Exercise 6 - Bayesian Inference

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1.1

1. P(11-side die | 6)

P(A) = 11-side die = ⅕
P(B) = 6 = Dependent on which die → P(A)
Bayes Theorem: P(A|B)=(P(B|A)*(P(A)))/P(B)
According to "Tutorial to Bayesian Inference: chapter 2-5 in
https://www.stat.auckland.ac.nz/~brewer/stats331.pdf:

- P(A) = "Think about prior beliefs"
- P(B|A) = "Think about what the experiment is doing: If A is true, what data would you expect to see, and with what probabilities?"
- P(B) = "The probability of obtaining the data *B* but without assuming that *A* is either true or false. This is obtained using the sum rule."

Considering this, I calculate these probabilities to be:

- P(A) = ⅕
- P(B|A) = 1/11
- P(B) = 0 + 0 + 1/7 + 1/11 + 1/20 = 0.28

Which gives: P(A|B) = 0.065

2.

Same procedure.
Bayes Theorem: P(A|B)=(P(B|A)*(P(A)))/P(B), using the same train of thought as stated above.

- P(A) = 20-side die = ⅕
- P(B) = 0 + 0 + 1/7 + 1/11 + 1/20 = 0.28
- P(B|A) = 1/20

Which gives: P(A|B) = 0.036

3.

Not possible to roll an 18 with an 11-sided die → Probability = 0

4.

- P(A) = 20-side die = ⅕
- P(B) = 0 + 0 + 0 + 0 + 1/20 = 0.05
- P(B|A) = 1/20

Which gives: P(A|B) = 0.2

1.2

a) Binomial distribution: (15 9) * $0,5^9$ * $(1 - 0,5)^{15-9}$ = <u>0,15</u>.
b)
   i) Maximum likelihood parameters: Using the P`(D|'theta') = 0 → 'theta'hat = N_H/N_H+N_T: 9 / (9+6) = 9 / 15 = <u>0,6</u>
   ii) Likelihood of the data with these parameters: $P(D|\theta) = \theta^{N_h}(1-\theta)^{N_t}$
   Which gives: 0,00004
   iii) Likelihood when the coin is fair, e.g. theta = 0,5:
   Which gives: 0.00003
   iv) Posterior distribution, i.e. P(theta|D) considering a uniform prior.
   Uniform prior → Uninformativ → alpha = beta = 1.
   Beta(alpha + N_H, beta + N_T) = (1 + 9, 1 + 6) = (10, 7).
   v) Uniform Prior: theta =
      1) P(X=H)
         (a) MLP: N_H/N_H+N_T: 1 / (1 + 0) = 1
         (b) Bayesian Inference: $P(D|\theta) = \theta^{N_h}(1-\theta)^{N_t}$ = 1
      2) P(X=H|D=H)
         (a) MLP: N_H/N_H+N_T: 2 / (2 + 0) = 1
         (b) Bayesian Inference: $P(D|\theta) = \theta^{N_h}(1-\theta)^{N_t}$ = 1
      3) P(X=T|D=HH)
         (a) MLP: N_H/N_H+N_T: 2 / (2 + 1) = 2/3
         (b) Bayesian Inference: $P(D|\theta) = \theta^{N_h}(1-\theta)^{N_t}$ = 0.148
      4) P(X=H|D=HHT)
         (a) MLP: N_H/N_H+N_T: 3 / (3 + 1) = 3/4
         (b) Bayesian Inference: $P(D|\theta) = \theta^{N_h}(1-\theta)^{N_t}$ = 0.105
      5) $P(X$=H|D=HHTH)
         (a) MLP: N_H/N_H+N_T: 4 / (4 + 1) = 4/5
         (b) Bayesian Inference: $P(D|\theta) = \theta^{N_h}(1-\theta)^{N_t}$ = 0.082
      6) $P(X$=T|D=HHTHH)
         (a) MLP: N_H/N_H+N_T: 4 / (4 + 2) = 4/6
         (b) Bayesian Inference: $P(D|\theta) = \theta^{N_h}(1-\theta)^{N_t}$ = 0.022

c) D = {TTTHHHHHHH} → N_T=3, N_H=7.
   i) Maximum likelihood parameters: Using the P`(D|'theta') = 0 → 'theta' hat = N_H/N_H+N_T: 7 / (7+3) = 7 / 10 = <u>0,7</u>
   ii) Likelihood of the data with these parameters: $P(D|\theta) = \theta^{N_h}(1-\theta)^{N_t}$

Which gives: 0,0022

iii) Likelihood when the coin is fair, e.g. theta = 0,5:
Which gives: 0.0009

iv) Posterior distribution, i.e. P(theta|D) considering a uniform prior.
Uniform prior → Uninformativ → alpha = beta = 1.
Beta(alpha + N_H, beta + N_T) = (1 + 7, 1 + 3) = (8, 4).

v) Assuming this is not relevant for this task, seeing as this has its own set of history, D.

1.3

p (sunny | yes) = 2/9
p (hot | yes) = 2/9
p (normal | yes) = 6/9
p (false | yes) = 6/9
p (yes) = 9/15

new data: y = (sunny, hot, normal, false) → p(y | yes) * p(yes):
2/9 * 2/9 * 6/9 * 6/9 * 9/15 = <u>0,013</u>

1.4

The good thing about Naive Bayes is that one can factor out unknown variables, according to https://www.youtube.com/watch?v=EqjyLfpv5oA.
y = (sunny, hot, ?, ?) → y = (sunny, hot) → p(y | yes) * p(yes)
2/9 * 2/9 * 9/15 = <u>0,0296</u>

1.5

In order to plot the standard deviation, change two lines:
From:

```
y_predict_bayesian = bayesian_reg.predict(X_poly)
```

To:

```
y_predict_bayesian, y_pred_bay_std = bayesian_reg.predict(X_poly, return_std=True)
```

And the plot function:
From:

```
plt.plot(X, y_predict_bayesian, color='navy', label='Bayesian regression fit')
```

To:

```
plt.errorbar(X, y_predict_bayesian, y_pred_bay_std, color='navy', label='Bayesian regression fit')
```