INF283
Exercise 1

Task 1

a) $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 + (-1) \cdot 3 & 2 \cdot 2 + (-1) \cdot 4 \\ 1 \cdot 1 + 3 \cdot 3 & 1 \cdot 2 + 3 \cdot 4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -1 & 0 \\ 10 & 14 \end{pmatrix}}}$

b) $A = \begin{pmatrix} -1 & 0 \\ 10 & 14 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$A^{-1} = \frac{1}{-1 \cdot 14 - (0 \cdot 10)} \begin{bmatrix} 14 & 0 \\ -10 & -1 \end{bmatrix}$

$= \frac{1}{-14} \begin{bmatrix} 14 & 0 \\ -10 & -1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 \\ \frac{10}{14} & \frac{1}{14} \end{bmatrix} = \underline{\underline{\begin{bmatrix} -1 & 0 \\ \frac{5}{7} & \frac{1}{14} \end{bmatrix}}}$

c) $\det(C - \lambda I) = 0, \quad C = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$

$\lambda I = \lambda \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$C - \lambda I = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & -2 \\ 1 & -1-\lambda \end{bmatrix}$

$\det \begin{bmatrix} 2-\lambda & -2 \\ 1 & -1-\lambda \end{bmatrix} \Rightarrow (2-\lambda)(-1-\lambda) - (-2 \cdot 1) = 0$

$-2 - 2\lambda + \lambda + \lambda^2 + 2 = 0$

$\lambda^2 - \lambda = 0 \Rightarrow \lambda(\lambda - 1) = 0 \Rightarrow \underline{\underline{\lambda = 0 \land \lambda = 1}}$

d) $(C - \lambda I) \cdot \vec{v} = \vec{0}$

$\lambda 1 = 0, \ \lambda 2 = 1$

$\underline{\lambda 1 = 0}$

$$(C - \lambda 1 \cdot I) = \begin{bmatrix} 2-0 & -2 \\ 1 & -1-0 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$

$$(C - \lambda 1 \cdot I) \cdot \vec{v} = \vec{0} \Rightarrow \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Row reduction

$$\left[\begin{array}{cc|c} 2 & -2 & 0 \\ 1 & -1 & 0 \end{array}\right] \rightarrow \begin{array}{c} \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & -1 & 0 \end{array}\right] \\ \begin{array}{cc} x_1 & x_2 \end{array} \end{array} \Rightarrow x_1 - x_2 = 0$$

$$\underline{x_1 = x_2}$$

$2R_2 - R_1 \to R_1$

Lar $x_1 = 1 \Rightarrow x_2 = 1$

Egenvektor for $C = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}}$, når $\lambda = 0$

$\underline{\lambda 2 = 1}$

$$(C - \lambda 2 I) = \begin{bmatrix} 2-1 & -2 \\ 1 & -1-1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$$

$$(C - \lambda 2 I) \cdot \vec{v} = \vec{0} \Rightarrow \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 1 & -2 & 0 \end{array}\right] \rightarrow \begin{array}{c} \begin{array}{cc} x_1 & x_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right] \end{array} \Rightarrow x_1 - 2x_2 = 0$$

$$\underline{x_1 = 2x_2}$$

$R_1 - R_2 \to R_2$

Lar $x_1 = 1 \Rightarrow x_2 = \frac{1}{2}$

Egenvektor for $C = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}}}$, når $\lambda = 1$

e) $f1(x,y) = x^2 + 2y^2 - xy$

$$\nabla f1(x,y) = \left[\frac{\partial f1}{\partial x}, \frac{\partial f1}{\partial y}\right] = \underline{\underline{[2x - y,\ 4y - x]}}$$

f) $\nabla f1(x,y) = 0$

$$\begin{bmatrix} 2x - y \\ 4y - x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \quad \text{I} \quad 2x - y = 0$$

$$y = 2x$$

$$\text{II} \quad 4(2x) - x = 0$$

$$7x = 0 \Rightarrow \underline{\underline{x = 0 \Rightarrow y = 0}}$$

h) $f1(x,y) = x^2 + 2y^2 - xy$

$g(x,y) = 2x + y - 22$

$L(x,y,\lambda) = x^2 + 2y^2 - xy - \lambda(2x + y - 22)$

I $\quad \frac{\partial L}{\partial x} = 0$

II $\quad \frac{\partial L}{\partial y} = 0$

III $\quad \frac{\partial L}{\partial \lambda} = 0$

I $\quad 2x - y - 2\lambda = 0$

II $\quad 4y - x - \lambda = 0$

III $\quad -2x - y + 22 = 0$

Assuming we can use tools to solve the system

$\underline{\underline{x = 9,\ y = 4,\ \lambda = 7}}$

g) $P(\text{Konge} \mid !\text{Ess}) = \frac{P(\text{Konge} \cap !\text{Ess})}{P(!\text{Ess})}$

Konge = B $\qquad = \frac{P(B)\,P(A \mid B)}{P(A)}$

!Ess = A

$$= \frac{\frac{4}{52} \cdot 1}{\frac{48}{52}} = \frac{4}{48} = \underline{\underline{\frac{1}{12}}}$$

h) Antar "mean" betyr "gjennomsnitt" i denne Konteksten

Mean $\Rightarrow \frac{1+2+3+4+5+6}{6} = \underline{\underline{3,5}} = \bar{X}$

$$Var(X) = \frac{\sum_{i=1}^{6} (X_i - \bar{X})^2}{6}$$

$$\frac{(1-3,5)^2 + (2-3,5)^2 + \dots + (6-3,5)^2}{6} = \frac{17,5}{6} = \underline{\underline{\frac{105}{36}}} = 2,917$$

Uniform Distribution: Alle verdier er like Sannsynlig.

$\Rightarrow$ for alle terningkast er det like Stor Sannsynlighet for hver mulige verdi.