INF283
Exercise 1

Task 1

a) $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2\cdot 1 + (-1)\cdot 3 & 2\cdot 2 + (-1)\cdot 4 \\ 1\cdot 1 + 3\cdot 3 & 1\cdot 2 + 3\cdot 4 \end{pmatrix} = \underline{\begin{pmatrix} -1 & 0 \\ 10 & 14 \end{pmatrix}}$

b) $A = \begin{pmatrix} -1 & 0 \\ 10 & 14 \end{pmatrix}$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$A^{-1} = \frac{1}{-1\cdot 14 - (0\cdot 10)}\begin{bmatrix} 14 & 0 \\ -10 & -1 \end{bmatrix}$

$= \frac{1}{-14}\begin{bmatrix} 14 & 0 \\ -10 & -1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 \\ \frac{10}{14} & \frac{1}{14} \end{bmatrix} = \underline{\begin{bmatrix} -1 & 0 \\ \frac{5}{7} & \frac{1}{14} \end{bmatrix}}$

c) $\det(C - \lambda I) = 0$, $C = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$

$\lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$C - \lambda I = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & -2 \\ 1 & -1-\lambda \end{bmatrix}$

$\det\begin{bmatrix} 2-\lambda & -2 \\ 1 & -1-\lambda \end{bmatrix} \Rightarrow (2-\lambda)(-1-\lambda) - (-2\cdot 1) = 0$

$-2 - 2\lambda + \lambda + \lambda^2 + 2 = 0$

$\lambda^2 - \lambda = 0 \Rightarrow \lambda(\lambda - 1) = 0 \Rightarrow \underline{\lambda = 0 \wedge \lambda = 1}$

d) $(C - \lambda I) \cdot \vec{v} = \vec{0}$

$\lambda_1 = 0, \quad \lambda_2 = 1$

$\underline{\lambda_1 = 0}$

$$(C - \lambda_1 \cdot I) = \begin{bmatrix} 2-0 & -2 \\ 1 & -1-0 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$

$$(C - \lambda_1 \cdot I) \cdot \vec{v} = \vec{0} \Rightarrow \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Row reduction

$$\left[\begin{array}{cc|c} 2 & -2 & 0 \\ 1 & -1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & -1 & 0 \\ x_1 & x_2 & \end{array}\right] \Rightarrow x_1 - x_2 = 0$$

$$\underline{x_1 = x_2}$$

$2R_2 - R_1 \rightarrow R_1$

Lar $x_1 = 1 \Rightarrow x_2 = 1$

Egenvektor for $C = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}}$, når $\lambda = 0$

$\underline{\lambda_2 = 1}$

$$(C - \lambda_2 I) = \begin{bmatrix} 2-1 & -2 \\ 1 & -1-1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$$

$$(C - \lambda_2 I) \cdot \vec{v} = \vec{0} \Rightarrow \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 1 & -2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \\ x_1 & x_2 & \end{array}\right] \Rightarrow x_1 - 2x_2 = 0$$

$$\underline{x_1 = 2x_2}$$

$R_1 - R_2 \rightarrow R_2$

Lar $x_1 = 1 \Rightarrow x_2 = \frac{1}{2}$

Egenvektor for $C = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}}}$, når $\lambda = 1$

e) $f1(x,y) = x^2 + 2y^2 - xy$

$\nabla f1(x,y) = \left[\frac{\partial f_1}{\partial x}, \frac{\partial f_1}{\partial y}\right] = \underline{\underline{[2x - y, \; 4y - x]}}$

f) $\nabla f1(x,y) = 0$

$$\begin{bmatrix} 2x - y \\ 4y - x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$$

I $2x - y = 0$

$y = 2x$

II $4(2x) - x = 0$

$7x = 0 \Rightarrow \underline{\underline{x = 0 \Rightarrow y = 0}}$

h) $f1(x,y) = x^2 + 2y^2 - xy$

$g(x,y) = 2x + y - 22$

$L(x,y,\lambda) = x^2 + 2y^2 - xy - \lambda(2x + y - 22)$

I $\frac{\partial L}{\partial x} = 0$

II $\frac{\partial L}{\partial y} = 0$

III $\frac{\partial L}{\partial \lambda} = 0$

I $2x - y - 2\lambda = 0$

II $4y - x - \lambda = 0$

III $-2x - y + 22 = 0$

[redacted]

Assuming we can use tools to solve the system.

$\underline{\underline{x = 9, \; y = 4, \; \lambda = 7}}$

g) $P(\text{Konge} \mid !\text{Ess}) = \frac{P(\text{Konge} \cap !\text{Ess})}{P(!\text{Ess})}$

Konge = B

!Ess = A

$= \frac{P(B)\,P(A \mid B)}{P(A)}$

$= \frac{\frac{4}{52} \cdot \frac{1}{1}}{\frac{48}{52}} = \frac{4}{48} = \underline{\underline{\frac{1}{12}}}$

h) Antar "mean" betyr "gjennomsnitt" i denne kontekksten

Mean $\Rightarrow$ $\frac{1+2+3+4+5+6}{6} = \underline{\underline{3,5}} = \bar{X}$

$\text{Var}(X) = \frac{\sum_{i=1}^{6} (X_i - \bar{x})^2}{6}$

$\frac{(1-3,5)^2 + (2-3,5)^2 + \ldots + (6-3,5)^2}{6} = \frac{17,5}{6} = \underline{\underline{\frac{105}{36}}} = 2,91\overline{6}$

Uniform Distribution: Alle verdier er like sannsynlig.

$\Rightarrow$ for alle tenningkast er det like stor sannsynlighet for hver mulige verdi.

2. Machine Learning Problems

2a. Grocery Store Problem

T : Maximize sale by having a sortiment consisting of only relevant groceries.

P : Sale, alternatively amount of sortiment that is thrown away.

E : Data from the other stores, namely sortiment, sale, geography, etc. Would assume it would be relevant to consider other similar datasets, such as the Rossman dataset (https://www.kaggle.com/c/rossmann-store-sales (https://www.kaggle.com/c/rossmann-store-sales)), in order to gain information about possible, relevant categories.

2b. Oil Drilling Problem

Assuming that the goal is to maximize oil production (as stated in the text), and therefore not considering cost, etc.

T : Maximize the production of each platform.

P : Size of the oil production at each platform.

E : Data about the tools (drill size, drill density, etc.), the platforms, the weather, the crew, time, etc.

2c. Autonomous Car Problem

T : Avoid crashing.

P : Number of crashes.

E : Data from the sensors, weather, GPS, etc.

3. K-Nearest-Neighbours

3a. Cat-dog Problem

In [2]:

```
import numpy as np
```

In [7]:

```
cat = np.array((2, 4))
dog = np.array((5, 1))

class_point = np.array((3.5, 2.5))

dist_cat = np.linalg.norm(cat-class_point)
dist_dog = np.linalg.norm(dog-class_point)
print (dist_cat, dist_dog)
```

```
2.1213203435596424 2.1213203435596424
```

NOTE Assuming that we do not need to show calculations for points that are obviously close/far away from classification point.

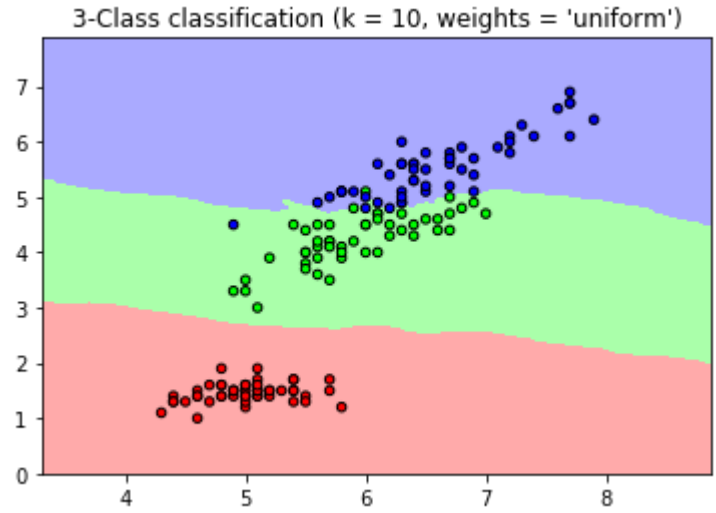
K = 3: Need a tie breaker, because of two points being equally far from the point, namely **[2, 4]** and **[5, 1]** (as shown in code above)
K = 9: Dog

3b. Iris dataset

In [2]:

```
print(__doc__)

# Not in the code from Scikit-learns websitea
%matplotlib inline

import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import ListedColormap
from sklearn import neighbors, datasets

import collections

n_neighbors = 10

# import some data to play with
iris = datasets.load_iris()

# we only take the first two features. We could avoid this ugly
# slicing by using a two-dim dataset
X = iris.data[:, [0,2]]
y = iris.target

h = .02  # step size in the mesh

# Create color maps
cmap_light = ListedColormap(['#FFAAAA', '#AAFFAA', '#AAAAFF'])
cmap_bold = ListedColormap(['#FF0000', '#00FF00', '#0000FF'])

for weights in ['uniform']:
    # we create an instance of Neighbours Classifier and fit the data.
    clf = neighbors.KNeighborsClassifier(n_neighbors, weights=weights)
    clf.fit(X, y)

    # Plot the decision boundary. For that, we will assign a color to each
    # point in the mesh [x_min, x_max]x[y_min, y_max].
    x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h),
                         np.arange(y_min, y_max, h))
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])

    # Put the result into a color plot
    Z = Z.reshape(xx.shape)
    plt.figure()
    plt.pcolormesh(xx, yy, Z, cmap=cmap_light)

    # Plot also the training points
    plt.scatter(X[:, 0], X[:, 1], c=y, cmap=cmap_bold,
                edgecolor='k', s=20)
    plt.xlim(xx.min(), xx.max())
    plt.ylim(yy.min(), yy.max())
    plt.title("3-Class classification (k = %i, weights = '%s')"
              % (n_neighbors, weights))

plt.show()

print(collections.Counter(y))
```

```
Automatically created module for IPython interactive environment
```




```
Counter({0: 50, 1: 50, 2: 50})
```

Correcting number of flowers in each group, manually(?)

In [6]:

```
type(iris)
```

Out[6]:

```
sklearn.utils.Bunch
```

In [2]:

```
print(iris.target_names)
```

```
['setosa' 'versicolor' 'virginica']
```

In [3]:

```
print(y)
```

```
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2
 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
 2 2]
```

In [4]:

```
n_setosa = np.count_nonzero(y == 0) # red
n_versicolor = np.count_nonzero(y == 1) # green
n_virginica = np.count_nonzero(y == 2) # blue
```

In [5]:

```
n_setosa_corrected = n_setosa
n_versicolor_corrected = n_versicolor - 1
n_virginica_corrected = n_virginica + 1
```

In [6]:

```
n_different_classes_corrected = [n_setosa_corrected, n_versicolor_corrected, n_virginic
a_corrected]
```

In [7]:

```
dict(zip(iris.target_names, n_different_classes_corrected))
```

Out[7]:

```
{'setosa': 50, 'versicolor': 49, 'virginica': 51}
```

NOTE : Fully aware that we are able to get the same data from the *collections.Counter(y)* however, I wrote this code before reading this, so I wanted to keep it.

5

Example of a problem where KNN would not work well as a classifier is problems were the dataset is big, and contains a lot of (unnormalized) noise, because

1. KNN stores all of the training data
2. is sensitive to irrelevant features and data scaling.

One example would be (raw) hospital records.