scale_color_viridis(discrete = TRUE, option = "H") +
geom_line(aes(group = knavn, colour = knavn, size =knavn)) +
scale_size_manual(values = c(seq(2.0, 0.5, by = -0.1))) +
geom_hline(yintercept = 0) +
theme(legend.position = 'bottom')
```



ii.

Stavanger-kommunene overvurderes (Hå, Klepp og Randaberg).

Modell for hvert år

i.

```
pm2_n <- pm2 %>%
  group_by(aar) %>%
  select(pm2, fnr, knr, aar, aar_f, Menn_ya_p, Kvinner_ya_p, Total_ya_p, inc_k1, inc_k5, uni_k_mf, uni_
  nest()
```

```
pm2_n
```

```
## # A tibble: 10 x 2
## # Groups:   aar [10]
##      aar data
##    <dbl> <list>
##  1  2008 <tibble [214 x 12]>
##  2  2009 <tibble [214 x 12]>
```

```
##  3  2010 <tibble [214 x 12]>
##  4  2011 <tibble [214 x 12]>
##  5  2012 <tibble [214 x 12]>
##  6  2013 <tibble [214 x 12]>
##  7  2014 <tibble [214 x 12]>
##  8  2015 <tibble [214 x 12]>
##  9  2016 <tibble [214 x 12]>
## 10  2017 <tibble [214 x 12]>
```

```
pm2_n$data[[1]] %>%
head(n = 5)
```

```
## # A tibble: 5 x 12
##     pm2 fnr   knr   aar_f Menn_ya_p Kvinner_ya_p Total_ya_p inc_k1 inc_k5
##   <dbl> <chr> <chr> <chr>     <dbl>        <dbl>      <dbl>  <dbl>  <dbl>
## 1 13427 01    0101  2008       59.7         56.8       58.3   24.5   13.6
## 2 18299 01    0104  2008       60.7         58.7       59.7   22.8   16.2
## 3 14981 01    0105  2008       60.9         58.1       59.5   22.2   13.6
## 4 15671 01    0106  2008       59.8         57.8       58.8   21.8   16.2
## 5 18844 01    0111  2008       61.7         61.3       61.5   17.8   19
## # ... with 3 more variables: uni_k_mf <dbl>, uni_l_mf <dbl>,
## #   Trade_pc_100K <dbl>
```

```
dim(pm2_n)
```

```
## [1] 10  2
```

1.

```
kom_model <- function(a_df) {
  lm(pm2 ~ fnr + Total_ya_p + inc_k1 + inc_k5 + uni_k_mf + uni_l_mf + Trade_pc_100K, data = pm2)
}
```

```
pm2_n <- pm2_n %>%
  mutate(model = map(data, .f = kom_model))
```

```
kom_model(pm2_n$aar) %>%
  summary()
```

```
##
## Call:
## lm(formula = pm2 ~ fnr + Total_ya_p + inc_k1 + inc_k5 + uni_k_mf +
##     uni_l_mf + Trade_pc_100K, data = pm2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10648.8  -1602.8   -168.1   1474.5  14320.1
##
## Coefficients:
```

```
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    6480.17    2872.86   2.256 0.024194 *
## fnr02           151.87     314.23   0.483 0.628913
## fnr03          7275.88     959.78   7.581 5.11e-14 ***
## fnr04         -2866.03     317.21  -9.035  < 2e-16 ***
## fnr05         -2728.80     305.31  -8.938  < 2e-16 ***
## fnr06         -2048.90     312.29  -6.561 6.70e-11 ***
## fnr07          -198.84     434.89  -0.457 0.647557
## fnr08         -3439.76     356.42  -9.651  < 2e-16 ***
## fnr09         -2211.31     367.84  -6.012 2.16e-09 ***
## fnr10         -1357.67     346.79  -3.915 9.33e-05 ***
## fnr11          -354.75     345.19  -1.028 0.304213
## fnr12         -1067.22     318.04  -3.356 0.000806 ***
## fnr14         -3685.59     483.41  -7.624 3.68e-14 ***
## fnr15         -3897.81     307.47 -12.677  < 2e-16 ***
## fnr16         -2039.39     304.44  -6.699 2.69e-11 ***
## fnr17         -3222.93     376.12  -8.569  < 2e-16 ***
## fnr18         -2229.33     316.67  -7.040 2.59e-12 ***
## fnr19         -2938.14     530.36  -5.540 3.40e-08 ***
## fnr20         -4283.12     477.15  -8.976  < 2e-16 ***
## Total_ya_p      136.67      42.46   3.219 0.001306 **
## inc_k1         -387.33      33.05 -11.720  < 2e-16 ***
## inc_k5           42.66      26.93   1.584 0.113318
## uni_k_mf        278.27      34.36   8.099 9.28e-16 ***
## uni_l_mf       1030.52      50.75  20.305  < 2e-16 ***
## Trade_pc_100K  1075.31     238.65   4.506 6.97e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2639 on 2115 degrees of freedom
## Multiple R-squared:  0.8209, Adjusted R-squared:  0.8189
## F-statistic: 403.9 on 24 and 2115 DF,  p-value: < 2.2e-16
```

i.

```
pm2_n %>%
  filter(aar%in% c("2008")) %>%
  .$model %>%
  map_df(glance) %>%
  print()
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic p.value    df  logLik    AIC    BIC
##       <dbl>         <dbl> <dbl>     <dbl>   <dbl> <dbl>   <dbl>  <dbl>  <dbl>
## 1     0.821         0.819 2639.      404.       0    24 -19883. 39818. 39965.
## # ... with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

```
mod_sum <- pm2_n %>%
  filter(aar %in% c("2008", "2009", "2010", "2011", "2012", "2013", "2014", "2015", "2016", "2017")) %>%
  mutate(mod_summary = map(.x = model, .f = glance)) %>%
  unnest(mod_summary) %>%
  print()
```

```
## # A tibble: 10 x 15
## # Groups:   aar [10]
##      aar data        model r.squared adj.r.squared sigma statistic p.value    df
##    <dbl> <list>      <lis>     <dbl>         <dbl> <dbl>     <dbl>   <dbl> <dbl>
##  1  2008 <tibble [2~ <lm>      0.821         0.819 2639.      404.       0    24
##  2  2009 <tibble [2~ <lm>      0.821         0.819 2639.      404.       0    24
##  3  2010 <tibble [2~ <lm>      0.821         0.819 2639.      404.       0    24
##  4  2011 <tibble [2~ <lm>      0.821         0.819 2639.      404.       0    24
##  5  2012 <tibble [2~ <lm>      0.821         0.819 2639.      404.       0    24
##  6  2013 <tibble [2~ <lm>      0.821         0.819 2639.      404.       0    24
##  7  2014 <tibble [2~ <lm>      0.821         0.819 2639.      404.       0    24
##  8  2015 <tibble [2~ <lm>      0.821         0.819 2639.      404.       0    24
##  9  2016 <tibble [2~ <lm>      0.821         0.819 2639.      404.       0    24
## 10  2017 <tibble [2~ <lm>      0.821         0.819 2639.      404.       0    24
## # ... with 6 more variables: logLik <dbl>, AIC <dbl>, BIC <dbl>,
## #   deviance <dbl>, df.residual <int>, nobs <int>
```